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AN OPTIMIZATION PROCEDURE FOR WATER RESERVOIRS IN CASCADE

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Abstract. A water resource system of up to four reservoirs is optimized. The planning problem is solved by a two-stage optimization procedure. At the first stage the optimal values of irrigation area and installed power capacity of system are determined subject to allowed deficits. The allowed deficits are given for irrigation and total primary energy production. At the second stage the multistate problem of optimal control is solved applying dynamic programming. The criterion for optimal control is maximum benefit. At the second stage the values of irrigation area and installed power are modified subject to allowed deficit, and the second stage is run again, until the computed deficits become smaller than or equal to the allowed.

Key words and phrases: optimization, water reservoirs, allowed deficits, dynamic programming.

1. INTRODUCTION

A formal optimization method requires an explicit mathematical formulation of objective and constraints. A relatively large amount of simplification might be involved in the application of an optimization method, which means that it should be used with a considerable insight. This insight should also be transferred to the decision maker in order to avoid meaningless or misleading results. On the other hand, it may provide, for a certain complex planning problem a good means to prepare a synthesis over a large number of alternatives. The aim in applying the optimization methods should be to make possible better decisions by supplying better information to the decision maker about the consequences and impacts of a chosen strategy and decisions.

The influential factors in the application of water resources optimization methods are the following: 1) less and less water of good quality has to be used by more and more consumers, so the water resource system has to be multipurpose; 2) the water resources systems are becoming more complex, including engineering, social and environmental elements; 3) the number and type of decision makers involved;

4) the attitude towards uncertainty and risk, associated with water resources development.

Dynamic programming is a powerful tool for optimizing many types of water resources systems. The limitations of dynamic programming (DP) are the number of state variables required and the level of discretization of these variables needed to achieve a satisfactory description of the physical system. For a multireservoir system there is a state variable for each reservoir storage. For this reason, DP can be applied to the system with up to three reservoirs.

There is extensive literature on the application of dynamic programming. The paper by Yeh (1985) reviewed the state-of-the-art of management models developed for reservoir operations. DP developed by Bellman (1957) requires discretizing the state and control variables. The main problem with the optimization of large-scale systems is the "curse of dimensionality". The application of DP for a multireservoir system is limited by the number of reservoirs (three to four). The optimal operation of multireservoir power system is considered by Turgeon (1980). Much of the research reported in the area of optimization for multireservoir system operation has been based on the assumption that the events (like inflows) are known (deterministic models). The use of stochastic dynamic programming for reservoir management is presented in the paper (Trezos and Yeh, 1987).

In the following text multireservoir system planning is considered as an extension of the optimization procedure described by Opricović et al. (1991) (for a single reservoir). The new optimization procedure is developed for water reservoirs in cascade.

2. PROBLEM STATEMENT

The multireservoir system planning is considered as the problem of determining the system parameters and the optimal control, according to given criterion and constraints. The planning (design) problem is solved for NC reservoirs in the cascade, but the optimal control problem is solved for NR reservoirs, where $NR \leq NC$. The controls of $NC - NR$ reservoirs are known (for example, run-of-river). A scheme of the system is presented in Figure 1.

