

COMPROMISE IN COOPERATIVE GAME AND THE VIKOR METHOD

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Abstract: Five approaches in conflict resolution are distinguished, based on cooperativeness and aggressiveness in resolving conflict. Compromise based on cooperativeness is emphasized here as a solution in conflict resolution. Cooperative game theory oriented towards aiding the conflict resolution is considered and the compromise value for TU (transferable utility)-game is presented. The method VIKOR could be applied to determine compromise solution of a multicriteria decision making problem with noncommensurable and conflicting criteria. Compromise is considered as an intermediate state between conflicting objectives or criteria reached by mutual concession. The applicability of the cooperative game theory and the VIKOR method for conflict resolution is illustrated.

Keywords: Multicriteria decision making, cooperative game, compromise, VIKOR method.

1. INTRODUCTION

Conflict is often considered as disagreement, opposition, or struggle between two or more people or groups, and an antagonistic interaction in which one party attempts to thwart the intentions or goals of another, or process in which people disagree over significant issues, thereby creating friction between parties, or a situation when people oppose views trying to prevent each other from accomplishing goals [4]. Conflict exists when there is goals conflict, or goal is not within capabilities. In this paper, values and needs based conflict is considered, as a situation different than violence. Conflict over the management of a shared resource arises because of differing objectives among different interest groups. Beside many disadvantages of a conflict situation, a conflict can

be good if based on issues, enhances constructive problem solving and creativity, and challenging complacency, status quo and stagnation.

There exist normative methods oriented towards aiding the conflict resolution by identifying and evaluating alternative strategies and solutions. This approach requires that various desirable goals be specified, and the normative method explores ways of reaching these goals through alternative paths and decision points. A normative approach in conflict resolution indicate ways that parties should deal with conflict, not how they actually do or will deal with it. The gap between normative and explicative (or predictive) decision theories has generated an immense literature [1], [8], [10], [14]. Bell et al. [1] proposed the prescriptive approach to bring the normative and explicative together. According to this approach, prescriptive decision makers consult rational, normative models for optimizing the interests of parties in a conflict; at the same time, they are willing to bend decisions around imperfect information or emotional factors. Hence normative methods can provide parties in a conflict with prescriptive frameworks to guide behavior so that courses of action, consequences, and risks and benefits become less uncertain. Among these methods, those that seem to predominate include multicriteria decision making (MCDM), and game theory.

Game theory is the study of the ways in which strategic interactions among rational players produce outcomes with respect to the preferences (or utilities) of those players. A game is defined by: players with conflict interests, preferences (benefit, cost) as a result of the game, and a set of strategies (alternatives, moves). Theorists define broad categories of games across a spectrum from pure competition to pure cooperation. A crucial aspect of the specification of a game involves the information that players have when they choose strategies. And a specific question in game theory is: should a player be rational and to cooperate in order to provide maximum mutual benefit or to be aggressive. Cooperative game theory (Pareto Game) assumes that each player is a member of a team willing to compromise his own objective to improve the solution as a whole [5]. In the cooperative solution, the team would reallocate the resources with the intent that all the players should be as optimal as possible, in other words, a Pareto optimal solution. It should be emphasized that cooperativeness is necessary for compromising.

Conflict resolution is considered here as multicriteria decision making problem. The criteria usually conflict with each other and there may be no solution maximizing all criteria simultaneously. Thus, the concept of Pareto optimality was introduced for a vector optimization problem [17], [23]. Pareto optimal (noninferior) solutions have the characteristic that, if one criterion is to be improved, at least one other criterion has to be made worse. In engineering and management practice there is a need to select a final solution to be implemented. An approach to determine a final solution as a compromise was introduced by Yu [21]. The method VIKOR has been developed to solve a discrete optimization problem with noncommensurable and conflicting criteria [15], [16].

