

**A FINITE TIME–HORIZON DETERMINISTIC EOQ  
MODEL WITH STOCK–DEPENDENT DEMAND, EFFECTS  
OF INFLATION AND TIME VALUE OF MONEY WITH  
SHORTAGES IN ALL CYCLES**

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**Abstract:** A finite time–horizon deterministic inventory model is developed, taking the demand rate at any instant to be a function of the on–hand inventory (stock–level) at that instant. Shortages in inventory are allowed. The effects of inflation and time value of money are considered. Two separate inflation rates: namely, the internal (company) and the external (general economy) are introduced. A numerical example of the model is discussed. A sensitivity analysis of the optimal solution with respect to the parameters of the model is examined.

**Keywords:** Inventory, stock–dependent demand, shortage, inflation, time value of money.

## 1. INTRODUCTION

Usually, the effect of inflation and the time value of money are not considered explicitly in analysing inventory systems, although inflation would influence the cost and price components to any significant degree. Recently, the economic condition of different countries has changed to such a manner that it has not been possible to ignore the effects of inflation and time–value of money any further. Among the few authors who considered inflation, Brown (1967) derived an expression for a step increase in purchase

price due to inflation and minimized the present value of all the future costs. Bierman and Thomas (1977) investigated the effect of inflation in an economic order quantity (EOQ) model with the aim of minimizing the present value of all future costs. But they did not provide a definite way of calculating the economic order quantity each time a new order is placed. Buzacott (1976) analysed an EOQ model with inflation having an effect on all cost parameters. He derived expressions for the optimal order quantity and showed that the choice of the inventory carrying fraction depends on the Company's pricing policy which inversely affects the optimal order quantity. Misra (1975) developed EOQ model for inflationary effects incorporation. He also considered a uniform inflation rate for all the associated costs and minimized the average annual cost. Several other researchers have extended their idea to other situations by considering the time-value of money, different inflation rates for the internal and external costs, finite replenishment rate, shortage, etc. The models of Van Hees and Monhemius (1972), Jeya Chandra and Bahner (1985), Aggarwal (1981), Misra (1979), Sarkar and Pan (1997), etc. considered the market demand rate to be a constant. Misra (1979) developed an EOQ model with time discounting and two different inflation rates, one for the internal costs and other for external costs. He remarked that though the optimal order quantities calculated with and without time discounting and inflation are quite different, the corresponding total costs per unit time are very close. Datta and Pal (1991) analysed a finite time-horizon inventory model considering the approach of Misra (1979) with a linearly time-dependent demand rate, shortages and considering the effect of inflation and time value of money. Bose et al. (1995) developed an EOQ model for a deteriorating item considering the inflationary effects, time-value of money, a linearly time dependent demand rate and shortages.

Many researchers have observed that the presence of more quantities of the same product tends to attract more customers. In other words, the consumption rate may be influenced by the stock levels. Modelers have made several attempts to analyze inventory models assuming a functional relation between the demand rate and the on-hand inventory (stock-level). This observation has attracted the interest of inventory modelers and has resulted in several papers such as Gupta and Vrat (1986), Mandal and Phaujdar (1989), Baker and Urban (1988), Datta and Pal (1990), Urban (1992), Goh (1992, 1994), Pal et al. (1993), Sarker et al. (1997), Ray and Chaudhuri (1997), Roy and Chaudhuri (2006), Dye and Ouyang (2005), Teng and Chang (2005), Chung and Tsai (2001), etc. They developed EOQ models with stock-level dependent demand, without considering the inflationary effects.

In this paper, we consider a deterministic inventory model for an item having a stock dependent demand and there is no inventory at the initial stage. Shortages are allowed and deterioration of the inventory over time is not taken into account. We consider two separate inflation rates; namely, the internal (company) and the external (general economy) and the time-value of money. The model is solved analytically for an finite time horizon. The objective of this paper is to minimize the total cost. Numerical examples are taken to illustrate the model and sensitivity of the model is analysed.

