MULTICRITERIA OPTIMIZATION IN A FUZZY ENVIRONMENT: THE FUZZY ANALYTIC HIERARCHY PROCESS[1]

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Abstract: In the paper the fuzzy extension of the Analytic Hierarchy Process (AHP) based on fuzzy numbers, and its application in solving a practical problem, are considered. The paper advocates the use of contradictory test to check the fuzzy user preferences during fuzzy AHP decision-making process. We also propose consistency check and deriving priorities from inconsistent fuzzy judgment matrices to be included in the process, in order to check if the fuzzy approach can be applied in the AHP for the problem considered. An aggregation of local priorities obtained at different levels into composite global priorities for the alternatives based on weighted-sum method is also discussed. The contradictory fuzzy judgment matrix is analyzed. Our theoretical consideration has been verified by an application of commercially available *Super Decisions* program (developed for solving multi-criteria optimization problems using AHP approach) on the problem previously treated in the literature. The obtained results are compared with those from the literature. The conclusions are given and the possibilities for further work in the field are pointed out.

Keywords: Fuzzy decision-making, fuzzy AHP, consistency check, military application.

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1. INTRODUCTION

Establishing criteria for decision-making is a difficult and responsible task. Today we almost always deal with multi-criteria optimisation, i.e. the decision making with respect to more than one criterion. The various mathematical methods have been developed for solving those problems. In those methods a decision maker (DM) has to define the structure preference for making a choice. The definition of the structure of preferences is a problem for itself within the multiple-criteria optimization.

Multi-criteria optimization, precisely multi-criteria decision-making (MCDM) is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints. It can be found in various fields, for example in an engineering design as well as in financial topics of an organization, precisely wherever optimal decisions need to be taken in the presence of trade-offs between the conflicting objectives. In each case we are looking for a solution that has optimized each objective in such a way that if we try to optimize the solution any further, then, as the result, the other objective(s) will suffer. When setting up and solving a multi-criteria optimization problem, the goal is to find a solution, and to quantify how much it is better than any other solution, [7].

One possible way to solve the problem of choosing alternatives is by using a multiple attribute decision making (MADM). A MADM is a branch in the decision engineering that operates in a discrete decision space – a space where there are a finite number of alternatives. Typically, a MADM aids the DM to evaluate and choose an alternative from a set of alternative with conflicting goals. Some examples of MADM models include the analytic hierarchy process (AHP), technique for order preference by similarity to ideal solution (TOPSIS), elimination and choice translating reality (ELECTRE), and preference ranking organization method for enrichment evaluation (PROMETHEE), [19].

Out of those methods, the AHP is one of the most popular, and has been applied widely in solving complex decision-making problems, [20]. The AHP was developed by *Saaty* [14] to solve complex decision making problems, which involves ranking and choosing of alternatives. In the AHP, the preferences of the DM are elicited in the form of ratios using a pair wise comparison matrix (PCM). The DM compares the elements in the PCM and assigns numerical values to them. A final aggregation of local weights is performed to rank and choose the best alternative. The first step in the decision making process using AHP is to break down a complex problem into a hierarchical structure. A typical AHP problem is comprised of several levels of hierarchy, the top most level consists of the goal or the objective of the decision problem while the bottom most part consists of alternatives that need to be chosen. In between there are several levels of hierarchy, which comprises of criteria, sub-criteria, sub-sub-criteria, and so on. The relative importance of the criteria could be approximated by introducing the significant factors in decision-making problems using pair wise comparisons. Those comparisons are done with respect to the higher level. Precisely, if there are two decision elements denoted by R_i and R_j respectively, then the *Saaty*'s method of approximating the weights is based on the scale of relative importance, for example the conventional scale, suggested in [14], although another scales exist (logarithmic, exponential, etc.). The main advantage of the AHP is its inherent ability to handle intangibles, which are predominant in any decision making process. Also, less cumbersome mathematical calculations and

comprehensibility makes the AHP to be an ideal technique that can be employed in the evaluation process.

However, the articles cited and a review of literature on the application of the AHP to the selection and evaluation problem reveal that most of them employ conventional or discrete AHP, which does not address the issue of uncertainty. There have been several questions raised about the theoretical validity, discrete numerical value [22] and rank reversal problems [3] in the AHP. One such criticism of the AHP is its inability to accommodate uncertainty in the decision making process. Critiques argue that it would be cognitively demanding to ask a DM to express preference as a discrete numerical value in the PCM. Interval based [1], fuzzy sets based [21], and probability based [15] approaches have been suggested to overcome the inability of AHP to handle uncertainty. A fuzzy AHP is an extension of the conventional AHP; it employs fuzzy sets theory to handle uncertainty [16] and is considered in this paper.

We advocate the use of contradictory test to check the fuzzy user preferences during fuzzy AHP decision-making process. In the paper the consistency check and deriving priorities from inconsistent fuzzy judgment matrices are studied. As a result, we propose both of them to be included in a decision-making process, in order to check if the fuzzy approach can be applied in the AHP for the problem considered. If the fuzzy matrix passes the contradictory test, the next decision-making step should be to check for inconsistencies and to estimate priorities from it.