Five approaches in conflict resolution are considered in Section 2. Cooperative game theory oriented towards aiding the conflict resolution is considered and the compromise value for TU (transferable utility)-game is presented in Section 3. Compromising by the VIKOR method is considered in Section 4, introducing VIKOR as a normative method for conflict resolution by compromising. The algorithmic steps of the VIKOR method are presented in Appendix.

2. CONFLICT RESOLUTION APPROACHES

In this section, values and needs based conflict is considered, as a situation different than violence. Five approaches to conflict resolution could be distinguished, based on cooperativeness and aggressiveness in resolving conflict.

Competition. Aggressive and noncooperative. Priority to your own goals. There is an intractable conflict, power-oriented, and it could arise to “war”. All means (force) are used appropriate to win (assuming others will lose). Power may take the form of a majority or of a persuasive minority. A final solution is imposed. Descriptive and “political” methods may be used to “justify” the solution. Competition may be used, as a conflict resolution approach, when there is an imbalance in power of decision makers and a decisive action is vital (emergencies) on important issues. The power must exist to impose solution. However, it is not proposed when issues are complex and others long-term solutions and commitment are needed.

Engagement (Collaborating). Cooperative but aggressive. Working together to achieve all parties objectives; by open and honest discussions; bargaining; negotiation. There is a will to identify disagreements and interests of major decision makers (players) in order to reach final solution, but there exists a polarization of conflicting relationships. Finally, a temporary solution could be reached. The MCDM methods may be used, such as generating noninferior solutions for negotiation. Engagement as a collaborating approach could be used when decision makers are interest to work out a solution and they have skills to reach a solution in a complex learning situation.

Compromise. Fair aggressive and cooperative. There is a will to reach a mutually acceptable solution in which each person gets part of what they want. A compromise is an agreement established by mutual concessions. A final solution could be sustainable. The MCDM methods may be used to determine compromise solutions, such as VIKOR method. Compromising is acceptable when decision makers have equal power and under existing circumstances all parties will give up something.

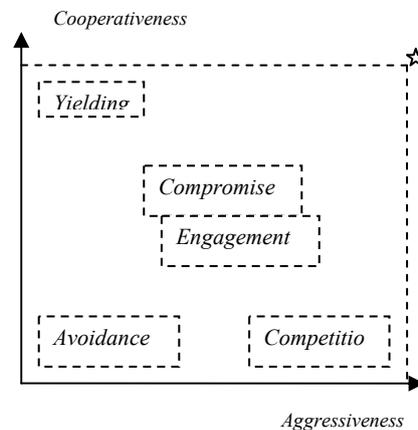


Figure 1: Conflict resolution approaches

Avoidance. Nonaggressive but noncooperative. Conflict consideration is avoided, or conflict resolution is postponed (for a “better time”), or players withdraw from conflict situations. There is no final solution. Descriptive methods may be used to “justify” the situation. Avoidance is used when conflict is too high, decision makers need to “cool off”, and/or gathering more information is necessary. A disadvantage could be an escalation of problem.

Yielding. Nonaggressive and cooperative. Opposite of competition. One party sacrifices his/her own interests (to give up), it could be in a form of generosity or mercy. A final solution is obvious. No methods are needed. Yielding is used when someone find issues more important to others, has no power “to fight”, and/or harmony and stability are especially important.

Fig. 1 illustrates the position of the approaches in two-dimension space, related to cooperativeness and aggressiveness. Assuming necessity of cooperativeness and strive for some degree of aggressiveness in reaching fair (good) solution, compromising and collaborating could be considered as better approaches acceptable in many cases (except violence).