## 2. ASSUMPTIONS AND NOTATION

The following assumptions and notation have been used in developing the model:

- (i) The replenishment rate is infinite, i.e., replenishment is instantaneous.
- (ii)  $A$  is the internal replenishment cost.
- (iii) The demand rate  $R(q)$  of the item, when the on-hand inventory level  $q$ , is considered in the form

$$\begin{aligned} R(q) &= \alpha q^\beta, & (k+j-1)T \leq t \leq jT, & \quad (j = 1, 2, \dots, n) \\ &= -D, & (j-1)T \leq t \leq (k+j-1)T, & \quad (j = 1, 2, \dots, n) \end{aligned}$$

where  $\alpha > 0$  and  $0 < \beta < 1$  are scale and shape parameters and  $D (> 0)$  is a constant. The shape parameter  $\beta$  is (Pal et al., 1993) the elasticity of the demand rate with respect to the stock level. We may briefly call it the “*stock elasticity of demand*”.

- (iv) Lead-time is zero.
- (v) Internal and external inflation rates are denoted by  $i_1$  and  $i_2$  respectively.
- (vi)  $r$  is the discount rate representing the time value of money.
- (vii) At time  $t = 0$ ,  $c_{11}$  and  $c_{12}$  are respectively the internal and external holding costs per unit item per unit time.
- (viii) Shortages are permitted.  $c_{21}$  and  $c_{22}$  are respectively the internal and external shortage costs per units item per unit time and  $p$  is the purchase cost.
- (ix)  $H$  is the fixed time horizon. The time-horizon  $H$  is divided into  $n$  equal parts, each of length  $T$ , so that  $T = H/n$ .
- (x) At time  $t = 0$ , there is no inventory.
- (xi)  $R_m = r - i_m$ ,  $m = 1, 2$ .
- (xii)  $Q$  is the optimal replenishment size in each cycle.

## 3. THE MODEL

The initial inventory is zero. The re-order time over the time-horizon  $H$  is  $(k+j-1)H$  ( $j = 1, 2, \dots, n$ ). We assume that there is no shortage in each interval  $[(k+j-1)T, jT]$ . Shortage occurs at times  $(j-1)T$  ( $j = 1, 2, \dots, n$ ) where  $(j-1)T < (k+j-1)T < jT$  ( $j = 1, 2, \dots, n$ ). Our purpose is to determine the optimal values of  $n$  and  $k$  that minimize the total cost over the time-horizon  $[0, H]$ . The pictorial representation of the system is given in Fig. 1. The differential equations governing the stock status for the period  $[0, H]$  are the following:

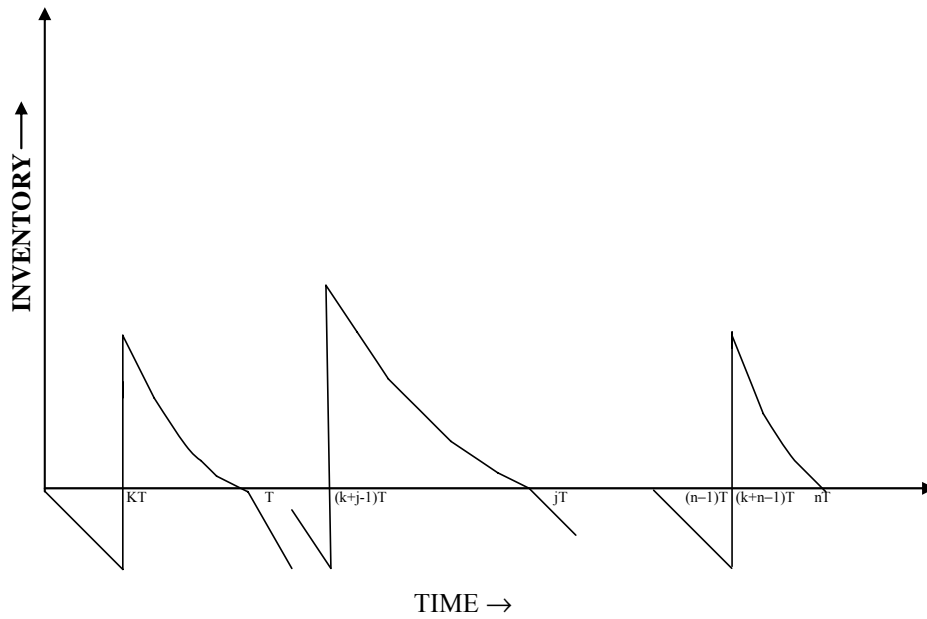
$$\frac{dq}{dt} = -D, \quad (j-1)T \leq t \leq (k+j-1)T, \quad (j = 1, 2, \dots, n) \quad (1)$$

and

$$\frac{dq}{dt} = -\alpha q^\beta, \quad (k+j-1)T \leq t \leq jT, \quad (j = 1, 2, \dots, n) \quad (2)$$