In [17] and [11] some of the earliest research addresses to the issue of inconsistency in fuzzy pairwise comparison matrices (PCM) is described. They used mathematical programming models to check for inconsistency in the fuzzy PCM. Besides [17] and [11], the fuzzy preference programming (FPP) described in [12] and two-stage logarithmic programming (TLGP) described in [24], also based on mathematical programming models, are able to provide both consistency check and to derive priorities concurrently for a fuzzy PCM. The objective of any prioritization approach in the case of a fuzzy PCM is to derive a satisfactory weight vector such that it satisfies the initial fuzzy judgment expressed in the fuzzy PCM.

Solving practical problems using the fuzzy AHP was considered in the literature, usually with modification needed when the uncertain environment is considered. In [16], a direct approach to processing of uncertain data - linguistically expressed expert experience and qualitative multi-criteria optimization requirements – was considered using the fuzzy AHP: three tactical missile systems A, B and C were considered. The objective was to choose the best one, evaluating quantitative and qualitative data about their properties. The case considered was characterized by absence of precise and certain information, elements of decision matrix were given as grades (of membership) by which "an alternative fulfils a criterion".

Our theoretical consideration about the fuzzy AHP decision-making process has been verified in the process of solving this problem of a multi-criteria optimization of a tactical missile system described by quantitative and qualitative features from [16]. The optimization method with contradictory test to check the fuzzy user preferences during the fuzzy AHP decision-making process, with consistency check and with derivation of priorities from inconsistent fuzzy judgment matrices is applied to this problem, in the case when a fuzzy description of the system's properties is given.

In the paper, after this Introduction, the fuzzy AHP is described in details. Section 3 deals with aggregation of final priorities from the fuzzy PCM. The results of

application of commercially available *Super Decisions* program on the problem considered in the paper are given in sections 4 and 5. The obtained results are compared with the results from the literature. The conclusions are given and the possibilities for further work in the field are pointed out.

2. FUZZY ANALYTIC HIERARCHY PROCESS

As it is known from the literature on fuzzy systems, for example [27], [8], [9], a kind of fuzzy numbers (fuzzy sets of real numbers as universes of discourse) can be expressed as triangular fuzzy numbers $\tilde{a}_{ij}$; which is a triplet denoted by $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$. An example of a triangular fuzzy number is shown in Figure 1, [9].
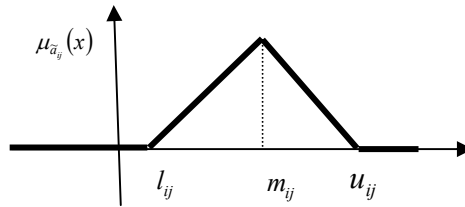
Figure 1: A triangular fuzzy number

Throughout this paper, triangular fuzzy numbers are used unless otherwise specified. The membership function of the triangular fuzzy number in Figure 1 is defined as follows:

$$\mu_{\tilde{a}_{ij}}(x) = \begin{cases} 0 & , x < l_{ij} \\ \dfrac{x - l_{ij}}{m_{ij} - l_{ij}}, & l_{ij} \le x \le m_{ij} \\ \dfrac{u_{ij} - x}{u_{ij} - m_{ij}}, & m_{ij} \le x \le u_{ij} \\ 0 & , x > u_{ij} \end{cases} \quad . \tag{1}$$

Since fuzzy sets theory was initially introduced in [26], and subsequently described the decision-making methods in fuzzy environments in [27] and [2], an increasing number of studies have dealt by uncertain fuzzy problems of applying fuzzy sets theory. Similarly, this study includes fuzzy decision making theory, considering the possible fuzzy subjective judgment during the evaluation process.

According to the known characteristics of triangular fuzzy numbers (TFN), [8], and the extension principle put forward in [27], the operational laws of two triangular fuzzy numbers $\widetilde{A} = (a_l, a_m, a_u)$ and $\widetilde{B} = (b_l, b_m, b_u)$ are as follows:

(1) Addition of two fuzzy numbers

$$(a_l, a_m, a_u) \oplus (b_l, b_m, b_u) = (a_l + b_l, a_m + b_m, a_u + b_u),$$ (2)

(2) Subtraction of two fuzzy numbers

$$(a_l, a_m, a_u) \ominus (b_l, b_m, b_u) = (a_l - b_u, a_m - b_m, a_u - b_l),$$ (3)

(3) Multiplication of two fuzzy numbers

$$(a_l, a_m, a_u) \otimes (b_l, b_m, b_u) = (a_l \cdot b_l, a_m \cdot b_m, a_u \cdot b_u),$$ (4)

(4) Multiplication of any real number k and a fuzzy number

$$k \otimes (a_l, a_m, a_u) = (k \cdot a_l, k \cdot a_m, k \cdot a_u),$$ (5)