Compromising and collaborating could be used to resolve complexity of problem and when there is a need for long term solution. Compromising is needed when goals are clearly incompatible and mutually exclusive, decision makers have equal power, and partial satisfaction maybe better and feasible. Compromising is not acceptable when there is an imbalance in power of decision makers, or when sets of concerns are too important to compromise. To reach a compromise solution, the decision makers (parties or players) must have appropriate skills and knowledge. The preference of each player (decision maker) is based on his/her “Habitual Domain” and “competence sets”, a concept proposed by Po-Lung Yu [22]. For instance, “gambler” and “risk aversion” decision maker could have different preferences. The authors in [11] argue that risk-aversion of at least one party explains the situation when conflict resolution ends in a bargained agreement. Cooperation and negotiation, emphasizing the similarities and reducing dissimilarities will help to solve problems [22]. In negotiations, the parties realize the potential of a compromise and can assess its main features. This can be supported with MCDM-based methods and tools (for example, the VIKOR method). When negotiations reach an impasse, final arbitration is often imposed to determine a settlement. But it seems that arbitration is rarely necessary in practice because of cost in time, effort or resources.

3. COOPERATIVE GAME

3.1. Cooperative game theory

The game theory is generally divided into two branches: noncooperative and cooperative. In the noncooperative theory, either the players are unable to communicate before decisions are made, or if such communication is allowed, the players are forbidden or are otherwise unable to make a binding agreement on a joint choice of strategy. In the cooperative theory, it is assumed that the players are allowed to communicate before the decisions are made. They may make proposals and counterproposals, and hopefully come to some compromise. Zero-sum games are those in

which each player benefits only at the expense of others, and the total benefit to all players in the game adds to zero. In non-zero-sum game any gain by one player doesn't necessarily correspond with a loss by another player, and some outcomes are good for all players or bad for all players.

Non-cooperative game is with solution concepts based on players maximizing their own utility functions subject to stated constraints. Each player selects his share of resources only with the view of optimizing his own objective and does not care for other players. A Nash equilibrium exists when neither party has an incentive to alter its strategy, taking the other's strategy as given. Complete information games are those in which each player has the same game-relevant information as every other player.

Cooperative games are those in which the players may freely communicate among themselves before making game decisions and may make bargains to influence those decisions [5]. The cooperative game theory assumes that binding agreements (contracts) between players can be made. Cooperative game theory (Pareto Game) assumes that each player is a member of a team willing to compromise his own objective to improve the solution as a whole. In the cooperative solution, the team would reallocate the resources with the intent that all the players should be as optimal as possible—in other words, a Pareto optimal solution. The cooperative theory itself breaks down into two branches, depending on whether or not the players have comparable units of utility and are allowed to make monetary side payments in units of utility as an incentive to induce certain strategy choices. The corresponding solution concept is called the TU cooperative value if side payments are allowed, and the NTU cooperative value if side payments are forbidden or otherwise unattainable. The initials TU and NTU stand for “transferable utility” and “non-transferable utility” respectively.

An interesting result of a cooperative game is the compromise value. In the theory of cooperative games the compromise value is a feasible compromise between upper and lower bounds of the core. In [3] special attention is given to relations between these bounds and compromise values for the class of convex fuzzy games. The compromise value for cooperative games with random payoffs is considered in [20] as a compromise between the utopia payoffs and the minimal rights of the players. A utopia payoff of a player is such that he/she will always accept it as a payoff since it is a very large payoff. On the contrary, a player's minimal right is the minimal amount he may rightfully claim from any payoff allocation.

In the classical model of cooperative games with transferable utility (TU game) one assumes that each subgroup of players can form “coalition” and cooperate to obtain its value. The worth of a coalition is interpreted as the maximal profit or minimal cost for the players in their own coalition. In n -person cooperative games there are no restrictions on the agreements that may be reached among the players. We assume that all payoffs are measured in the same units and that there is a transferable utility which allows side payments to be made among the players (TU-game). Side payments may be used as inducements for some players to use certain mutually beneficial strategies. The coalitional form of an n -person game is given by the pair (N, v) , where $N = \{1, 2, \dots, n\}$ is the set of players and v is a real-valued function, called the characteristic function of the game, defined on the set, 2^N , of all coalitions (subsets of N) [9]. In the paper [2], the authors define the feasible coalitions by using combinatorial geometries called matroids. In the case of cooperative game with two players only one (practical) coalition is possible.