The initial condition is

$$q(jT) = 0, \quad j = 0, 1, 2, \dots, n. \quad (3)$$



**Figure 1.**

The solution of equations (1) and (2) subject to the condition (3) are respectively

$$q(t) = D(jT - t), (j - 1)T \leq t \leq (k + j - 1)T, (j = 1, 2, \dots, n) \tag{4}$$

and

$$q(t) = \{\alpha(1 - \beta)(jT - t)\}^{\frac{1}{1 - \beta}}, (k + j - 1)T \leq t \leq jT, (j = 1, 2, \dots, n) \tag{5}$$

The present worth of the total holding cost during the entire time horizon H is given by (see Appendix - I)

$$C_{HOL} = \sum_{j=1}^n H_j = \sum_{j=1}^n \left[ \sum_{m=1}^2 I_{jm} \right] \tag{6}$$

The present worth of the total purchase cost is (see Appendix - II)

$$C_{PUR} = \sum_{j=1}^n P_j \tag{7}$$

The present worth of the total shortage cost is (see Appendix - III)

$$C_{SHO} = \sum_{j=1}^n S_j = \sum_{j=1}^n \left[ \sum_{m=1}^2 J_{jm} \right] \tag{8}$$

The present worth of total replenishment cost is (see Appendix - IV)

$$C_R = A \frac{e^{-R_1 k T} (1 - e^{-R_1 H})}{(1 - e^{-R_1 T})} \tag{9}$$

Now the present value of all costs of the system during the whole time period  $H$  is given by

$$\begin{aligned} C(n, k) &= C_{HOL} + C_{PUR} + C_{SHO} + C_R \\ &= \sum_{j=1}^n \sum_{m=1}^2 [c_{1m} \alpha^{\frac{1}{1-\beta}} (1-\beta)^{\frac{2-\beta}{1-\beta}} e^{-R_m j T} [\frac{1}{2-\beta} \{(1-k)T\}^{\frac{2-\beta}{1-\beta}} + \frac{R_m}{(2-\beta)(3-2\beta)} \times \\ &\quad \{(1-k)T\}^{\frac{3-2\beta}{1-\beta}} + \frac{R_m^2}{2(3-2\beta)(4-3\beta)} \{(1-k)T\}^{\frac{4-3\beta}{1-\beta}}] + \frac{Dc_{2m}}{R_m^2} [R_m k T e^{-R_m(j-1)T} \\ &\quad + e^{-R_m(k+j-1)T} - e^{-R_m(j-1)T}]] + \sum_{j=1}^n [DpkT e^{-R_m(k+j-1)T} + p\{\alpha(1-\beta)\}^{\frac{1}{1-\beta}} \times \\ &\quad e^{-R_m(k+j-1)T} \{(1-k)T\}^{\frac{1}{1-\beta}}] + A \frac{e^{-R_1 k T} (1 - e^{-R_1 H})}{(1 - e^{-R_1 T})}. \end{aligned} \tag{10}$$

Now

$$\begin{aligned} C(n+1, k) - C(n, k) &= \sum_{m=1}^2 [c_{1m} \alpha^{\frac{1}{1-\beta}} (1-\beta)^{\frac{2-\beta}{1-\beta}} e^{-R_m(n+1)T} [\frac{1}{2-\beta} \{(1-k)T\}^{\frac{2-\beta}{1-\beta}} \\ &\quad + \frac{R_m}{(2-\beta)(3-2\beta)} \{(1-k)T\}^{\frac{3-2\beta}{1-\beta}} + \frac{R_m^2}{2(3-2\beta)(4-3\beta)} \times \\ &\quad \{(1-k)T\}^{\frac{4-3\beta}{1-\beta}}] + \frac{Dc_{2m}}{R_m^2} [R_m k T e^{-R_m n T} + e^{-R_m(k+n)T} \\ &\quad - e^{-R_m n T}]] + DpkT e^{-R_2(k+n)T} + p\{\alpha(1-\beta)\}^{\frac{1}{1-\beta}} \times \\ &\quad e^{-R_2(k+n)T} \{(1-k)T\}^{\frac{1}{1-\beta}}. \end{aligned}$$

