LIQUIDITY MANAGEMENT AND FUTURES HEDGING UNDER DEPOSIT INSURANCE: AN OPTION-BASED ANALYSIS*

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Abstract: Theories on financial futures hedging are generally based on a portfolio-choice approach. This paper presents an alterative: a firm-theoretic model of bank behavior with financial futures under deposit insurance. Assuming that the bank is a certificate of deposit (CD) rate-setter and faces random CDs, expressions for the optimal futures hedge are derived under the option-based valuation. When the bank is in a bad state of the world, a decrease in the short position of the futures decreases the loan rate and increases the CD rate; an increase in the deposit insurance premium increases the loan rate and decreases the CD rate. We also show that the bank's amount of futures increases with a lower expected futures interest rate.

Keywords: Liquidity, futures, deposit insurance, Black-Scholes valuation.

1. INTRODUCTION

It is widely recognized that liquidity management is on the front line of the banking business. A number of money-center banks have experienced liquidity problems

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that inevitably involve estimating fund needs, which is related to net deposit flows and varying levels of loan commitments, or meeting liquidity needs, which is related to asset liquidity and liability liquidity. Bank asset-liability managers are especially concerned with stabilizing their margin, or the spread between their bank's interest revenues and interest expenses. As liquidity management is important to bank profitability, the issues of how it is optimally determined and how it adjusts to changes in the banking environment deserve closer scrutiny.

Two divergent approaches have been employed in the literature to model the liquidity management. A number of writers, for example, Flannery (1994), Qi (1998), Diamond and Rajan (2001), and Kashyap, Rajan, and Stein (2002), have adopted the concept of the coexistence of lending and deposit-taking as their analytical apparatus. The principal advantage of this concept is the explicit treatment of intermediary behavior in estimating fund needs, which has long played a prominent role in bank liquidity management discussions. More specifically, Kashyap, Rajan, and Stein demonstrate that since banks often lend via commitments, their lending and deposit-taking may be two manifestations of one primitive function: the provision of liquidity on demand. Their demonstration emphasizes banks as liquidity providers and that there is a real synergy between lending and deposit taking.

However, if one takes the view that there is no synergy, the fact that banks engage in both activities might be explained as resulting either a past or current distortion in the regulatory environment. For example, Gorton and Pennacchi (1992) argue that deposit insurance has encouraged an artificial gluing together of those two activities when banks attempt to maximize the value of the insurance put option by engaging in risky lending. Under this argument, one naturally recognizes a "narrow banking" proposal of asset liquidity and liability liquidity, which effectively calls for the breaking up of banks into separate lending and deposit-taking operations that assemble finance companies and mutual funds, respectively. Koppenhaver (1985), Bryan (1988), Litan (1988), Mullins and Pyle (1994), and Lin (2000) contributed to the literature on narrowbanking arguments along these lines.

The liquidity management principal is based on a portfolio-theoretic approach. This approach has an advantage of explicitly treating uncertainty. It however, omits a key aspect of either the provision of liquidity on demand or the narrow-banking management on liquidity as mentioned previously. This paper presents an alternative: a firm-theoretic model of a money-center bank's liquidity management with financial futures under deposit insurance. The firm-theoretic approach to banking is used here to develop an anticipatory hedging strategy specific to loan commitment funding risks. We thus argue that whether or not there is a real synergy between deposit-taking and lending is no longer a critical issue in our model, and thus that large inefficiencies that switch to banks as liquidity provider from narrow banking are avoidable.¹

The comparative static properties of this model are investigated to determine the influence of futures contract and deposit insurance on optimal rate-setting decisions. The results from this model show how the bank's anticipatory a state of the world, the expectation of the futures rate, and deposit insurance policy jointly determine the bank's

¹ Kashyap, Rajan, and Stein (2002, p.34) pointed out that if there is a real synergy between deposittaking and lending, a forced switch to narrow banking (no synergy) could lead to large inefficiencies.

loan rate, certificate of deposit (CD) rate, and futures position. We demonstrate that when a bank anticipates a bad state of the world, a decrease in the short position of the futures contract decreases its loan rate and increases its CD rate. When the bank is in a bad state of the world, an increase in the deposit insurance premium increases the bank's loan rate and decreases the bank's CD rate. In addition, the optimal hedge decreases with higher expected futures interest rates. Thus, this paper contributes to the literature on the anticipatory hedging of bank liquidity under the option-based valuation because deposit quantity risk in the anticipatory hedging strategy.