An important issue in cooperative game theory is the allocation of the value of the grand coalition of a game to the players of this game [18]. To this aim various solution concepts have been developed, among them is the compromise value. In paper [3] two compromise values for cooperative game, the σ -value and the τ -value, are defined as the efficient convex combination of the utopia vector $M(v)$ and lower bounds

$$\sigma(v) = \alpha M(v) + (1 - \alpha)b(v)$$

$$\tau(v) = \alpha M(v) + (1 - \alpha)m(v)$$

where $\alpha \in [0,1]$ is such that $\sum_{i \in N} \sigma_i(v) = v(N)$ or $\sum_{i \in N} \tau_i(v) = v(N)$.

The utopia vector $M(v)$ of a game (N,v) consists of the utopia demands of all players. The utopia payoff for player $i \in N$ in the grand coalition N is given by:

$$M_i(v) = v(N) - v(N \setminus \{i\})$$

The lower bound $b(v)$ represents the stand-alone vector $b(v) = (v(\{1\}), \dots, v(\{n\}))$.

The minimum right $m_i(v)$ of player i corresponds to the value this player can achieve by satisfying all other players in a coalition S by giving them their utopia demands:

$$m_i(v) = \max_{S: i \in S} [v(S) - \sum_{j \in S \setminus \{i\}} M_j(v)].$$

A transferable utility game is said to be compromise admissible if:

$$m(v) \leq M(v) \text{ and } \sum_{i \in N} m_i(v) \leq v(N) \leq \sum_{i \in N} M_i(v).$$

In the paper [19] a survey on several well-known compromise values in cooperative game theory and its applications are presented, with special attention to the τ -value for TU-games (transferable utility), and the compromise value for NTU-games.

3.2. Illustrative example

Consider the two-person TU-game with players I and II , and resulting profits for the game are given by the following payoff matrix:

$$\begin{array}{cc} (0,0) & (920,630) \\ (120,830) & (70,80) \end{array}$$

Two pairs of strategies are Nash equilibria, with the outcome (920,630) and (120,830). The cooperative outcome would maximize joint payoffs, here 1550, with the outcome (920,630). Player I benefits most from cooperation. The difference between its best payoff under cooperation and the next best payoff is $920 - 120 = 800$. To persuade Player II to choose I 's best option, Player I must offer at least the 200 (830-630). However, II realizes that I benefits much more from cooperation and should try to extract as much as it can from I (up to 800).

The associated game in coalitional form is determined by finding the characteristic function v . There are 4 coalitions, $\{\emptyset, \{1\}, \{2\}, N\}$. We automatically have $v(\emptyset) = 0$, and $v(N)$ is the largest sum in the eight cells as total payoff $v(N) = 1550$. To find $v(\{1\})$, compute the payoff matrix for the winnings of I against II :

$$v(\{1\}) = -920 \times 120 / 70 - 920 - 120 = 113.81$$

To find $v(\{2\})$, compute the payoff matrix for the winnings of II against I :

$$v(\{2\}) = -630 \times 830 / 80 - 630 - 830 = 378.91$$

The utopia vector $M(v)$ of the game is

$$M(v) = (1171.09, 1436.19)$$

The vector of minimum rights is

$$m(v) = (113.81, 378.91)$$

$$\alpha = 1550 - 492.72 / 2114.56 = 0.5$$

The τ -value for this TU-game is

$$\tau = (642.45, 907.55)$$

This is a compromise allocation of the value of the grand coalition to the players of this game. Player I benefits 642.45 and II benefits 907.55 from cooperation.