Similarly,

$$\begin{aligned} C(n, k) - C(n-1, k) &= \sum_{m=1}^2 [c_{1m} \alpha^{\frac{1}{1-\beta}} (1-\beta)^{\frac{2-\beta}{1-\beta}} e^{-R_m n T} [\frac{1}{2-\beta} \{(1-k)T\}^{\frac{2-\beta}{1-\beta}} \\ &\quad + \frac{R_m}{(2-\beta)(3-2\beta)} \{(1-k)T\}^{\frac{3-2\beta}{1-\beta}} + \frac{R_m^2}{2(3-2\beta)(4-3\beta)} \times \\ &\quad \{(1-k)T\}^{\frac{4-3\beta}{1-\beta}}] + \frac{Dc_{2m}}{R_m^2} [R_m k T e^{-R_m(n-1)T} + e^{-R_m(k+n-1)T} \\ &\quad - e^{-R_m(n-1)T}]] + DpkT e^{-R_2(k+n-1)T} + p\{\alpha(1-\beta)\}^{\frac{1}{1-\beta}} \times \\ &\quad e^{-R_2(k+n-1)T} \{(1-k)T\}^{\frac{1}{1-\beta}}. \end{aligned}$$

For minimum value of  $C(n, k)$ , the inequality  $C(n-1, k) \geq C(n, k) \leq C(n+1, k)$  must hold. This gives  $F(n-1) \leq \psi \leq F(n)$  where

$$F(n) = e^{-R_m n T} / e^{-R_2 n T}$$

and

$$\begin{aligned} \psi = & -e^{-R_2 k T} [DpkT + p\{\alpha(1-\beta)\}^{\frac{1}{1-\beta}} \{(1-k)T\}^{\frac{1}{1-\beta}}] / \left[ \sum_{m=1}^2 [c_{1m} \alpha^{\frac{1}{1-\beta}} (1-\beta)^{\frac{2-\beta}{1-\beta}} \times \right. \\ & e^{-R_m T} \left[ \frac{1}{2-\beta} \{(1-k)T\}^{\frac{2-\beta}{1-\beta}} + \frac{R_m}{(2-\beta)(3-2\beta)} \{(1-k)T\}^{\frac{3-2\beta}{1-\beta}} \right. \\ & \left. \left. + \frac{R_m^2}{2(3-2\beta)(4-3\beta)} \{(1-k)T\}^{\frac{4-3\beta}{1-\beta}} \right] + \frac{Dc_{2m}}{R_m^2} [R_m k T + e^{-R_m k T} - 1] \right]. \end{aligned}$$

Consequently, we may formulate a Lemma as follows :

**Lemma 1:** For fixed  $k$ , if  $F(n)$  satisfies  $F(n-1) \leq \psi \leq F(n)$ , then  $C(n,k)$  has minimum value at  $n = n^*$ .

### 4. SOLUTION PROCEDURE

The total variable cost  $C(n, k)$  given above is a function of two variables  $n$  and  $k$  where  $n$  is a discrete variable and  $k$  is a continuous variable. For any given value of  $n$ , the necessary condition for  $C(n, k)$  to be minimum is  $\frac{dC}{dk} = 0$  which gives