This paper is organized as follows. Section 2 develops the basic structure of the model. Section 3 derives the model solution and the comparative static analysis. Section 4 presents the firm-theoretic model with futures trading. Section 5 concludes the paper.

2. THE MODEL

The framework used here is related to the single-period model developed by Crouhy and Galai (1991). Similar to Crouhy and Galai's model, the original depositors are offered a market rate R_D on their demand deposits. The promised payment to depositors at the end of the period is $(1+R_D)D$ when the bank raises D in deposits at the beginning of the period. In the model below, it is assumed that the bank obtains funds in the form of an imperfectly competitive large denomination negotiable CDs, B. We follow Slovin and Sushka (1983) and assume that the bank is a rate setter in its CD market. The CD supply is an upward-sloping function of the CD rate R_B , $\partial B / \partial R_B > 0$. The bank is fully insured by the Federal Deposit Insurance Corporation (FDIC) and it pays an insurance premium P per dollar of total deposits.²

We assumed that the variable operative costs associated with serving loans and deposits and the fixed costs are omitted for simplicity. This assumption is frequently used in the literature.³ The total gross costs at the end of the period are given using:

$$Z = (1 + R_B)B(R_B) + (1 + R_D)D + P(B(R_B) + D)$$
(1)

The bank is assumed to hold a homogeneous class of loans. The bank acts as a rate setter in the loan market so that loan demand *L* is a downward-sloping function of the loan rate R_L , $\partial L / \partial R_L < 0$. This assumption implies that the bank exercises some monopoly in its loan market. This would be characteristic of an imperfect loan market in which a bank has a reason to exist.⁴

During the period, the total number of loans operated by the bank is $L(R_L)$, which is invested in risky lending assets. Loans granted by the bank belong to the fixed-

 $^{^2}$ For simplicity, we modeled a fixed fee paid by the bank for deposit insurance. Gianmarino, Lewis, and Sappington (1993) show how more sophisticated fee schemes can be used to reduce the moral hazard.

³ See, for example, Crouhy and Gailai (1991).

⁴ See, for example, Wong (1997).

rate claims with an unspecified maturity greater than one period. At any time t $(0 \le t \le 1)$, the value of the bank's risky assets is:

$$V(R_L) \begin{cases} = (1+R_L)L(R_L) & \text{if loan losses} = 0 \\ < (1+R_L)L(R_L) & \text{if loan losses} > 0 \end{cases}$$
(2)

A futures market decision is also a choice variable to the bank at the beginning of the period. The futures decision has an uncertain return since the price of the futures contract at the end of the period is unknown. The bank can hedge loan repayments by taking a long (buy) position in the futures market. In addition, the cost of obtaining a given amount of funds in the CD market can be hedged by the sale of futures contracts.

The principle relationships in the model are expressed as follows. The bank's ex ante balance sheet constraint is:

$$L = B + D \tag{3}$$

where *B* is CDs sold (purchased), B > 0(< 0). We note that the balance sheet constraint in equation (3) does not treat futures trading profits (losses) as a source (use) of funds. In reality, a bank's futures position is treated as an off-balance sheet item with trading profits and losses appearing in its income statement.

The bank's equity value at the end of the period is the sum of gross revenues from the loan and futures position minus the total gross costs in equation (1). The total gross revenues in the model are expressed as:

$$A = V(R_L) + [(1 - R_X) - (1 - R_X)]X$$
(4)

where X is the size of the futures position, R_X is the futures contract interest rate at the beginning of the period (and thus its price of the futures contract is $1-R_X$), and \tilde{R}_X is the futures contract interest rate at the end of the period (and thus its price is $1-\tilde{R}_X$). Thus, we can state the bank's equity value at the end of the period as the residual value of the bank after meeting all of its debt obligations and show as:

$$S = \max\left\{0, A - Z\right\} \tag{5}$$

The sequence of decision-making is assumed to occur in two distinct stages. In the first stage, the futures contract is determined and remains fixed for the remainder of the bank management period. In the second stage, the bank's futures contract is revealed and its rate-setting takes place. As we see below, the bank's futures contract influences its rate-setting decisions. The rate-setting decisions are made considering the futures choices made in the first stage. Conversely, in the first stage, a futures contract decision is made bearing in mind its impact on the equilibrium of the rate-setting stage. The equilibrium for the framework is sequential since a sub-stage is characterized by equilibrium and the solution of the first stage takes into account the fact that equilibrium occurs in the second stage. A multistage framework is usually solved using backward induction. Applying that method here, we solve for the rate-setting stage first, and then examine the futures contract stage. Accordingly, the bank's objective is to choose R_L and R_B as well as X sequentially to maximize the expected market value of its equity S, subject to the balance sheet constraint in equation (3) and its expectations about the future.