4. COMPROMISE SOLUTION BY THE VIKOR METHOD

4.1 VIKOR Framework

Consensus is becoming popular as a democratic form of decision making and as a process of nonviolent conflict resolution [4]. By the definition it is general agreement or the judgment arrived at by most or all of those concerned. Often, the consensus process is informal and vague. This paper aims to formal consensus solution through normative compromise. It focuses on preparing consensus proposal based on a compromise solution obtained by an MCDM method. The most methods are based on comparisons and outranking or ranking the alternatives. A good compromise could be reached by a method based on the concordance-discordance principle. This principle consists in declaring that an alternative a is at least as good as an alternative b if:

- a majority of the criteria (attributes) supports this assertion (concordance condition), and
- the opposition of the other criteria (the minority) is not "too strong" (non-discordance condition).

The VIKOR method $\{vikor\}$ was developed as an MCDM method to solve a discrete decision problem with noncommensurable and conflicting criteria

$$mco_j\{(f_{ij}(A_j), j = 1, \dots, J), i = 1, \dots, n\} \quad (1)$$

where: J is the number of feasible alternatives; $A_j = \{x_1, x_2, \dots\}$ is the j -th alternative obtained (generated) with certain values of system variables x ; f_{ij} is the value of the i -th criterion function for the alternative A_j ; n is the number of criteria; mco denotes the MCDM operator. Alternatives can be generated and their feasibility can be tested by mathematical models (determining variables x), physical models, and/or by experiments on the existing system or other similar systems. Constraints are seen as high-priority objectives, which must be satisfied in the alternatives generating process.

Development of the VIKOR method started with the following form of L_p -metric

$$L_{p,j} = \left\{ \sum_{i=1}^n [w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-)]^p \right\}^{1/p}, \quad 1 \leq p \leq \infty; \quad j = 1, 2, \dots, J \quad (2)$$

The measure $L_{p,j}$ was introduced by Duckstein and Opricovic [6] and it represents the distance of the alternative A_j to the ideal solution. Previously, the L_p -metric has been introduced in compromise programming method [21], [7]. Here, L_1 is the sum of all individual regrets (disutility), and L_∞ is the maximal regret that an individual could have. Developing the VIKOR method, the author (Opricovic) integrated these two measures in one aggregating index (see Q in Eq. (a3) in Appendix). Aggregating (compound) function should be used with extreme caution since that involves comparing potentially incomparable quantities (noncommensurable criteria or indicators). To add values of noncommensurable indicators, first we have to convert them into the same units. Normalization could be used to eliminate the units of indicators, so that all the indicators are dimensionless. The normalized values by the linear normalization do not depend on the evaluation unit of an indicator [15].

The compromise solution F^c is a feasible solution that is the “closest” to the ideal F^* . Here, compromise means an agreement established by mutual concessions, as illustrated in Fig. 2 by $\Delta f_1 = f_1^* - f_1^c$ and $\Delta f_2 = f_2^* - f_2^c$. Assuming that each alternative is evaluated according to all criteria, the ranking could be performed by comparing the measure of closeness to the ideal solution F^* (the best values of criteria).

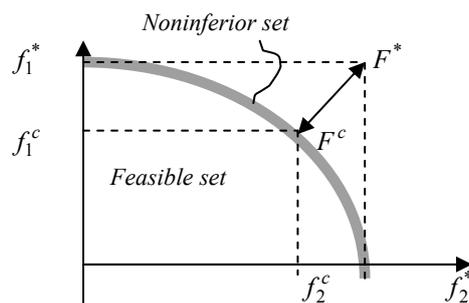


Figure 2. Ideal and Compromise solutions

The extended VIKOR method in comparison with three multicriteria decision making methods TOPSIS, PROMETHEE, and ELECTRE is presented in the work of Opricovic and Tzeng [16]. The ranking algorithm VIKOR (from [16]) is presented in the Appendix. The VIKOR method focuses on ranking and selecting from a set of alternatives, and determines compromise solutions for a problem with conflicting criteria, which can help the decision makers to reach a final decision. The obtained compromise solution could be accepted by the decision makers because it provides a maximum group utility of the “majority” (represented by $\min S$, Equation (a1) in Appendix), and a minimum individual regret of the “opponent” (represented by $\min R$). The compromise solutions could be the base for negotiation, involving the decision makers’ preference by criteria weights. The trade-offs determined in step (vii) (Appendix) could help the decision maker to assess new values, although that task is very difficult. Trade-off assessment is the most difficult issue in MCDM, and many methods have been developed to alleviate this problem.