$$\begin{aligned} \sum_{j=1}^n & \left[ (1-R_2 T k) D p T e^{-R_2 (k+j-1) T} - p \alpha^{\frac{1}{1-\beta}} (1-\beta)^{\frac{\beta}{1-\beta}} T^{\frac{1}{1-\beta}} \left\{ (1-k)^{\frac{\beta}{1-\beta}} \right. \right. \\ & \left. \left. e^{-R_2 (k+j-1) T} + (1-\beta) R_2 T e^{-R_2 (k+j-1) T} (1-k)^{\frac{1}{1-\beta}} \right\} \right] \\ & + \sum_{j=1}^n \sum_{m=1}^2 \left[ \frac{Dc_{2m} T}{R_m} \left\{ e^{-R_m (j-1) T} - e^{-R_m (k+j-1) T} \right\} \right] \\ & - \sum_{j=1}^n \sum_{m=1}^2 \left[ c_{1m} \alpha^{\frac{1}{1-\beta}} (1-\beta)^{\frac{1}{1-\beta}} e^{-R_m j T} \left\{ T^{\frac{2-\beta}{1-\beta}} (1-k)^{\frac{1}{1-\beta}} + \frac{R_m}{2-\beta} T^{\frac{3-2\beta}{1-\beta}} \right. \right. \\ & \left. \left. (1-k)^{\frac{2-\beta}{1-\beta}} + \frac{R_m^2}{2(3-2\beta)} T^{\frac{4-3\beta}{1-\beta}} (1-k)^{\frac{3-2\beta}{1-\beta}} \right\} \right] - \frac{R_1 T A e^{-R_1 k T} (1 - e^{-R_1 H})}{(1 - e^{-R_1 T})} = 0. \end{aligned} \tag{11}$$

Now,

$$\begin{aligned} \frac{d^2C}{dk^2} = & \sum_{j=1}^n \sum_{m=1}^2 [c_{1m} \alpha^{\frac{1}{1-\beta}} (1-\beta)^{\frac{2-\beta}{1-\beta}} e^{-R_m jT} [\frac{T^2}{(1-\beta)^2} \{(1-k)T\}^{\frac{\beta}{1-\beta}} + \frac{R_m T^2}{(1-\beta)^2} \times \\ & \{(1-k)T\}^{\frac{1}{1-\beta}} + \frac{R_m^2 T^2}{2(1-\beta)^2} \{(1-k)T\}^{\frac{2-\beta}{1-\beta}}] + D c_{2m} T^2 e^{-R_m (k+j-1)T} ] \\ & + \sum_{j=1}^n T^2 p e^{-R_m (k+j-1)T} [DR_2^2 T k + \frac{R_2}{(1-\beta)} \{\alpha(1-\beta)\}^{\frac{1}{1-\beta}} + \frac{\beta}{(1-\beta)^2} \{\alpha(1-\beta)\}^{\frac{1}{1-\beta}} \times \\ & \{(1-k)T\}^{-\frac{1}{1-\beta}} + R_2^2 \{\alpha(1-\beta)\}^{\frac{1}{1-\beta}} \{(1-k)T\}^{\frac{1}{1-\beta}} + \frac{R_2}{(1-\beta)} \{\alpha(1-\beta)\}^{\frac{1}{1-\beta}} \times \\ & \{(1-k)T\}^{\frac{\beta}{1-\beta}} - 2R_2 D]. \end{aligned}$$

From the above analysis, we have a Lemma as follows:

**Lemma 2:** For fixed value of  $n$ , if

$$\begin{aligned} F = & DR_2^2 T k + \frac{R_2}{(1-\beta)} \{\alpha(1-\beta)\}^{\frac{1}{1-\beta}} \\ & + \frac{\beta}{(1-\beta)^2} \{\alpha(1-\beta)\}^{\frac{1}{1-\beta}} \{(1-k)T\}^{-\frac{1}{1-\beta}} + R_2^2 \{\alpha(1-\beta)\}^{\frac{1}{1-\beta}} \{(1-k)T\}^{\frac{1}{1-\beta}} \\ & + \frac{R_2}{(1-\beta)} \{\alpha(1-\beta)\}^{\frac{1}{1-\beta}} \{(1-k)T\}^{\frac{\beta}{1-\beta}} - 2R_2 D > 0, \end{aligned}$$

then  $\frac{d^2C}{dk^2} > 0$  so that  $C(n, k)$  has a minimum value at  $k^*$ .

Equation (11) is a non-linear equation. This equation is solved by Newton-Raphson method (iterative method) when the values of the parameters are prescribed.

If  $n = 1, 2, 3, \dots$  is substituted in eq. (11), then the corresponding values of  $k(0 < k < 1)$  have to be found which give the minimum value of  $C(n, k)$ , provided  $\frac{d^2C}{dk^2} > 0$ .

