RELIABILITY ANALYSIS OF A TWO-UNIT STANDBY SYSTEM BY COMPUTER SIMULATION*

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Abstract: We study the two-unit standby system with repair and with preventive maintenance. Preventive maintenance is introduced in order to make the lifetime of the system longer. Using Monte-Carlo method we simulate the work of the two-unit system and we analyze the influence of different types of preventive maintenance on reliability of the system. Monte-Carlo method enables us to find estimates of various parameters relevant to the system for which there exist no explicit formulas in the literature.

Keywords: Reliability, two-unit standby system, maintenance, Weibull distribution, simulation.

1. INTRODUCTION

In Reliability Theory systems consisting of two units with repair and preventive maintenance were investigated by methods of embedded semi-Markov processes or by recurrent equations which gave formulas, in terms of Laplace transforms, for some random variables which characterize the work of the system. There exists extensive bibliography concerning this topic (see [1], [2], [7], [8]), and the interest to investigate various variants of two-unit systems continues ([3], [4], [5], [11], [15]). Two-unit standby system with repair is important from a theoretical point of view too, because it stimulated the study of limit theorems for a random number of random variables, which appear naturally in this context. Chapter 2 of a recent monograph [10] has the title: "Doubling with repair" and it deals with a mathematical model of a two-unit system and with limit theorems related to it. Similar problems are investigated in [6], [9], [12], [13], [14], too. Also, in connection with the two-unit standby system, the possibility of accelerated repair and preventive maintenance (by employing more staff, for instance) and the situation when the repaired units are not as good as new were investigated in [1, 13-15].

We suppose that the active unit works until it breaks down, while the standby unit is inactive and begins to operate only in case of the breakdown of the active unit. The unit that breaks down goes to repair. The reserve is cold, i.e. the unit does not change its properties (it can not fail or deteriorate) while being in the standby state. On order to prolong the lifetime of the system, the active unit is submitted to preventive maintenance

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(which includes inspection and preventive repair) at moments fixed in advance which can be constant or variable (random).

Here we consider a two-unit standby system with repair and with three different types of preventive maintenance: rigid, sliding and economical. Rigid preventive maintenance is applied independent of the state of the standby unit, while the other two maintenance types both depend on the state of the standby unit.

2. DESCRIPTION OF THE SYSTEM

The two-unit system which we are going to investigate by simulating its work on the computer satisfies the following conditions:

- The system consists of two equal units with the same failure characteristics.
- At the beginning, at $t = 0$ one unit starts to work (we shall call it active unit), while the other one is in the standby state (cold reserve). This means that the unit remains unchanged (in particular it can not fail) while it is in the standby.
- In the case of rigid preventive maintenance, at moments (which can be random or non random) fixed in advance the active unit stops to work and undergoes preventive maintenance, independent of the state of the standby unit at that moment. In the case of sliding and economical preventive maintenance, the active unit stops with work in order to undergo preventive maintenance only in case when the standby unit is ready (i.e. when it is not on repair or on preventive maintenance). Otherwise, in the sliding case, preventive maintenance is postponed until the reserve unit is ready, and in the economical case preventive maintenance is rejected (not performed) and the active unit continues to work until it fails.
- After repair and preventive maintenance the unit is as good as new so that all probability distributions characterizing the system are identical to those at the beginning.
- After repair or preventive maintenance the unit remains in the standby as long as the active unit works.
- We assume that the sensing and switch over devices are absolutely reliable.
- We assume that the switch over times, from failure to repair, from repair completion to the standby state and from the standby state to the active state, of each unit is instantaneous, and such are the switch over times occurring in the preventive maintenance too.
- The repair time distribution and the preventive maintenance time distribution are independent of the failure time distribution and of the preventive maintenance time distribution.

Random variables characterizing the two-unit standby system with repair and preventive maintenance are the following:

Z - Time interval of the work without failure of the active unit;

R - Duration of repair;

P - Time interval starting from the beginning of work of the active unit until the preventive maintenance time;

Q - Duration of preventive maintenance.