The VIKOR result depends on an ideal solution, and stands only for the given set of alternatives. Inclusion (or exclusion) of an alternative could affect the VIKOR ranking of new set of alternatives. Giving the best f_i^* and the worst f_i^- values of criteria, this effect could be avoided, but that would mean that a fixed ideal solution could be defined by the decision maker.

The main contributions of VIKOR to conflict resolution are: consideration of the decision making process in addition to the result (outcome) which is the predominant focus of game theory; the use of criteria which is more meaningful for decision makers than utilities; search for the set of efficient compromise solutions rather than one solution; and, interactivity which allows decision makers to participate in and control the decision process (by weights).

4.2. Illustrative example

For bimatrix game in Section 3.2 with pair strategies (s_i^I, s_j^II) , $i=1, \dots, m$; $j=1, \dots, n$, resulting to the pay off (benefit) (b_{ij}^I, b_{ij}^{II}) of players I and II, an appropriate model for the VIKOR method is the following:

- Alternatives:

$$a_1 = (s_1^I, s_1^{II}), a_2 = (s_2^I, s_1^{II}), \dots, a_m = (s_m^I, s_1^{II}), a_{m+1} = (s_2^I, s_1^{II}), \dots$$

$$a_{m \times n} = (s_m^I, s_n^{II})$$

- Criterion functions:

$$f_{11} = b_{11}^I, f_{12} = b_{21}^I, \dots, f_{1m} = b_{m1}^I, f_{1,m+1} = b_{12}^I, \dots, f_{1,m \times n} = b_{mn}^I$$

$$f_{21} = b_{11}^{II}, f_{22} = b_{21}^{II}, \dots, f_{2m} = b_{m1}^{II}, f_{2,m+1} = b_{12}^{II}, \dots, f_{2,m \times n} = b_{mn}^{II}$$

Both functions represent benefits, and they have equal importance, equal weights in VIKOR, $w_1 = w_2 = 0.5$.

Applying VIKOR method the following ranking list was obtained: a_3 ($Q=0.0$, Q in Eq. (a3) in Appendix), a_2 ($Q=0.593$), a_4 ($Q=0.901$), a_1 ($Q=1.0$), and a_3 is the compromise solution with the outcome (920,630).

Let us introduce new pairs strategies as alternatives a_5, a_6, a_7, a_8 with outcome $(1550/2, 1550/2)=(775, 775)$, $(920-100, 830-100)=(820, 730)$, $(720, 830)$, $(650, 900)$, respectively, representing fair allocation, a_9 with outcome (642.45, 907.55) from Section 3.2, and the ideal $F^*=(1171.09, 1436.19)$ as the utopia from Section 3.2. New ranking list by VIKOR is: a_7 ($Q=0.019$), a_5 (0.031), a_8 (0.036), a_9 (0.039), a_6 (0.040), a_3 (0.060), a_2 (0.539), a_4 (0.905), a_1 (1.0). The compromise solution for final decision is the set of alternative outcomes:

a_7 (720, 830)

a_5 (775, 775)

a_8 (650, 900)

a_9 (642.45, 907.55)

a_6 (820, 730)

a_3 (920, 630)

The VIKOR result is the set of compromise solutions, and it could be a base for negotiation. The solution with outcome (720,830) is fair compromise; Player *II* gets his/her ideal, extracting 200 from player's *I* ideal (original 920). An attempt of Player *II* to extract from *I* 400 or more is not a compromise solution.

This example illustrates the compromise by VIKOR based on cooperativeness.