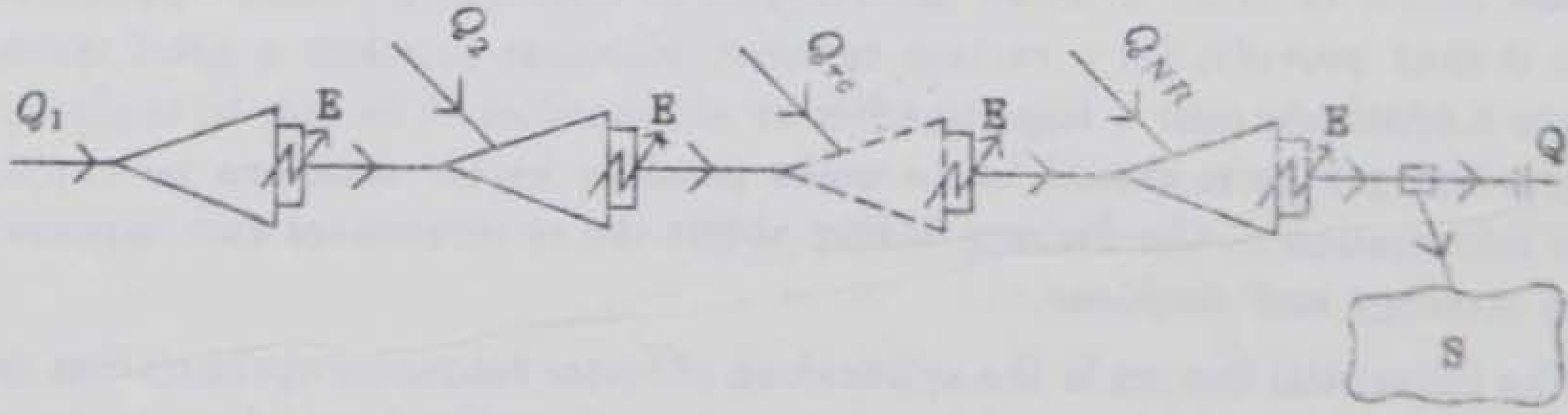

Fig. 1. The cascade

The system consists of a river with a deterministic flow, water reservoirs, tributaries (natural inflows), hydropower stations, and irrigation area. The river flow

(Q_1) is the inflow into the upstream reservoir. The water from each reservoir is delivered for hydroenergy production (E). Downstream of the cascade the water is used for irrigation of the area S, and for downstream purposes (QM). The rc-th reservoir is a run-of-river one.

The optimization task is to determine the installed power P for the cascade and the area S for irrigation, as the "design" parameters (design problem), and optimal values of water delivered for hydroenergy and irrigation in each month m, $m = 1, \ldots, M$ (control problem). In the design problem the values of S and P are determined subject to allowed deficit for irrigation and energy production. In the control problem the sum of benefits by irrigation and energy production is maximized for certain S and P.

The optimization problem is formulated as follows:

$$\max_{S,P,U} \sum_{m=1}^{M} B_m(S,P,U) \qquad \text{subject to the constraints.}$$

This problem is decomposed into two problems:

I — Design problem:

$$\max_{S,P} B^*(S,P)$$

subject to allowed deficits, which introduce "design" constraints;

II — Control problem:

$$B^*(S,P) = \max_{U} \sum_{m=1}^{M} B_m(S,P,U)$$

subject to "control" constraints (constrained water storages and water releases).

This decomposition enables the development of a two-stage optimization procedure. The control problem is a "subproblem" of design problem. The optimization model with constraints is described in the following section.

In developing the mathematical model for optimization the following symbols are used:

Q — natural inflow into water reservoir,
V — water volume in the reservoirs (state vector),
U — water delivered from the reservoirs (control vector),
B — benefit of multireservoir system management,
S — irrigation area (unknown),
P — installed system power capacity (unknown),
m — month (index in the time series),
M — number of months in the time horizon,
r — reservoir index.

NC — number of reservoirs in the cascade,

rc — order number of reservoir with constant storage (run-of-river),

NR — number of reservoirs without rc-th reservoir,

E — hydroenergy,

h_m — the number of operational hours for primary (peak) energy production in m-th month,

DE — primary energy demand,

ED — allowed deficit in primary energy production,

D_m — unitary water demand for irrigation of area S (m^3/ha),

ID — allowed deficit in irrigation of area S,

QM — release for downstream purposes.

$*$ — indicates the optimal value.

3. OPTIMIZATION METHODOLOGY

A multireservoir system is considered. It is assumed that the water from each reservoir is used for energy production. If there is the water demand with a high priority from whichever reservoir, it has to be subtracted from the inflow. The last (downstream) reservoir is a multipurpose, one.

The water resource of cascade has to be used for energy production, for irrigation of area S, and for completely satisfying the demands with high priority.

A two-stage optimization procedure is developed. At the first stage the optimal values of irrigation area S and of installed power P of cascade are determined subject to allowed deficits. The allowed deficits are given for irrigation and primary energy production in the cascade. At the second stage the problem of optimal control of reservoirs is solved applying dynamic programming. The structure of the optimization algorithm OPREC (OPtimization of REservoirs in Cascade) is presented in Figure 2. At the first stage (SOLDEF) the optimal parameters S and P are determined by an iterative procedure, satisfying the constraints of allowed deficits ("design" constraints). At the second stage (DYNPRO) the control problem is solved for the given values of S and P from the stage SOLDEF. The input data are given for: inflows, allowed deficits, demands, unit benefits, and system constants.

At the first stage the system of four reservoirs is optimized, but at the second stage the control problem is solved for three reservoirs, since the control for the rc-th reservoir is given and known. The algorithm OPREC may be applied to a cascade with more reservoirs with known controls ($NC - NR > 1$) and the optimal control problem is solved by dynamic programming for NR reservoirs.