Let these expectations be described by Black and Scholes' (1973) option pricing model. In the Black-Scholes valuation, we treat the market value of the bank's equity capital as the call option effectively purchased by the bank's shareholders. Following Mullins and Pyle (1994), our model prices the bank's equity in two parts: (i) the risk adjusted present value of the bank's total gross revenues minus the promised payment to depositors (only holding CDs), (ii) the risk adjusted present value of the bank's promised payment to depositors (only holding demand deposits) plus deposit insurance costs. Under the circumstances, the current market value during the period can be presented as:

$$\max_{R_L, R_B} S = [A - (1 + R_B)B(R_B)]N(d_1) - \{(1 + R_D)[L(R_L) - B(R_B)] + PL(R_L)\}N(d_2)$$
(6)

where,

$$d_{1} = \frac{1}{\hat{\sigma}} \left[\ln \frac{A - (1 + R_{B})B(R_{B})}{(1 + R_{D})[L(R_{L}) - B(R_{B})] + PL(R_{L})} + \mu + \frac{1}{2}\hat{\sigma}^{2} \right]$$

$$d_{2} = d_{1} - \hat{\sigma}$$

$$\hat{\sigma}^{2} = \hat{\sigma}_{v}^{2} + \hat{\sigma}_{1}^{2} - 2\rho_{v,1}\sigma_{v}\sigma_{1}$$

$$\mu = R - R_{D}$$

In the objective function, the cumulative standard normal distribution of $N(d_1)$ and $N(d_2)$ represent the risk adjusted factors of (i) and (ii) above, respectively. $\hat{\sigma}^2$ is the variance with $\hat{\sigma}_{\nu}$ and $\hat{\sigma}_1$, which is the instantaneous standard deviation of the rates of returns on the risky loan and the certificates of deposit, respectively. $\rho_{\nu,1}$ is the instantaneous correlation coefficient between the two components in part (i). μ is treated as the net spread rate, which is defined as the difference between the opportunity costs expressed by the default-free security rate (*R*) and the promised demand deposit rate to the initial depositors.

3. RATE-SETTING STRATEGIES

The first-order conditions for an optimum of equation (6) in the rate-setting stage are given using:

$$[L + (1 + R_L)\frac{\partial L}{\partial R_L}]N(d_1) - (1 + R_D + P)\frac{\partial L}{\partial R_L}N(d_2) = 0$$
(7)

$$[B + (1 + R_B)\frac{\partial B}{\partial R_B}]N(d_1) - (1 + R_D)\frac{\partial B}{\partial R_B}N(d_2) = 0$$
(8)

Sufficient conditions for an optimum are $\Delta_{R_L} = \partial^2 S / \partial R_L^2 < 0$ and $\Delta_{R_B} = \partial^2 S / \partial R_B^2 < 0$. In this section, the optimal loan rate is determined when the CD rate is fixed, and the optimal CD rate is determined when the loan rate is fixed. These non-simultaneous results are obtained for the following reasons. First, a bank frequently encounters situations where loan rate decisions must be made in the presence of fixed deposit rates. Sealey (1980), for example, modelled this behavioral mode. Second, a bank also frequently encounters situations where it has fixed loan rates and must set deposit rates. Stigum (1976) and Sealey, for example, explored this behavioral mode for savings and loan associations.⁵

The first-order conditions in equations (7) and (8) determine the optimal loan rate, CD, and demand deposits of the bank. Equilibrium conditions for the optimum demonstrate that the bank's risk-adjusted present value of marginal revenue of loan rate equals that of marginal cost of loan rate in equation (7), and the bank's risk-adjusted present value of marginal cost (only holding CDs) of CD rate equals that (only holding demand deposits) of CD rate in equation (8). Thus, the optimum integrates the risk conditions of the portfolio-theoretic approach with the market conditions and rate-setting behavioral modes of the firm-theoretic approach.

Having examined the solution to the bank's optimization problem, in the following section, we explore the effects on the optimal rates from changes in the parameters in the rate-setting stage of the model.