The system with rigid preventive maintenance has 7 states, while systems with sliding and economical preventive maintenance have 5 states. For all three types of maintenances active states are the following:

E_0 - Both units are in order;

E_1 - One unit is in order and the other one is being repaired;

E_2 - One unit is in order and the other one undergoes preventive maintenance,

In the system with rigid preventive maintenance the remaining four states are those that cause the breakdown of the system:

E_3 - Active unit has failed during the repair of the reserve unit;

E_4 - Active unit has failed during the preventive maintenance of the reserve unit;

E_5 - The time of preventive maintenance of the active unit comes while the reserve unit is on repair;

E_6 - The time of preventive maintenance of the active unit comes while the reserve unit is on preventive maintenance.

For systems with sliding and economical preventive maintenance states that cause the breakdown of the system are only E_3 and E_4. Those two types of preventive maintenances do not allow breakdowns at states E_5 and E_6. Transition Graphs for two-unit standby system with repair and the three types of preventive maintenances are presented on Figure 1.

Rigid preventive maintenance

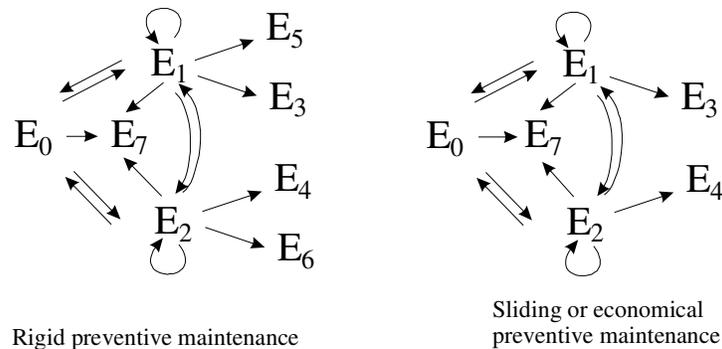Sliding or economical preventive maintenance

Figure 1: Transition graphs

By E_7 we denote the state where we stop to follow the work of our two-unit system because the system has been working without failure during the period of time τ, fixed in advance. We fix in advance the number of simulations N (in the example below $N = 10000$) as well as the time interval (τ) $(\tau = 20000)$ during which we observe the work of our system. This number of simulations allows us to obtain good enough estimates of values we are interested in, because, by Central Limit Theorem, the error (i.e. the absolute value of the difference between the actual value and its estimate) in 99.7% of cases is less or equal than $3\sigma/\sqrt{N}$, where σ is a standard deviation of the given variable calculated from the sample of the size N.

The lifetime of the system is the interval of time until two-unit standby system definitely stops performing its function, i.e. the interval of time until the breakdown of the

system. It is equal to the sum of sojourn times in the states E_0, E_1, E_2 beginning at $t_0 = 0$, from the state E_0.

We are interested in the estimates of the following variables, which are relevant to the work of two-unit system:

- mean lifetime of the system (T_m);
- mean number of failures (i_{rm});
- mean sojourn time under repair (R_m);
- mean number of preventive maintenances (i_{qm});
- mean sojourn time under preventive maintenance (Q_m);
- mean number of postponed preventive maintenances (i_{pos});
- mean number of rejected preventive maintenances (i_{ref});
- the probability $P(E_i)$ that the system breaks down at the state E_i, $i = 3,4,5,6$ (rigid), and the probability $P(E_i)$ that the system breaks down at the state E_i, $i = 3,4$ (sliding and economical);
- reliability function of the system, i.e. $P(T > t)$ (the probability that the system lives longer than t).

In order to analyze the two-unit standby system, we have made the program which simulates its work. The input data are: parameters of probability distributions relevant to the system, number of simulations (N) and length (τ) of time interval during which we observe the work of our system.

The simulation starts by generating random variables using random number generator. We simulate the work of the system under different setups and we estimate the expected lifetime of the system as well as other values characterizing the system.