3.1. The first optimization stage — SOLDEF

The main objective of the first optimization stage is satisfying the allowed deficits for irrigation and energy production.

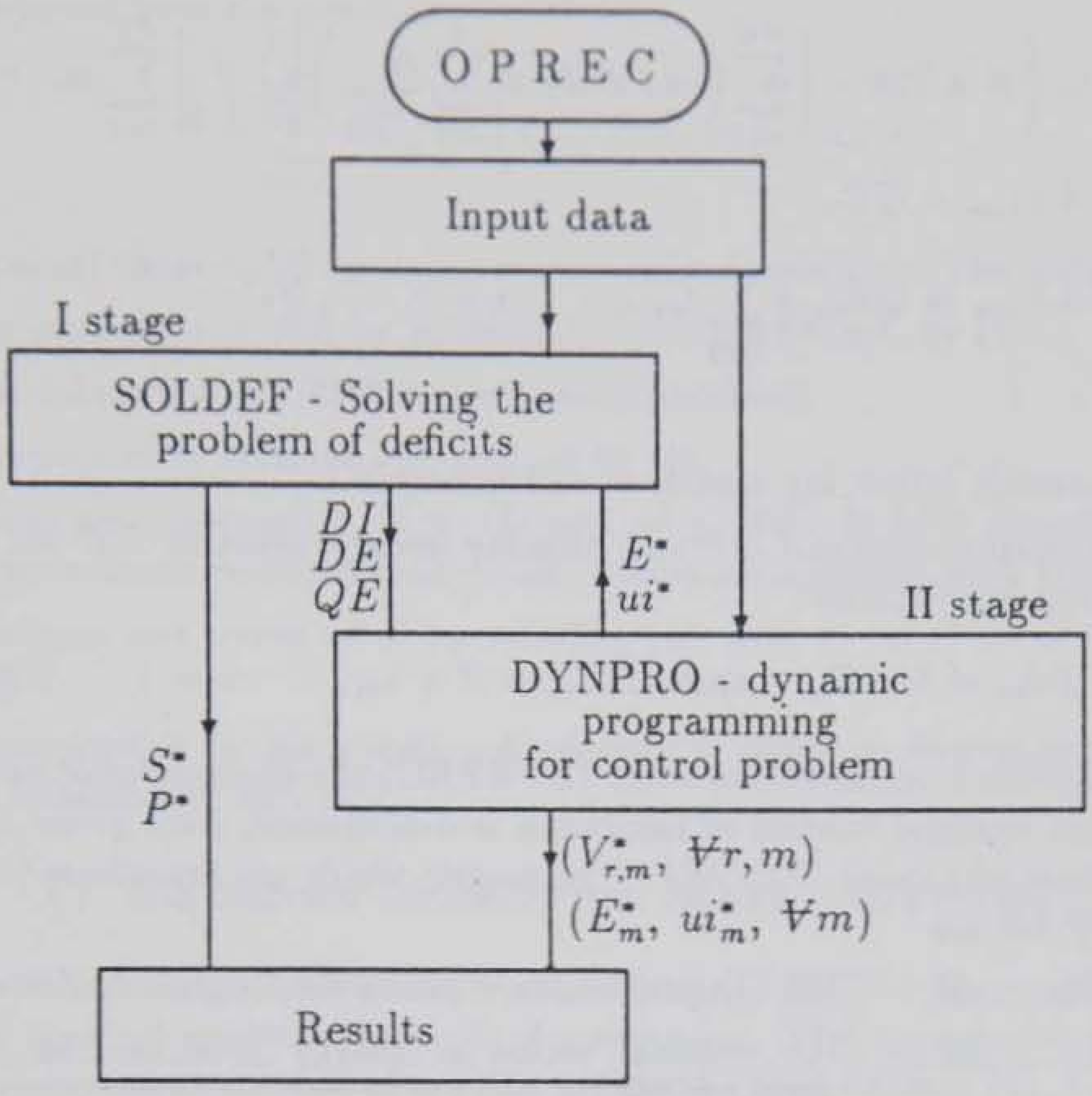

Fig. 2. The structure of OPREC

At the first optimization stage S and P are unknown (parameters in "design" problem). The allowed deficits are given: ID — for irrigation, and ED — for primary energy production.