Implicit differentiation of equation (7) with respect to the amount of the futures yields:

$$\frac{\partial R_L}{\partial X} = -\frac{1}{\Delta_{R_L}} (1 + R_D + P) \frac{\partial L}{\partial R_L} (\frac{N(d_2)}{N(d_1)} \frac{\partial N}{\partial d_1} - \frac{\partial N}{\partial d_2}) \frac{\partial d_1}{\partial X}$$
(9)

The effect of a change in the amount of the bank's futures on its loan rate depends on two conditions. The first condition is the difference between the distribution elasticity of the risk-adjusted value of the bank's total gross revenues minus the promised payment to CD holders, $\varepsilon_{d_1} = (\partial N / \partial d_1) / (N(d_1) / d_1)$, and the distribution elasticity of the risk-adjusted value of the bank's total gross costs minus the promised payment to CD holders, $\varepsilon_{d_2} = (\partial N / \partial d_2) / (N(d_2) / d_2)$. Both the distribution elasticity represents the bank's risk magnitude for its equity market value. If $\varepsilon_{d_1} > \varepsilon_{d_2}$, we demonstrate that the bank has an increasing risk magnitude for its equity market value, or the bank's equity market value is expected to decrease. Under these observations, we recognize that the

⁵ The dichotomy between the asset and liability sides of bank operations may be broken. As Krasa and Villamil (1992, a, b) pointed out, the bank's "two-sided problem" of loan-rate and CD-rate settings is recognized as an interactive operation in its optimization. This paper emphasizes that the essence of bank management is the integration of the rate-setting strategies and the futures hedging instrument choices. For simplicity, this paper features the dichotomy, rather than the "two-sided problem", to analyze the bank's integration management.

bank is in *a bad state* of the world. However, the bank is in *a good state* of the world if $\varepsilon_{d_1} < \varepsilon_{d_2}$. The observation of this result follows a similar argument as in the case of a bad state of the world.

The second condition is the difference between the futures contract interest rate at the beginning of the period and that at the end of the period. This difference captures the influence of expected futures trading profits on the optimal loan rate in equation (9). If the expectation of the futures rate is greater then the current futures rate, $\tilde{R}_X > R_X$, then the bank will decrease the hedge: that is the bank takes a short (sell) position in the futures market. Under these circumstances, the short position is expected to be more profitable than the long position. But if $\tilde{R}_X < R_X$, the long position is more profitable than the short position, then X increases accordingly.

Proposition 1 *When the bank is in a bad state of the world, a decrease in the short position of the futures decreases the loan rate.*

An explanation of the results for Proposition 1 is possible in terms of the distribution elasticity condition and the expectation of the futures rate as discussed previously. Intuitively, as the bank anticipates a higher uncertainty about its equity value at the end of the world, it is likely to decrease a short futures position to increase its expected futures revenue, and also decrease its loan rate to increase its expected loan revenue. If loan demand is relatively rate-elastic, larger loan revenue is possible at a reduced rate.

Implicit differentiation of equation (8) with respect to the amount of the futures yields:

$$\frac{\partial R_B}{\partial X} = -\frac{1}{\Delta_{R_D}} (1 + R_D) \frac{\partial B}{\partial R_B} (\frac{N(d_2)}{N(d_1)} \frac{\partial N}{\partial d_1} - \frac{\partial N}{\partial d_2}) \frac{\partial d_1}{\partial X}$$
(10)

Proposition 2 *When the bank is in a bad state of the world, a decrease in the short position of the futures increases the CD rate.*

A decrease in the short position for the futures forces the bank to increase its CD rate in order to keep the equity value optimal. The interpretation of this result follows a similar argument as in the case of a change in the loan rate. Basically, the bank will decrease a short futures position to increase its CD rate to decrease the expected interest cost when the bank expects a higher risk about its equity value at the end of period.

Implicit differentiation from equation (7) with respect to the premium paid for deposit for deposit insurance yields:

$$\frac{\partial R_L}{\partial P} = \frac{1}{\Delta_{R_L}} \frac{\partial L}{\partial R_L} \left[\frac{1}{\hat{\sigma}} \left(\frac{N(d_2)}{N(d_1)} \frac{\partial N}{\partial d_1} - \frac{\partial N}{\partial d_2} \right) + N(d_2) \right]$$
(11)

Proposition 3. When the bank is in a bad state of the world, an increase in the deposit insurance premium increases the bank's loan rate.