(5) Division of two fuzzy numbers

$$(a_l, a_m, a_u) \oslash (b_l, b_m, b_u) = \left(\frac{a_l}{b_u}, \frac{a_m}{b_m}, \frac{a_u}{b_l} \right),$$ (6)

and

(6) Inverse of triangular fuzzy number

$$(a_l, a_m, a_u)^{-1} = \left(\frac{1}{a_u}, \frac{1}{a_m}, \frac{1}{a_l} \right)$$ (7)

An AHP approach based on fuzzy sets theory should be employed when we want to capture subjective preferences of the DM and handle uncertainty, [7]. Most of the basic steps involved in the fuzzy AHP are similar to those in the discrete AHP. However, the use of fuzzy numbers instead of discrete numbers and the process of extracting priorities from the PCM differentiate the fuzzy AHP from the discrete AHP.

Five key decision elements or terminologies common to both the discrete AHP and the fuzzy AHP are as follows, [4]:

1. Alternatives: available choice of alternatives for the DM;
2. Criteria/attributes: aspects on which alternatives are assessed;
3. Evaluations: assessment of the performance of alternatives based on the criteria;
4. Weights: assessment of relative importance of the criteria;
5. Aggregation method: algorithm for synthesis of above information.

The first step in the decision making process involves the construction of reciprocal pairwise comparison matrix **R** to assess the performance of alternatives and the importance of criteria. The DM expresses preferences in terms of (real or fuzzy) numerical values by comparing the decision element at a particular level in the hierarchy. This comparison is done with respect to the higher level. If there are two decision

elements R_i and R_j, then a reciprocal PCM is constructed based on the following two criteria:

1. If R_i is r_{ij} times preferred by R_j, then R_j is $\dfrac{1}{r_{ij}}$ preferred by R_i, and

2. if an element is compared with itself, then a value of 1 is assigned.

Table 1: Conventional and fuzzy scale for evaluating preferences of R_i over R_j

$S \;\; = \;\; \left\{ \dfrac{1}{9}, \dfrac{1}{8}, \dfrac{1}{7}, \dfrac{1}{6}, \dfrac{1}{5}, \dfrac{1}{4}, \dfrac{1}{3}, \dfrac{1}{2}, 1, 2, 3, 4, 5, 6, 7, 8, 9 \right\}$		
Conventional scale	Preference of R_i over R_j	Fuzzy scale
1	Equality preferred	$\widetilde{1} = (1-\delta, 1, 1+\delta)$
3	Weakly preferred	$\widetilde{3} = (3-\delta, 3, 3+\delta)$
5	Strongly preferred	$\widetilde{5} = (5-\delta, 5, 5+\delta)$
7	Very strongly preferred	$\widetilde{7} = (7-\delta, 7, 7+\delta)$
9	Absolutely preferred	$\widetilde{9} = (9-\delta, 9, 9+\delta)$
2,4,6,8	Intermediate values	$\widetilde{2}, \widetilde{4}, \widetilde{6}, \widetilde{8}$

In the case of conventional AHP the number r_{ij} is an element from the set S (defined in Table 1.), comparison matrix **R** is positive, and elements from "upper triangular sub-matrix" of the matrix **R** are reciprocal to those from its "lower triangular sub-matrix". If the judgment is perfect in all comparisons, then $r_{ij} r_{jk} = r_{ik}$ for all i, j, k and we call the matrix **R** consistent. If we denote the priority vector by $w = (w_1 \quad w_2 \quad w_3 \quad ... \quad w_n)^T$, then the next formula holds:

$$\mathbf{R} = \begin{bmatrix} 1 & r_{12} & . & . & . & r_{1n} \\ \dfrac{1}{r_{12}} & 1 & . & . & . & r_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \dfrac{1}{r_{1n}} & \dfrac{1}{r_{2n}} & . & . & . & 1 \end{bmatrix} = \begin{bmatrix} \dfrac{w_1}{w_1} & \dfrac{w_1}{w_2} & . & . & . & \dfrac{w_1}{w_n} \\ \dfrac{w_2}{w_1} & \dfrac{w_2}{w_2} & . & . & . & \dfrac{w_2}{w_n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \dfrac{w_n}{w_1} & \dfrac{w_n}{w_2} & . & . & . & \dfrac{w_n}{w_n} \end{bmatrix}. \tag{8}$$