An attempt of comparing cooperative game theory and VIKOR method results is presented in this section, initiating a comparison of conflict resolution by VIKOR and Game Theory. The VIKOR method could be compared with N-person cooperative game, but it is necessary to extend VIKOR to group decision making, and this could be a task for future research. A cooperative game could be considered as N-person decision problem [13]. A similar problem has been considered in MCDM as group decision making [12]. Tremendous uncertainty and unknown is involved in N-person decision problem for each player. A prediction cannot always be accurate because of the existing gaps in the players' perceptions, information inputs, and judgments [22].

5. CONCLUSION

Conflict resolution approach and solution are related to the degree of the cooperativeness and aggressiveness in resolving conflict. Cooperation and negotiation, emphasizing the similarities and reducing dissimilarities will help to solve problems. Conflict resolution is considered here as multicriteria decision making problem. Basic elements involved in decision processes are alternatives (strategies, scenarios), criteria (pay off) and preferences, and they are similar in MCDM and in the game theory.

In the classical model of cooperative games with transferable utility (TU game) one assumes that each subgroup of players can form "coalition" and cooperate to obtain its value. An important issue in cooperative game theory is the allocation of the value of the grand coalition of a game to the players of the game. To this aim various solution

concepts have been developed, among them is the compromise value. In the theory of cooperative games the compromise value is a feasible compromise between upper and lower bounds of the core.

Compromising is considered as fair aggressive and cooperative approach to conflict resolution. A compromise is an agreement established by mutual concessions. Compromising is needed when goals are clearly incompatible and mutually exclusive, decision makers have equal power, and partial satisfaction maybe better and feasible. Compromising is not acceptable when there is an imbalance in power of decision makers, or when sets of concerns are too important to compromise. In negotiations, the parties realize the potential of a compromise and can assess its main features. This can be supported with MCDM-based methods and tools, for example, the VIKOR method. When negotiations reach an impasse, final arbitration is often imposed to determine a settlement. But it seems that arbitration is rarely necessary in practice because of cost in time, effort or resources.

Applying the VIKOR method (as a normative method) could help in conflict resolution, and the compromise solution could be the base for negotiation. It is emphasized that cooperativeness is necessary for compromising. The VIKOR method assumes all parties acting as one decision maker in compromising, and his/her preference is expressed as weights of criteria. Here, the compromise solution is a feasible solution which is the closest to the ideal, and a compromise means an agreement established by mutual concessions. The obtained compromise solution could be accepted by the decision makers because it provides a maximum group utility of the "majority" and a minimum individual regret of the "opponent". The main contributions of VIKOR to conflict resolution are: consideration of the decision making process in addition to the result (outcome) which is the predominant focus of game theory; the use of criteria which is more meaningful for decision makers than utilities; search for the set of efficient compromise solutions rather than one solution; and, interactivity which allows decision makers to participate in and control the decision process (by weights).

Comparison of MCDM and game theory is a challenging research area and selecting and integrating ideas could help in developing new approaches to conflict resolution.

Acknowledgments

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APPENDIX

The VIKOR method has been developed to solve the following problem

$$mco_j\{(f_{ij}(A_j), j = 1, \dots, J), i = 1, \dots, n\}$$

where: J is the number of feasible alternatives; $A_j = \{x_1, x_2, \dots\}$ is the j -th alternative obtained (generated) with certain values of system variables x ; f_{ij} is the value of the i -th criterion function for the alternative A_j ; n is the number of criteria; mco denotes the

operator of a multicriteria decision making procedure for selecting the best (compromise) alternative in multicriteria sense.

The algorithm VIKOR has the following steps:

(i) Determine the best f_i^* and the worst f_i^- values of all criterion functions, $i = 1, 2, \dots, n$;

$$f_i^* = \max_j f_{ij}, \quad f_i^- = \min_j f_{ij}, \quad \text{if the } i\text{-th function represents a benefit;}$$

$$f_i^* = \min_j f_{ij}, \quad f_i^- = \max_j f_{ij}, \quad \text{if the } i\text{-th function represents a cost.}$$

(ii) Compute the values S_j and R_j , $j=1, 2, \dots, J$, by the relations

$$S_j = \sum_{i=1}^n w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-) \quad (\text{a1})$$

$$R_j = \max_i [w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-)] \quad (\text{a2})$$

where w_i are the weights of criteria, expressing the DM's preference as the relative importance of the criteria.