An explanation of the results of Proposition 3 depends on the distribution elasticity condition discussed previously. An increase in the premium for deposit insurance increases the bank's loan rate if the bank expects higher uncertainty about its market value of equity at the end of the period. In an imperfect loan market, the bank must increase the loan rate to decrease its lending business and thus the expected loan losses. This is because the bank anticipates its equity value with a higher uncertainty. However, if the bank expected its equity value to be a lower uncertainty (the bank is in a good state of the world), then an increase in the deposit insurance premium has an indeterminate effect.

Implicit differentiation of equation (8) with respect to the premium paid for deposit insurance yields:

$$\frac{\partial R_B}{\partial P} = \frac{1}{\Delta_{R_B}} \frac{\partial B}{\partial R_B} \left(\frac{N(d_2)}{N(d_1)} \frac{\partial N}{\partial d_1} - \frac{\partial N}{\partial d_2} \right) \frac{\partial d_1}{\partial P}$$
(12)

Proposition 4 *When the bank is in a bad state of the world, an increase in the deposit insurance premium decreases the bank's CD rate.*

The interpretation of the results of Proposition 4 follows a similar argument as in the case of Proposition 3. Basically, when the bank expects a higher risk about its equity, increases in the cost of deposit insurance encourage the bank to shift funding to its demand deposits from the CDs. In an imperfect certificate of deposit market, the bank must reduce the CD rate in order to decrease the amount of the CDs.

4. FUTURES STRATEGY

The bank maximizes its expected equity capital market value anticipating resolution in the rate-setting stage. In choosing the size of its futures position, the bank takes into account the effect on the rate-setting decisions because these decisions affect the bank's futures contracts (Koppenhaver, 1985). The offered futures contract maximizes the bank's equity value along its first-order conditions in equations (7) and (8). Using the sufficient conditions in $\Delta_{R_L} < 0$ and $\Delta_{R_B} < 0$, we know that $R_L(X)$ and $R_B(X)$ characterize the bank's rate-setting equilibrium as a function of its futures contract. We can substitute in $S(R_L, R_B, X)$ to obtain $S(R_L(X), R_B(X), X)$. This new objective function for the bank incorporates its optimally chosen futures position as it forces the bank to stay on its rate setting reaction functions. This also guarantees that we solve for a sequential equilibrium in the game. Accordingly, the bank solves the following problem, max $S(R_L(X), R_B(X), X)$, when $S(\cdot)$ is defined in equation (6).

The first-order condition of the above objective function shows that the futures contract interest rate at the end of the period is equal to that at the beginning of the period. We note that this equilibrium condition is valid whether or not the bank's "two-sided problem" in Krasa and Villamil's (1992, a, b) sense is recognized as an interactive operation in its asset-liability management. In this first-order condition, the size of the futures position increases with lower expected futures interest rates or with higher current

futures interest rates. The results presented here can be given an intuitive interpretation. In either situation, a long position is more profitable than a short position: the futures position increases accordingly. However, a short position is more profitable than a long position if the current futures interest rate is less than the expected futures interest rate: the futures amount decreases accordingly. To summarize, we have the following proposition:

Proposition 5 *When both the bank's rate and CD rate are optimally pre-determined, its size of the futures position increases with lower expected futures interest rates, or with higher current futures interest rates.*

Koppenhaver shows that expectations of higher futures rates decrease the optimal hedge under constant absolute risk aversion, but has an indeterminate effect under constant relative risk aversion. Proposition 5 is consistent with Koppenhaver's finding in the case of constant absolute risk aversion.

5. CONCLUSION

The purpose of this paper was to apply the hedging theory concepts to the optimal loan rate and the optimal CD rate determinations under deposit insurance. In particular, Propositions 1 and 2 show that a decrease in the short position for the futures decreases the optimal loan rate, and increases the optimal CD rate when the bank anticipates the market value of its equity capital with increasing risk. Changes in the bank's regulatory parameters, such as deposit insurance, have a direct effect on the bank's loan rate and CD rate. Our model in Propositions 3 and 4 show that an increase in the deposit insurance premium increases the bank's loan rate and decreases the CD rate when the bank realizes the market of its equity capital in a bad state of world. The results in Proposition 5 indicate that the bank's futures position size increases with a lower expected futures interest rate or higher current futures interest rate. Our findings provide an alternative explanation for the coexistence of lending and deposit-taking concerning narrow banking hedging behavior under deposit insurance.

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