The following equation could be constructed:

$$\mathbf{R}w = nw. \tag{9}$$

A solution of this equation is the right eigenvector of the matrix **R**, which consists of positive elements of the matrix **R** and is unique, disregarding possible multiplicative constants. An additive normalization could be done in order to get a unification of the eigenvector. If all the elements of the matrix **R** are known, the evaluation system can be established, and the solution of (9) is the normalized version of any column of matrix **R**. In practice, the matrix **R** is very often inconsistent. It means that the expert evaluations are given with a small account error. The appropriate eigenvalue λ_{max} is not n, furthermore it holds that $\lambda_{max} \geq n$ (equality holds in case of consistency). Deviation of expert evaluation for consistent approximation is expressed by $\lambda_{max} \geq n$. For a measure of inconsistency, *Saaty* suggests the consistency index, denoted by *CI*, defined as follows:

$$CI = \frac{\lambda_{max} - n}{n-1} \quad ,$$

$$(10)$$

where λ_{max} is *the maximum eigenvalue* and n is the size of the PCM. *Saaty* compares this value with the random consistency index, denoted by *RI*, obtained as an average *CI* of a large number of randomly generated reciprocal matrixes of the same order (Table 2.). The calculated vector w is accepted if the ratio *CI/RI* is less than or equal to 0.1, otherwise the preferences are considered not to be consistent enough to serve as a basis for decision- making.

Table 2: Random consistency index for matrices of size n

n	1	2	3	4	5	6	7	8	9	10	11	12
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48

In the fuzzy AHP, comparison of R_i with R_j is presented by a fuzzy number $\widetilde{r}_{ij}$. According to *Zadeh* [27], it is very difficult for conventional quantification to express reasonably those situations that are overtly complex or hard to define; thus the notion of a linguistic variable is necessary in such situations. A linguistic variable is a variable whose values are words or sentences in a natural or artificial language. We compare two evaluation criteria by linguistic variables in a fuzzy environment: if the criterion is "absolutely preferred" we are going to express this fact by a linguistic variable "high" or a fuzzy number $\widetilde{9}$, "very strongly preferred" criterion we will represent by a linguistic variable "high" or a fuzzy number $\widetilde{7}$, "strongly preferred" by a linguistic variable "fair" or a fuzzy number $\widetilde{5}$, "weakly preferred" by a linguistic variable "low" or a fuzzy number $\widetilde{3}$, and "equally preferred" by a linguistic variable "very low" or a fuzzy number $\widetilde{1}$, having in mind fuzzy five levels scale (see Table 1.). Furthermore, linguistic variables are used as a way to measure performance values of alternatives for each criterion as "very low", "low", "fair", "high", and "very high". In this paper, we employ triangular fuzzy numbers to express the fuzzy scale given by linguistic variables, as in Table 1.

If there are n $(i,j=1,2,...,n)$ decision elements, a DM need to provide $\frac{n(n-1)}{2}$ comparisons. The positive fuzzy reciprocal PCM is represented by its elements $\widetilde{r}_{ij} = (l_{ij}, m_{ij}, u_{ij})$, $\frac{1}{\widetilde{r}_{ij}} = \left(\frac{1}{u_{ij}}, \frac{1}{m_{ij}}, \frac{1}{l_{ij}} \right)$ (equation (7)) and $\widetilde{r}_{ii} = (1,1,1)$. Table 1 presents the conventional and fuzzy scales used in AHP. In the case of fuzzy AHP, some researchers use the value of δ equal to 0.25, 0.5, and 0.75, [7], [13], and [28].

It is shown in [22] that the liberal use of scales instead of the discrete scale may considerably reduce the failure rates in the AHP. Hence, the DM can be given the liberty of choosing his/her own fuzzy scale and is not confined to the scales presented in Table 1. We cannot assume that each evaluation criterion is of equal importance because the evaluation criteria have diverse meanings. Many methods then can be employed to determine weights such as the eigenvector method, weighted least square method, entropy method and the AHP, as well as linear programming techniques for multi-dimensions of analysis preference (LINMAP). The selection of method depends on the nature of the problems. Our choice is to use the fuzzy eigenvector method to determine the criteria weights.

A matrix $\mathbf{R} = (\widetilde{r}_{ij})_{n\times n}$ is said to be perfectly consistent if the following condition holds, [5]:

$$\widetilde{r}_{ik} \otimes \widetilde{r}_{kj} = \widetilde{r}_{ij} \,, \forall i,j,k = 1,2,...,n \;. \tag{10}$$

In a practical situation, it is highly unlikely to expect that the DM will provide a perfectly consistent PCM. Hence, there should be some deviations allowed to accept a PCM.

Accordingly to definition of consistency in the case of conventional AHP and definition of a fuzzy number and multiplication of two fuzzy numbers, we can consider (10) as the following:

$$(l_{ik}, m_{ik}, u_{ik})(l_{kj}, m_{kj}, u_{kj}) = (l_{ij}, m_{ij}, u_{ij}) \qquad \forall i,j,k = 1,2,...,n \;; \tag{11}$$

$$(l_{ik}l_{kj}, m_{ik}m_{kj}, u_{ik}u_{kj}) = (l_{ij}, m_{ij}, u_{ij}) \qquad \forall i,j,k = 1,2,...,n \;. \tag{12}$$

So, checking the consistency in the fuzzy case could be considered as a checking triplet consistency, triplet consisting of three types of values: lower, middle and upper values.