(iii) Compute the values Q_j , $j = 1, 2, \dots, J$, by the relation

$$Q_j = v(S_j - S^*) / (S^- - S^*) + (1-v)(R_j - R^*) / (R^- - R^*) \quad (\text{a3})$$

where $S^* = \min_j S_j$, $S^- = \max_j S_j$, $R^* = \min_j R_j$, $R^- = \max_j R_j$; and v is introduced as a weight for the strategy of "the majority of criteria" (or "the maximum group utility"), whereas $1-v$ is the weight of the individual regret. These strategies could be compromised by $v = 0.5$, and here v is modified as $v = (n+1)/2n$ (from $v + 0.5(n-1)/n = 1$) since the criterion (1 of n) related to R is included in S , too.

(iv) Rank the alternatives, sorting by the values S , R and Q in decreasing order. The results are three ranking lists.

(v) Propose as a compromise solution the alternative ($A^{(1)}$) which is the best ranked by the measure Q (minimum) if the following two conditions are satisfied:

C1. "Acceptable Advantage":

$$Q(A^{(2)}) - Q(A^{(1)}) \geq DQ$$

where: $A^{(2)}$ is the alternative with second position in the ranking list by Q ;

$$DQ = 1/(J-1).$$

C2. "Acceptable Stability in decision making":

The alternative $A^{(1)}$ must also be the best ranked by S or/and R . This compromise solution is stable within a decision making process, which could be the strategy of maximum group utility (when $v > 0.5$ is needed), or "by consensus" $v \approx 0.5$, or "with

veto" ($v < 0.5$). Here, v is the weight of decision making strategy of maximum group utility.

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives $A^{(1)}$ and $A^{(2)}$ if only the condition C2 is not satisfied, or
- Alternatives $A^{(1)}, A^{(2)}, \dots, A^{(M)}$ if the condition C1 is not satisfied; $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A^{(1)}) < DQ$ for maximum M (the positions of these alternatives are "in closeness").

(vi) Determine the weight stability interval $[w_i^L, w_i^U]$ for each (i -th) criterion, separately, with the initial (given) values of weights. The compromise solution obtained with initial weights ($w_i, i=1, \dots, n$), will be replaced at the highest ranked position if the value of a weight is out of the stability interval. The stability interval is only relevant concerning one-dimensional weighting variations.

(vii) Determine the trade-offs, $tr_{ik} = |(D_i w_k) / (D_k w_i)|, k \neq i, k = 1, \dots, n$, where tr_{ik} is the number of units of the i -th criterion evaluated the same as one unit of the k -th criterion, and $D_i = f_i^* - f_i^-, \forall i$. The index i is given by the VIKOR user.

(viii) The decision maker may give a new value of $tr_{ik}, k \neq i, k = 1, \dots, n$ if he or she does not agree with computed values. Then, VIKOR performs a new ranking with new values of weights $w_k = |(D_k w_i tr_{ik}) / D_i|, k \neq i, k = 1, \dots, n; w_i = 1$ (or previous value). VIKOR normalizes weights, with the sum equal to 1. The trade-offs determined in step (vii) could help the decision maker to assess new values, although that task is very difficult.

(ix) The VIKOR algorithm ends if the new values are not given in step (viii).

The results by the VIKOR method are rankings by S, R, and Q, proposed compromise solution (one or a set), weight stability intervals for a single criterion, and the trade-offs introduced by VIKOR.

The extended VIKOR method in comparison with three multicriteria decision making methods TOPSIS, PROMETHEE, and ELECTRE is presented in the work of Opricovic and Tzeng [16].

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