The algorithm at the first optimization stage (SOLDEF) starts with a maximum possible value for irrigation area S and for installed power P. The initial values of S and P are:

$$S = \left[\sum_{m=1}^{M} \sum_{r=1}^{NC} Q_{r,m} \right] \Big/ \sum_{m=1}^{M} D_m$$
$$P = \left[\sum_{m=1}^{M} \sum_{r=1}^{NC} \left(ce_r \times H_r \times \sum_{i=1}^{r} Q_{i,m} \right) \right] \Big/ \left[\sum_{m=1}^{M} h_m \right] \tag{1}$$

where

ce — efficiency (constant)

H_r — the power head.

With the computed values of S and P, the discharge capacity of turbines

QEMAX is determined.

$$QE_m = \left\{ P \times 720 - \left[\sum_{r=2}^{NC} \left(ce_r \times H_r \times \sum_{i=2}^{r} \overline{Q}_{i,m} \right) \right] \right\} / \left[\sum_{r=1}^{NC} ce_r \times H_{r,m} \right] \tag{2}$$

$$QEMAX_{1,m} = QE_m$$

$$QEMAX_{r,m} = QE_m + \sum_{i=2}^{r} \overline{Q}_{i,m}, \qquad r = 2, \ldots, NC$$

where

$\overline{Q}_{i,m}$ — average inflow for month m and reservoir i.

The irrigation demand DI and primary energy demand DE for the cascade are computed in SOLDEF:

$$DI_m = S \times D_m \quad \text{and} \quad DE_m = P \times h_m, \qquad m = 1, \ldots, M \tag{3}$$

At the second optimization stage (DYNPRO) the time series of net inflows are used and the optimal control of reservoirs is determined, with given DI, DE and $QEMAX$ from the stage $SOLDEF$. The results which are transferred to SOLDEF from $DYNPRO$ are

ui^*_m, $m = 1, \ldots, M$ — the optimal water volumes for irrigation of area S,

E^*_m, $m = 1, \ldots, M$ — the optimal values of energy produced in the cascade, $E^*_m = \sum_{r=1}^{NC} E^*_{r,m}$.

The deficits are computed by the following relations:

$$id(S) = 1 - \sum_{m=1}^{M} ui^*_m \Big/ \left(S \times \sum_{m=1}^{M} D_m \right) \tag{4}$$

$$ed(P) = 1 - \sum_{m=1}^{M} EP^*_m \Big/ \left(P \times \sum_{m=1}^{M} h_m \right) \tag{5}$$

where EP^*_m is the optimal primary energy:

$$EP_m* = \begin{cases} E^*_m, & \text{if } E^*_m < DE_m \\ DE_m, & \text{if } E^*_m \geq DE_m. \end{cases}$$

The primary energy demand DE is determined by relation (3), with given operational hours h_m for primary energy. The energy surplus $E^*_m - EP^*_m$ is considered as secondary energy.

Then the constraint $id \leq ID$ is checked, ID is a given allowed deficit. If this constraint is not satisfied, the new value of S is computed from the following relation

$$S = \sum_{m=1}^{M} ui^*_m \Big/ \left((1 - ID) \times \sum_{m=1}^{M} D_m \right). \tag{6}$$

Also, the constraint $ed \leq ED$ is checked. If this is not satisfied, the new value of P is computed from the relation:

$$P = \sum_{m=1}^{M} EP_m^* \Big/ \left((1 - ED) \times \sum_{m=1}^{M} h_m \right). \tag{7}$$

Each (next) iteration of optimization procedure consists of the following steps:

1. Compute DI and DE by relations (3), and $QEMAX$ by (2).
2. Run subroutine $DYNPRO$ (for optimal control).
3. Compute id by relation (4), and ed by (5).
4. Check the constraints: $id \leq ID$ and $ed \leq ED$. If both are satisfied, the optimization procedure terminates, otherwise continue with step 5.
5. Compute new value of S by relation (6), and P by (7). Continue with step 1.

The convergence of this iterative procedure is based on decreasing value of S and P (by relations (6) and (7)).

3.2. The Second Optimization Stage — DYNPRO

At the second optimization stage (subroutine DYNPRO) the problem of optimal control is solved applying dynamic programming. The state vector is the water storage in reservoirs at the end of months within the time horizon $(V_1, V_2, \ldots, V_M)$. The water storages in the rc-th reservoir are known (given). The algorithm with there state variables is developed.