A list of corresponding costs  $C$  can be obtained from (10) and the minimum value of  $C$  in this list would be the optimum cost  $C^*$ . The corresponding values of  $n$  and  $k$  for the optimum  $C^*$  are the optimum values of  $n (=n^*)$  and  $k (=k^*)$ .

### 5. NUMERICAL EXAMPLE

Let us take the values of the parameters of the inventory system as  $A = 30$ ,  $i_1 = 0.1$ ,  $i_2 = 0.14$ ,  $r = 0.2$ ,  $c_{11} = 0.2$ ,  $c_{12} = 0.4$ ,  $c_{21} = 0.8$ ,  $c_{22} = 0.6$ ,  $\alpha = 15$ ,  $\beta = 0.1$ ,  $p = 6$ ,  $H = 10$ ,  $D = 20.92$  in appropriate units.

Equation (11) is solved for  $k$  ( $0 < k < 1$ ) when  $n = 1, 2, 3, \dots, 16$ . If we substitute these values of  $n$  and  $k$  in (10), we get the corresponding values of the cost  $C$ . The results obtained are shown in Table – 1. From the table, we observe that for  $n = 12$ , the cost  $C$  becomes minimum and the corresponding optimal values of  $n$  and  $k$  are respectively  $n^* = 12$  and  $k^* = 0.77139$  and the minimum cost  $C^*$  becomes  $C^*(n^*, k^*) = 277.02$ . We have  $T^* = H/n^* = 0.833$  and  $F = 13.8014$ .

**Table – I**

$n$	$k$	$T$	$Q$	$C(n, k)$
2	0.75300	5.000	258.879	2020.900
3	0.86666	3.333	115.674	939.290
4	0.83456	2.500	64.9099	602.130
5	0.87218	2.000	41.6360	452.860
6	0.88863	1.667	28.9508	374.800
7	0.88976	1.428	21.2430	330.379
8	0.87955	1.250	16.2635	304.218
9	0.86079	1.111	12.8268	288.971
10	0.83555	1.000	10.3661	280.720
11	0.80536	0.909	8.5359	277.201
12*	0.77139*	0.833*	7.1363*	277.020*
13	0.73454	0.769	6.0480	279.276
14	0.69551	0.714	5.1787	283.362
15	0.65487	0.667	4.4836	288.856
16	0.61306	0.6250	3.9010	295.452

## 6. SENSITIVITY ANALYSIS

We now study the effects of changes in the values of input parameters  $A, i_1, i_2, r, c_{11}, c_{12}, c_{21}, c_{22}, \alpha, \beta, p, H$  and  $D$  on the optimal total cost  $C(n, k)$  and each of the decision variables  $n, k, T$ . The sensitivity analysis is performed by changing each of the parameters by 50%, 20%, -20% and -50%, taking one parameter at a time and keeping the remaining parameters unchanged. Then we calculate the percentage change of  $C(n, k)$  with respect to the base value. This analysis is shown in Table –II.

**Table – II**

Parameters	% change in the parameter	$n^*$	$k^*$	$T^*$	% change in $C^*(n^*, k^*)$
$A$	+50	11	0.867631	0.90909	28.2280
	+20	11	0.830466	0.90909	11.3300
	-20	12	0.743348	0.83333	-12.1991
	-50	13	0.657323	0.76923	-32.0192
$i_1$	+50	11	0.80536	0.90909	0.06529
	+20	12	0.77139	0.83333	0.00000
	-20	12	0.77139	0.83333	0.00000
	-50	13	0.73454	0.76929	0.81449
$i_2$	+50	11	0.80536	0.90909	0.6529
	+20	11	0.80536	0.90909	0.06529
	-20	12	0.77139	0.83333	0.00000
	-50	13	0.73454	0.76923	0.81450