We can state that the fuzzy matrix is consistent if and only if all three types of consistency are satisfied. Precisely, the fuzzy matrix is consistent if and only if the next formulas hold:

$$l_{ik}l_{kj} = l_{ij}, m_{ik}m_{kj} = m_{ij}, \; u_{ik}u_{kj} = u_{ij} \quad \forall i,j,k = 1,2,...,n \;. \tag{13}$$

So, the fuzzy matrix $\mathbf{R} = (\widetilde{r}_{ij})_{n\times n}$ is consistent if and only if three crisp matrices $\mathbf{L} = (l_{ij}) \in R^{n\times n}, \mathbf{M} = (m_{ij}) \in R^{n\times n}, \; \mathbf{U} = (u_{ij}) \in R^{n\times n}, \; \forall i,j,k = 1,2,..., n$ are consistency matrices.

Once the PCM passes the consistency check, the next step is to derive priority vector $w = (w_1, w_2, ..., w_n)^T$ from the PCM such that it satisfies the initial judgment $\tilde{r}_{ij}$. One of the most common methods employed in the conventional AHP is the right eigenvector based method. In the case of the fuzzy PCM, there are two distinct ways to obtain the priorities. The first method obtains crisp priority vector w similar to the conventional AHP, and the second approach obtains the fuzzy priority vector $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, ..., \tilde{w}_n)^T$.

3. AGGREGATION OF FINAL PRIORITIES

The final step involves aggregation of local priorities obtained at different levels into composite global priorities for the alternatives based on weighted-sum method. If there are i alternatives and j criteria then the final global priority of alternative is given as:

$$R_i = \sum_{j=1}^{n} w_j r_{ij} \tag{15}$$

where w_j is the weight of criterion j, and r_{ij} is the evaluation of alternative R_i against criterion j. Higher value of R_i means that the alternative is more preferred.

The above equation can be applied for the fuzzy AHP provided the priorities are crisp. However, if the priorities are fuzzy, then the appropriate aggregation will yield a fuzzy number and ranking procedures have to be used to defuzzify and rank the alternatives.

Once the preference of the DM is elicited, the next important step in a decision-making process is to check for any contradiction present in the PCM.

Suppose $r_{ij} = \tilde{r}_{ij} = (l_{ij}, m_{ij}, u_{ij})$. If $l_{ij} \geq 1$ then we say that the r_{ij} is more than one, and if $u_{ij} \leq 1$ then r_{ij} is less than one. The issue of contradiction in a PCM was addressed in [10] where it is defined: a PCM **R** is going to be contradictory if and only if for $i, j, k = 1, 2, ..., n$ any of the following detailed cases holds:

$$
\begin{aligned}
&1. r_{ij} > 1 \wedge r_{ik} < 1 \wedge r_{kj} > 1, \\
&2. r_{ij} < 1 \wedge r_{ik} > 1 \wedge r_{kj} < 1, \\
&3. r_{ij} = 1 \wedge r_{ik} > 1 \wedge r_{kj} < 1, \\
&4. r_{ij} = 1 \wedge r_{ik} < 1 \wedge r_{kj} > 1, \\
&5. r_{ij} = 1 \wedge r_{ik} = 1 \wedge r_{kj} < 1, \\
&6. r_{ij} = 1 \wedge r_{ik} = 1 \wedge r_{kj} > 1.
\end{aligned} \tag{16}
$$

It is shown, in [10], by numerical example, that even if the PCM has acceptable consistency it may still be contradictory. If a matrix is contradictory then it is impossible to derive weights, such that it satisfies all opinions expressed in the PCM.

Hence, it is imperative to remove any such contradictory matrices from the decision making process. They also formulate the following algorithm to check the presence of any contradiction in the PCM.

```
FOR (i,j,k =1 AND i ≠ j ≠ k) TO n DO
IF  log r_ij log r_ik ≤ 0 AND log r_ik log r_jk ≤ 0 THAN STOP - CONTRADICTORY
ELSE
IF  r_ij = 0 AND r_ik = 0 AND r_jk ≠ 0 THAN STOP - CONTRADICTORY
ELSE OK
```

Once the fuzzy matrix passes the contradictory test, the next step is to check for inconsistencies and to estimate priorities from it. The objective of any prioritization approach in the case of a fuzzy PCM is to derive a satisfactory weight vector such that it satisfies the initial fuzzy judgment $\tilde{r}_{ij} = (l_{ij}, m_{ij}, u_{ij})$ expressed in the fuzzy PCM.

4. ON USING SUPER DECISIONS PROGRAM

The commercial software package, *Super Decisions* program is developed for solving multi-criteria optimization problems using the AHP approach. In order to verify our theoretical analysis, we have used *Super Decisions* program to solve the following problem: three tactical missile systems A, B, and C are given. The objective is to choose the best system, in an evaluation procedure based on quantitative and qualitative data of given systems. Data are tactical specifications of systems, and qualitative part of data represents declaratively expressed expert's opinions about characteristics of three systems.