The criterion for optimal control of cascade is maximum benefit. The criterion (objective) function has the following form:

$$F = \sum_{m=1}^{M} (BI_m + BE_m)$$

where

$$BI_m = bi_m \times ui_m$$

$$BE_m = \begin{cases} bep_m \times E_m, & \text{if } E_m \leq DE_m \\ bep_m \times DE_m + bes_m \times (E_m - DE_m), & \text{if } E_m > DE_m \end{cases}$$

$$E_m = \sum_{r=1}^{NC} E_{r,m}$$

$$E_{r,m} = ce_r \times ue_{r,m} \times H_{r,m}$$

$$H_{r,m} = H_r(0.5 \times (V_{r,m-1} + V_{r,m})) - HTV_r;$$

BI_m — denotes the irrigation benefit within the m-th mouth.

BE — energy benefit

b_i — unitary irrigation benefit

u_i — water volume used for irrigation

bep — unitary benefit of primary energy

bes — unitary benefit of secondary energy $E_m - DE_m$ (if positive)

E_r — energy from r-th hydrostation

V_r — water storage in r-th reservoir

ue — water volume used for energy production

$H(V)$ — "the storage curve"

HTW — tailwater elevation

The dynamics of the reservoir storages are described by the following equations:

$$V_{r,m} = V_{r,m-1} + Q_{r,m} + u_{r-1,m} - u_{r,m} - g_{r,m}; \qquad \forall r, m.$$

The initial water storages in reservoirs ($V_{r,0}$) are given as input data.

By rearranging the state equations, the control variables $u_{r,m}$ can be expressed as function of the reservoir states $V_{r,m}$

$$u_{r,m} = \sum_{i=1}^{r}(V_{i,m-1} - V_{i,m} + Q_{i,m} - g_{i,m}); \qquad \forall r, m.$$

The evaporation losses are computed by the following relation

$$g_m = ev_m \times A(0.5(V_m + V_{m-1}))$$

where

ev — evaporation unit (mm/ha)

$A(V)$ — relationship between surface and storage of reservoir.

The state constraints are:

$$WMIN_r \leq V_{r,m} \leq WMAX_{r,m}; \qquad \forall m, r \in NR$$

where

$WMIN$ — minimal storage ("dead space")

$WMAX$ — maximal storage (the reservoir capacity, or less if some storage is reserved for flood control).

The conditions and constraints in the control space are:

$$ue_{r,m} \leq QEMAX_{r,m}, \qquad \forall r$$
$$u_{NR,m} \geq QM_m + ui_m$$
$$ui_m \leq DI_m.$$

This set of constraints has to be satisfied for all m.

At the second optimization level the control problem is

$$\max_{[V_{r,m}]} F(V_{r,m}, \forall r, m)$$

subject to a set of state and control constraints.

The cascade trajectory is the state vector $(V_{r,m}, \forall r, m)$ within the time horizon, and for NR reservoirs.

By the principle of optimality the recurrence relation is derived in the following form

$$F_m(V_{r,m}, \forall r) = \max_{(V_{r,m-1}, \forall r)} [B_m(V_{r,m-1}, V_{r,m}, \forall r) + F_{m-1}(V_{r,m-1}, \forall r)] \qquad (8)$$
$$m = 2, \dots, M$$

$$F_1(V_{r,1}, \forall r) = B_1(V_{r,0}, V_{r,1}, \forall r)$$

where $B_m = BI_m + BE_m$ is the total benefit within the m-th month.

The forward dynamic programming algorithm is applied.

Let $V_{m-1}^{+}(V_m)$ denote the value of vector V_{m-1} for which the maximum in (8) is obtained. All the $V_{m-1}^{+}(V_m)$, $m = 1, \dots, M$, are stored in the external computer memory.

The optimal value of state vector at the end of time horizon V_M^* is determined by

$$F = \max_{V_M} F_M(V_M)$$

subject to $V_{r,M}^* \geq V_{r,0}$, $r \in NR$.

The optimal values of state vector are determined from the stored data by the following vector relation

$$V_m^* = V_m^{+}(V_{m+1}^*), \qquad m = M-1, M-2, \dots, 1. \qquad (9)$$

After the optimal trajectory of cascade is determined, which is $(V_{r,1}^*, V_{r,2}^*, \dots, V_{r,M}^*; r \in NR)$, the optimal control is determined as follows:

$$u_{r,m}^* = V_{r,m-1}^* + Q_{r,m} + u_{r-1,m}^* - g_{r,m} - V_{r,m}^*,$$
$$ue_{r,m}^* = \begin{cases} 0 & \text{if } 0.5 \times (V_{r,m}^* + V_{r,m-1}^*) < WMIN_r \\ u_{r,m}^* & \text{if } 0 \leq u_{r,m}^* \leq QEMAX_{r,m} \\ QEMAX_{r,m} & \text{if } u_{r,m}^* > QEMAX_{r,m} \end{cases}$$
$$E_{r,m}^* = ce_r \times ue_{r,m}^* \times (H_r(0.5 \times (V_{r,m-1}^* + V_{r,m}^*)) - HTW_r)$$
$$ue_m^* = u_{NR,m}^* - QM_m$$

for $m = 1, \dots, M$ and $r = 1, \dots, NC$.