Parameters	% change in the parameter	$n^*$	$k^*$	$T^*$	% change in $C^*(n^*, k^*)$
$r$	+50	11	0.91676	0.90909	-9.08608
	+20	12	0.78398	0.83333	-5.37920
	-20	13	0.61458	0.76923	8.73890
	-50	13	0.41974	0.76923	25.0562
$c_{11}$	+50	11	0.81044	0.90909	0.33449
	+20	12	0.774109	0.8333	0.08911
	-20	12	0.76857	0.8333	0.08912
	-50	13	0.72559	0.76923	0.62723
$c_{12}$	+50	11	0.831216	0.90909	0.60369
	+20	12	0.816225	0.90909	0.28065
	-20	12	0.75719	0.8333	-0.17823
	-50	13	0.69047	0.76923	0.43995
$c_{21}$	+50	11	0.69599	0.90909	0.06529
	+20	12	0.72570	0.8333	0.00000
	-20	12	0.82427	0.8333	0.00000
	-50	13	0.87497	0.76923	0.81449
$c_{22}$	+50	11	0.80536	0.90909	0.06529
	+20	12	0.77139	0.83333	0.0000
	-20	13	0.73454	0.76923	0.8145
	-50	13	0.73454	0.76923	0.8145
$\alpha$	+50	11	1.3444	0.90909	24.9587
	+20	11	1.1923	0.90909	9.88740
	-20	12	-1.6058	0.8333	-8.5650
	-50	13	-17.1725	0.7692	-18.0615
$\beta$	+50	11	0.94061	0.90909	4.4540
	+20	12	0.84669	0.8333	1.3970
	-20	13	0.62099	0.7692	-0.3024
	-50	13	0.36414	0.7692	-1.8585
$p$	+50	11	0.83346	0.90909	21.1277
	+20	11	0.81769	0.90909	8.4920
	-20	12	0.76639	0.8333	-7.5335
	-50	13	0.74690	0.76923	-16.1971
$H$	+50	11	0.8875	1.3636	19.4304
	+20	11	0.8569	1.0909	6.1348
	-20	12	0.6548	0.6667	-1.5022
	-50	13	0.1829	0.3846	7.2966
$D$	+50	11	-1.17736	0.90909	0.06529
	+20	12	0.11667	0.8333	0.0000
	-20	12	1.259	0.8333	0.0000
	-50	13	1.8625	0.7692	0.8149

The following inferences can be made from the sensitivity analysis based on Table – II.

- (1) In Table – II, as the internal replenishment cost  $A$  increases by 50%, the optimal cost  $C^*(n^*, k^*)$  increases by 28.23%. For a 50 % decrease in  $A$ , the optimal cost  $C^*(n^*, k^*)$  also decreases by 32%. This is a very natural phenomenon in practical situations.
- (2) When internal and external inflation rates  $i_1$  and  $i_2$  increase by 50%, the optimal cost  $C^*$  increases by 0.06%. But when  $i_1$  and  $i_2$  decreases by 50%, the optimal cost  $C^*$  decreases by 0.81%. Thus the changes in  $i_1$  and  $i_2$  have very little effect on  $C^*$ .
- (3) We observed that when discount rate  $r$  increases by 50%, the optimal cost  $C^*$  decreases by 9% and when  $r$  decreases by 50%, then  $C^*$  increases by 25%.
- (4) When scale parameter  $\alpha$  is increasing, the optimal cost  $C^*$  is increasing. But when  $\alpha$  is decreasing, the optimal cost  $C^*$  is decreasing.
- (5) If the shape parameter  $\beta$  is increasing (or decreasing), then the optimal cost  $C^*$  is increasing (or decreasing).
- (6) When purchase cost  $p$  and time horizon  $H$  are increasing (or decreasing), the optimal cost  $C^*$  is increasing (or decreasing). Thus increase (or decrease) of purchasing cost and time horizon causes increase (or decrease) of cost  $C^*$ .

The percentage change in the total cost indicates that the model is moderately sensitive in the values of the parameters  $A$ ,  $r$ ,  $\alpha$ ,  $p$  and  $H$ , while it has very low sensitivity in  $\beta$ . It is insensitive to changes in the parameters  $i_1$ ,  $i_2$ ,  $c_{11}$ ,  $c_{12}$ ,  $c_{21}$ ,  $c_{22}$  and  $D$ .