If we use this program to analyze a concrete two-unit standby system, then the first step is to determine (using statistical methods) failure time distribution, repair time distribution and preventive repair time distribution and then to apply our procedure. Finally, by varying different preventive maintenance strategies, the most adequate preventive maintenance can be chosen.

3. SIMULATION

The program for simulation is illustrated by the following example, which resembles cases that happen often in practice. We suppose that failure time distribution, repair time distribution and preventive repair time distribution which characterize the work of the system have Weibull distribution which is of the form

$$F(x) = 1 - e^{-\lambda x^{\alpha}},$$

where λ, $\alpha > 0$, $0 \leq x < +\infty$, with the following parameters

$$Z : \alpha = 4, \qquad \lambda = 0.002, \qquad E(Z) = 4.286;$$
$$R : \alpha = 6, \qquad \lambda = 1, \qquad E(R) = 0.928;$$
$$Q : \alpha = 7, \qquad \lambda = 20, \qquad E(Q) = 0.61.$$

We have chosen Weibull distribution because it is used in Reliability Theory to describe "aging" elements. Aging of an element means that the failure rate, which, for a given probability distribution $F(t)$, is defined by

$$-\frac{(1-F(t))'}{1-F(t)},$$

increases. Weibull distribution, for $\alpha > 1$, satisfies this condition.

Using pseudo random numbers uniformly distributed in the interval (0,1), we compute random numbers having Weibull distribution in the standard way: let y_i, $i = 1,2,...$ be random numbers uniformly distributed in the interval (0,1), then

$$x_i = \left(\frac{-\ln(1-y_i)}{\lambda}\right)^{\frac{1}{\alpha}}$$

are realizations of a random variable having Weibull distribution with parameters λ, α.

The behavior of the two-unit standby system (described above), but without preventive maintenance is also simulated and obtained results are presented in Table 1, which contains means and corresponding errors ($3\sigma/\sqrt{N}$) for observed variables.

Table 1: Simulation results without preventive maintenance

T_m	R_m	i_{rm}
2555.78	488.71	526.95
Error T	error R	error i_r
95.44	14.67	15.81

In order to establish the appropriate preventive maintenance strategy providing longer life of the system, we tested three different types of preventive maintenances (rigid, sliding and economical) and for each of them we tested three different types of distributions for preventive maintenance time (Weibull (CASE I), Constant (CASE II) and Uniform (CASE III)) and we analyzed the behavior of the system under all these preventive maintenances.

(CASE I): P - time interval from the beginning of work of the active unit until the preventive maintenance time (for short, preventive maintenance) has Weibull distribution with parameters: α ranging from 0.0002 to 0.0048 with the step 0.0004; and λ ranging from 2 to 10 with the step 0.5.
(CASE II): Preventive maintenance is performed at fixed (non random) intervals of lengths starting from 0.5 to 9.5 with the step 0.5.
(CASE III): Preventive maintenance time has Uniform distribution with left endpoints ranging from 0.5 to 9.5 with the step 0.5, and lengths of support intervals range from 1 to 8 with the step 1.

For this particular example preventive maintenance with Weibull distribution (CASE I) was worse than preventive maintenances in the other two cases and in the sequel we concentrate on CASEs II and III.

Figure 2 contains mean values of the lifetime of the system with preventive maintenances described in the CASEs II and III. Constant preventive maintenance can be seen as a degenerated to a point Uniformly distributed preventive maintenance. As can be seen from Figure 2, if preventive maintenance is performed too often, it can shorten the lifetime of the system. Also, if preventive maintenance is rare, then the system behaves as if the preventive maintenance does not exist.
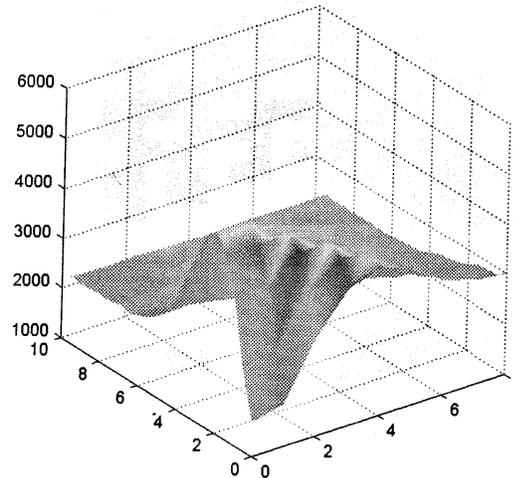