Characteristics of three considered systems are given by tables 3 and 4. Those characteristics are interpreted as criteria C_i, i =1,2, ... , 23 (see Table 5.) and we have three alternatives – three considered systems: system A, system B and system C. The feasibility of applying the AHP to the problem, using the fuzzy scale defined in Table 1, is considered.

Table 3: The tactical data specification

Characteristics	Unit of measurement	System			Criterion
		A	B	C	
Range	*km*	43	36	38	C_1
Flight height	*m*	25	20	23	C_2
Flight velocity	*M, Mach number*	0.72	0.8	0.75	C_3
Reliability	%	80	83	76	C_4
Firing accuracy	%	67	70	63	C_5
Destruction rate	%	84	88	86	C_6
Kill radius	m	15	12	18	C_7
Missile dimensions	cm	521 × 35-135	381 × 34-105	445 × 35-120	C_8
Reaction time	min	1.2	1.5	1.3	C_9
Fire rate	round/ min	0.6	0.6	0.7	C_{10}
Anti-jam	%	68	75	70	C_{11}
System cost	10 000/ unit	800	755	785	C_{12}
System life	years	7	5	5	C_{13}

Table 4: Expert's opinion about system characteristics

Characteristics	System A	System B	System C	Criterion
Safety	Good	Standard	Standard	C_{14}
Defilade	Standard	Good	Standard	C_{15}
Simplicity	Standard	Standard	Standard	C_{16}
Assembility	Standard	Standard	Poor	C_{17}
Combat capability	Good	Standard	Standard	C_{18}
Operation condition requirements	High	Standard	Standard	C_{19}
Material limitations	High	Standard	High	C_{20}
Modulization	Standard	Good	Standard	C_{21}
Mobility	Poor	Good	Standard	C_{22}
Standardization	Standard	Standard	Standard	C_{23}

The model deals by linguistic variables, denoted by a_{ij}, such that $a_j \in \{\tilde{1},..,\tilde{9}\}$.

In the problem, for each alternative, according to the considered criterion, a fuzzy preference is given as a fuzzy number. That fuzzy number provides us opportunity to express a score, i.e. a degree by which an alternative fulfils the criterion.

Table 5 The real decision matrix

Criterion	Weight	System A	System B	System C
C_1	$\tilde{7}$	$\tilde{7}$	$\tilde{3}$	$\tilde{5}$
C_2	$\tilde{1}$	$\tilde{6}$	$\tilde{2}$	$\tilde{4}$
C_3	$\tilde{9}$	$\tilde{6}$	$\tilde{8}$	$\tilde{7}$
C_4	$\tilde{8}$	$\tilde{6}$	$\tilde{7}$	$\tilde{4}$
C_5	$\tilde{9}$	$\tilde{2}$	$\tilde{4}$	$\tilde{1}$
C_6	$\tilde{7}$	$\tilde{5}$	$\tilde{7}$	$\tilde{6}$
C_7	$\tilde{6}$	$\tilde{6}$	$\tilde{3}$	$\tilde{9}$
C_8	$\tilde{4}$	$\tilde{6}$	$\tilde{4}$	$\tilde{5}$
C_9	$\tilde{9}$	$\tilde{5}$	$\tilde{2}$	$\tilde{4}$
C_{10}	$\tilde{9}$	$\tilde{4}$	$\tilde{4}$	$\tilde{3}$
C_{11}	$\tilde{5}$	$\tilde{5}$	$\tilde{8}$	$\tilde{6}$
C_{12}	$\tilde{7}$	$\tilde{4}$	$\tilde{1}$	$\tilde{3}$

Criterion	Weight	System A	System B	System C
C_{13}	$\tilde{8}$	$\tilde{7}$	$\tilde{5}$	$\tilde{5}$
C_{14}	$\tilde{6}$	$\tilde{7}$	$\tilde{5}$	$\tilde{5}$
C_{15}	$\tilde{2}$	$\tilde{5}$	$\tilde{7}$	$\tilde{5}$
C_{16}	$\tilde{3}$	$\tilde{5}$	$\tilde{5}$	$\tilde{5}$
C_{17}	$\tilde{3}$	$\tilde{5}$	$\tilde{5}$	$\tilde{1}$
C_{18}	$\tilde{9}$	$\tilde{7}$	$\tilde{5}$	$\tilde{5}$
C_{19}	$\tilde{5}$	$\tilde{1}$	$\tilde{5}$	$\tilde{5}$
C_{20}	$\tilde{5}$	$\tilde{1}$	$\tilde{5}$	$\tilde{1}$
C_{21}	$\tilde{5}$	$\tilde{5}$	$\tilde{7}$	$\tilde{5}$
C_{22}	$\tilde{7}$	$\tilde{1}$	$\tilde{7}$	$\tilde{5}$
C_{23}	$\tilde{3}$	$\tilde{5}$	$\tilde{5}$	$\tilde{7}$

Descriptively expressed expert's opinions about the characteristics of the considered systems, based on experience are given in Table 5. More precisely, the DM assigns the fuzzy scores to the specification data and characteristics of the considered systems with the respect to the criteria given in tables 3 and 4. Those scores are given in Table 5. The weights (fuzzy numbers, as well) for criteria are also assigned, by the DM, according to his/her attitude about the importance of the criteria, see Table 5. The data in Table 5 are similar to those in [16]. Here they are modified so that the alternatives are evaluated in the range $[\tilde{1}, ... , \tilde{9}]$, rather than $[\tilde{1}, … , \tilde{3}]$ as in [16], i.e. we have changed those data to make conditions to apply AHP.