## 7. CONCLUSION

A contemporary area of inventory research involves situations in which the demand rate is dependent on the level of inventory. This paper investigates a model for inventory systems with an inventory – level – dependent demand rate. At the present time, the presence of inventory has motivational effect on the people around it. This observation has attracted the interest of researchers in marketing and behavioural sciences.

The occurrence of shortages in inventory is a very natural phenomenon in real situations. Shortages in the inventory are considered in this model. Today the economy of different countries is in the grip of large-scale inflation and consequently there is sharp decline in the purchasing power of money. Effect of inflation and time-value of money can no longer be ignored in the present economy. For that reason, the effect of inflation and time-value of money are taken into account in this model. A numerical example shows that our algorithm is rather accurate and rapid. The sensitivity of the solution to changes in the values of different parameters has been discussed.

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### APPENDIX – I

The present worth of the holding cost over the period  $[(j-1)T, jT]$ , ( $j = 1, 2, 3, \dots, n$ ), is given by  $H_j = I_{j1} + I_{j2}$ .

Where

$$\begin{aligned} I_{jm} &= c_{1m} \int_{(k+j-1)T}^{jT} \{t - (k+j-1)T\} \alpha i^\beta e^{-R_m t} dt \\ &= c_{1m} \int_{(k+j-1)T}^{jT} \{t - (k+j-1)T\} \alpha \{\alpha(1-\beta)(jT-t)\}^{\frac{\beta}{1-\beta}} e^{-R_m t} dt \\ &= c_{1m} \alpha^{\frac{1}{1-\beta}} (1-\beta)^{\frac{2-\beta}{1-\beta}} e^{-R_m jT} \left[ \frac{1}{2-\beta} \{(1-k)T\}^{\frac{2-\beta}{1-\beta}} + \frac{R_m}{(2-\beta)(3-2\beta)} \right. \\ &\quad \left. \{(1-k)T\}^{\frac{3-2\beta}{1-\beta}} + \frac{R_m^2}{2(3-2\beta)(4-3\beta)} \{(1-k)T\}^{\frac{4-3\beta}{1-\beta}} \right] \end{aligned}$$

neglecting small quantities above the second order where  $R_m = r - i_m$ ,  $m = 1, 2$ .

### APPENDIX – II

The present worth of the purchase cost for purchasing at time  $t = (k+j-1)T$  for the period  $[(j-1)T, (k+j-1)T]$  and at time  $t = (k+j-1)T$  for the period  $[(k+j-1)T, jT]$ , ( $j = 1, 2, \dots, n$ ), is

$$\begin{aligned} P_j &= p e^{-R_2(k+j-1)T} \int_{(j-1)T}^{(k+j-1)T} D dt + p e^{-R_2(k+j-1)T} \int_{(k+j-1)T}^{jT} \alpha i^\beta dt \\ &= D p k T e^{-R_2(k+j-1)T} + p \{\alpha(1-\beta)\}^{\frac{1}{1-\beta}} e^{-R_2(k+j-1)T} \{(1-k)T\}^{\frac{1}{1-\beta}} \end{aligned}$$

where  $R_2 = r - i_2$ .

### APPENDIX – III

The present worth of the shortage cost during  $[(j-1)T, (k+j-1)T]$ , ( $j = 1, 2, \dots, n$ ), is given by

$$S_j = J_{j1} + J_{j2}$$

Where

$$\begin{aligned} J_{jm} &= c_{2m} \int_{(j-1)T}^{(k+j-1)T} \{(k+j-1)T - t\} D e^{-R_m t} dt \\ &= \frac{D c_{2m}}{R_m^2} \left[ R_m k T e^{-R_m(j-1)T} + e^{-R_m(k+j-1)T} - e^{-R_m(j-1)T} \right]; \quad m = 1, 2 \end{aligned}$$

**APPENDIX – IV**

Since there are  $n$  replenishments in the entire time horizon  $H$ , the present worth of the total replenishment cost is given by

$$C_R = A \sum_{j=1}^n e^{(i_1 - r)(k+j-1)T} = A \sum_{j=1}^n e^{-R_1(k+j-1)T},$$

where  $R_1 = r - i_1$ .

On simplification and summation, we get

$$C_R = A \frac{e^{-R_1 k T} (1 - e^{-R_1 H})}{(1 - e^{-R_1 T})}$$