For the rc-th reservoir the storages $(V_{rc,1}, \dots, V_{rc,M})$ are known (given).

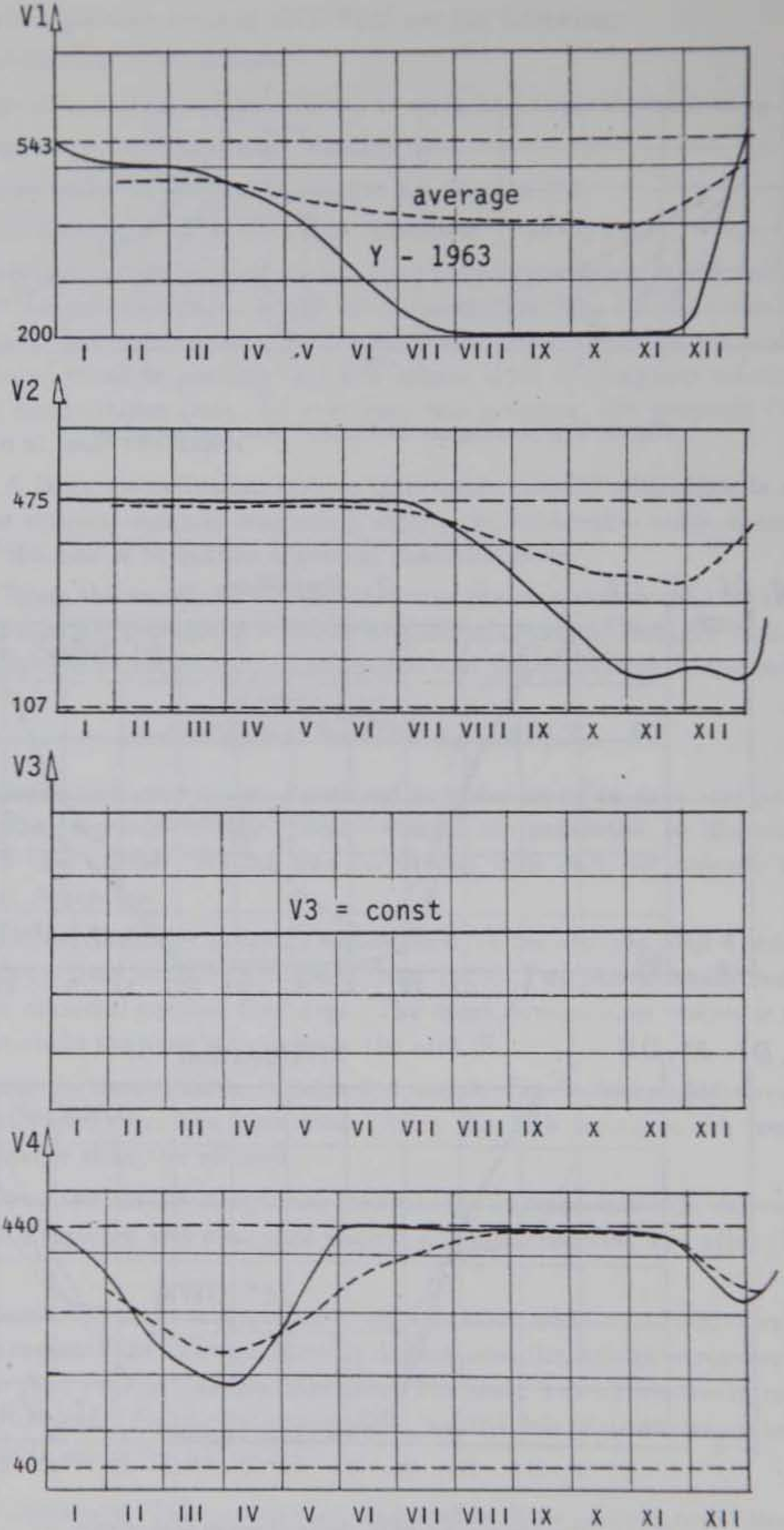

Fig. 5. The water volumes in reservoirs

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