5. RESULTS

Described data are inputs for the program, *Super Decisions*, and the basic hierarchical structure has been made.

First, expert's opinions on systems characteristics and tactical specifications are ordered according to the importance, Table 6.

Table 6. Expert's opinions and tactical specifications

	Expert's opinions	Tactical specifications
Expert's opinions	1.00000	2.00000
Tactical specifications	0.50000	1.00000

After that, mutual comparisons of sub criteria inside two basic criteria are given, as well as comparisons of alternatives according to given sub criteria C_1 to C_{23} (not given here because the tables are very large and unsuitable for presentation). For data from tables 3, and 4, expressed as fuzzy preferences (fuzzy numbers) and given by Table 5, conditions of applying AHP are fulfilled. The program *Super Decisions* is used, and the rank of alternatives is obtained by right eigenvector method, presented by Table 7. The colon *Total* gives numeric values of eigenvectors of preferences entered, while the colon *Normal* presents the normalized values of those eigenvectors, and the colon *Ideal* presents the percents of the fulfilled best solution. Those colons represent selection based on preferences of the DM. She/he made decisions based on comparisons of pairs of elements.

Table 7. Rank of alternatives

Alternatives	Total	Normal	Ideal	Ranking
system A	0.1159	0.3477	0.9717	2
system B	0.1193	0.3578	1.0000	1
system C	0.0982	0.2945	0.8231	3

This result intuitively is expected by the person that makes comparisons, but it is now the numerical result, that is, the objective result obtained on the base of given (subjective) preferences. The missile system B is the best choice, and that is in accordance with result in [16], no matter the scale (that is data) here has been changed. The order of systems has also been preserved, (B, A, C). But, in [16] the system B is distinctively the best, and the evaluation values for systems A and C are close. Table 5 differs from the similar table in [16] because the alternatives in Table 5 are evaluated in the range $[\tilde{1}, ... , \tilde{9}]$, rather than $[\tilde{1}, ... , \tilde{3}]$, as in [16]. For the scale in the range $[\tilde{1}, ... , \tilde{9}]$, the second choice, A, is relatively closer to the choice, B, than it is in [16].

6. CONCLUSION

In this paper a kind of fuzzy AHP approach is considered. In previously considered fuzzy AHP approaches, different methods were applied. For example, in [16] the decision-making process was based on the weighted aggregation of numerical and linguistic data: each specification or characteristic of a system was treated as criterion, so that the each specification or characteristic had its relative importance, assigned by the DM. The priority among the alternatives was derived on the base of fuzzy arithmetic for fuzzy triangular numbers and through a kind of a defuzzification. There was no need for complicated entropy weight calculations as in some other approaches, so the computer implementation and calculations are less complicated and the approach is more suitable for application. The influence of some specification or characteristic is not masked by grouping it with others under one criterion.

In our approach presented in this paper, the fuzzy eigenvector method is used to determine the criteria weights. The fuzzy extension of the AHP, based on fuzzy numbers, is considered, similarly as in [16]. In [16] priorities and criteria weights have been used mainly on intuitive base. The approach considered in our paper is more formal and more general, according to our best knowledge, than previous fuzzy AHP approaches, because our approach offers the possibility of checking the fulfillment of conditions for applying the fuzzy AHP. The consistency check is also used for derivation of priorities from fuzzy pairwise comparison matrices (PCM). This allows the existing software tools, like *Super Decisions* program, to be used in the AHP context, even in the case of existing uncertainty. Constrains (given in [16]) about the level of importance of criteria in the aggregation process are only uncertain variant of usual (crisp) AHP constraints.

We have included the contradictory test in decision-making process to check the fuzzy user preferences during fuzzy AHP decision-making process. If the fuzzy matrix passes the contradictory test, the next decision-making step should be to check for inconsistencies and to estimate priorities from it.

In the case when the conditions are fulfilled, it is possible to apply the fuzzy AHP, and the possibility of solving a multi-criteria optimization problem described by both quantitative and qualitative parameters exists. It seems that calculations and the software implementation in [16] are simpler compared with those in this paper, but that fact is not crucial when commercial software packages, like *Super Decisions,* exist.

It might be of interest to investigate further generalization of the applied selection procedure having in mind some diverse aggregation operators, and in due course the possibility of treating the considered problem in the context of soft computing.

REFERENCES

[1]   Arbel, A., and Vargas, L., "The analytic hierarchy process with interval judgements", in: Goicoechea A. et al. (eds.). *Multiple criteria decision making.* Springer, Berlin Heidelberg New York, 1992.

[2]   Bellman, R.E., and Zadeh, L.A., "Decision-making in a fuzzy environment", *Management Science,* 17 (1970) 141–164.

[3]   Belton, V., and Gear, T., "On a short-coming of Saaty's method of analytic hierarchies", *Omega,* 11 (1983) 228−230.

[4]   Belton, V., and Pictet, J., "A framework for group decision using a MCDA model: Sharing, aggregating or comparing individual information", *J Dec Sys,* 6 (1997) 283−303.

[5]   Buckley, J.J., "Ranking alternatives using fuzzy numbers", *Fuzzy Sets and Systems,* 15 (1985) 1–31.

[6]   Byun, B. S., and Lee, K.H., "A decision support system for the selection of a rapid prototyping process using the modified TOPSIS method", *Int J Adv Manf Technol*, 26 (2004) 1338−1347.

[7]   Deng, H., "Multicriteria analysis with fuzzy pairwise comparison", *Int. J. Approx. Reas.,* 21 (1999) 215−231.

[8]   Dubois, D., and Prade, H., "Operations on fuzzy numbers", *International Journal of Systems Science* 9 (1978) 613–626.

[9]   Kaufmann, A., and Gupta, M.M., *Introduction to fuzzy arithmetic: Theory and application*, Van Nostrand, New York, 1991.

[10] Kwiesielewicz, M, and Van Uden, E., "Inconsistent and contradictory judgments in pairwise comparison method in the AHP", *Comp. Oper. Res.,* 31 (2004) 713−719.

[11] Leung, L.C., and Cao, D., "On consistency and ranking of alternatives in fuzzy AHP", *European Journal of Opeartions Research,* 124 (2003) 102−113.

[12] Mikhailov, L., "Deriving priorities from fuzzy pairwise comparison judgments", *Fuzzy Sets Syst.,* 134 (2003) 365−385

[13] Prabhu, R.T., and Vizayakumar, K., "Technology choice using FHDM: A case of iron-making technology", *IEEE Trans. Eng. Mgm.t,* 48 (2001) 209−222.

[14] Saaty, T.L., *The Analytic Hierarchy Process.* McGraw-Hill, (1980)

[15] Saaty, T.L., Vargas, L., "Uncertainity and rank order in the analytic hierarchy process", *European Journal of Operations Research,* 32 (1987) 107−117.

[16] Šaletic, D.Z., and Velaševic, D.M., "Missile system selection based on the fuzzy sets Theory", *Yugoslav Journal of Operations Research,* 10 (1) (2000) 47-61.

[17] Salo, A.A., "On fuzzy ratio comparisons in hierarchical models", *Fuz Sets Syst,* 84 (1996) 21−32.

[18] Sambasivarao, K.V., and Deshmukh, S.G., "A decision support systems for selection and justification of advanced manufacturing", *Production Planning & Control*, 8 (1997) 270-284.

[19] Triantaphyllou, E., *Multi-criteria Decision Making Methods: A Comparitive Study,* Kluwer, Drodrecht, 2000.

[20] Vaidya, O.S., Kumar, S., "Analytic hierarchy process: An overview of application", *Euoropean Journal of Operations Research,* (2004) 4-28.

[21] Van Laarhoven, P., and Pedrycz, W., "Fuzzy extension of Saaty's priority theory", *Fuz Sets Syst,* 11 (1983) 229−241.

[22] Vinod, N., Ganesh, L.S., "An empirical analysis of the use of the analytic hierarchy process for estimating membership values in a fuzzy set", *Fuz Sets Syst,* 82 (1996) 1−16.

[23] Wabalickis, R.N., "Justification of FMS with the analytic hierarchy process", *Manf. Sys,* 17, (1988) 175−182 4.

[24] Wang, Y.M., Yang J.B., and Xu, D.L., "A two-stage logarithmic goal programming method for generating weights from interval comparison matrices", *Fuz Sets Syst,* 152 (2005) 475−498.

[25] Zahedi, F., "The analytic hierarchy process − A survey of the method and its applications", *Interfaces,* 16 (1986) 96−108.

[26] Zadeh, L.A., "Fuzzy Sets", *Information and Control,* 8 (1965) 338–353.

[27] Zadeh L.A., "The concept of a linguistic variable and its application to approximate reasoning", Parts 1–3. *Information Sciences* 8 (1975) 199–249, 301–357, 9 (1975), 43–80.

[28] Zhu, K.J., Jing, Y., and Chang, D.Y., "A discussion on extent analysis method and applications of fuzzy AHP", *European Journal of Oprerations Research,* 116 (1999) 450−456.