Figure 2: Mean values of the lifetime of the system in CASEs II and III

The best results (displayed in the Table 2 below) are obtained in the case of sliding preventive maintenance with constant (non random) preventive maintenance which is performed after 2.5 time units have elapsed since the beginning of the work of the active unit.

Table 2: Best results

T_m	R_m	Q_m	i_{pos}	i_{rm}	i_{qm}
5051.33	142.20	1157.17	0.06	154.33	1897.57
error T	error R	error Q	error i_p	error i_r	Error i_q
147.76	4.01	32.47	0.01	4.32	53.26

For this case probabilities of the breakdown at states E_3 and E_4 (under the assumption that the system lived less than $\tau = 2000$) are $P(E_3) = 0.287$ and $P(E_4) = 0.713$. Comparing the cases without and with preventive maintenance (see the first columns of Tables 1 and 2 and also Figure 2), we see that with the appropriate preventive maintenance the lifetime of the system can become significantly longer.
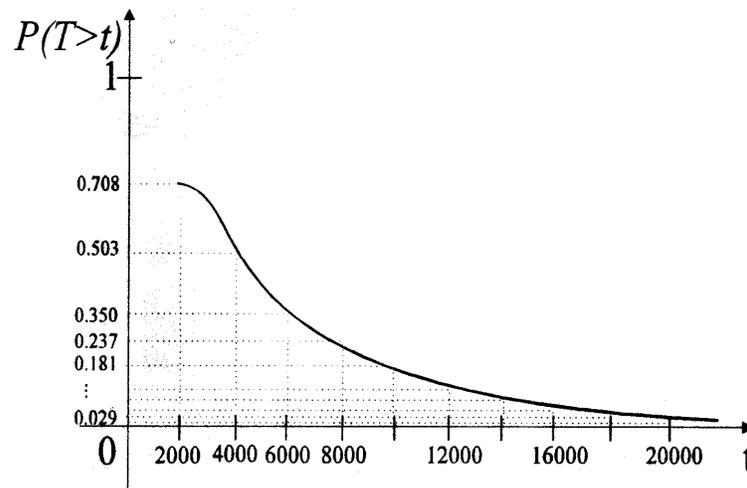
Figure 3: Reliability function for best results

        Obtained results from the example show that the lifetime of this system is longer in the case of non-random preventive maintenance and it is the longest in the case when the preventive maintenance time is less than the expectation of the failure time. Also it can be observed that the majority of breakdowns occur at the state E_4. Reliability function for that system is presented in Figure 3. Flow charts of the simulation of the work of the two-unit standby system are given in the Appendix.

4. CONCLUSION

        For the two-unit standby system with repair and with different preventive maintenances we made the program which simulates its work and which allows us to analyze the reliability of the system depending on different maintenance strategies. The possibilities of the simulation program are illustrated by a concrete example. The program allows the change of probability distributions which describe the system and the change of preventive maintenance in order to find the most adequate one. The advantage of this approach is that it gives estimates for various parameters which are interesting for users and for which there exist no explicit formulas (such as: expected number of repairs, expected number of preventive maintenances, expected sojourn time in the states E_0, E_1, E_2, etc.), which is important in investigation and planning of the system. For the given example we have found optimal variant of preventive maintenance, which in average almost doubles the lifetime of the system comparing to the one without preventive maintenance.

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APPENDIX

Flow chart of the simulation of the work of the two-unit standby system with the rigid preventive maintenance is on the Figure 4. Figure 5 contains left-hand sides of flow charts of the simulation of the work of the two-unit standby system with economical (a) and sliding (b) preventive maintenances (the right hand-sides of these flow charts are the same as in the rigid case and are omitted).
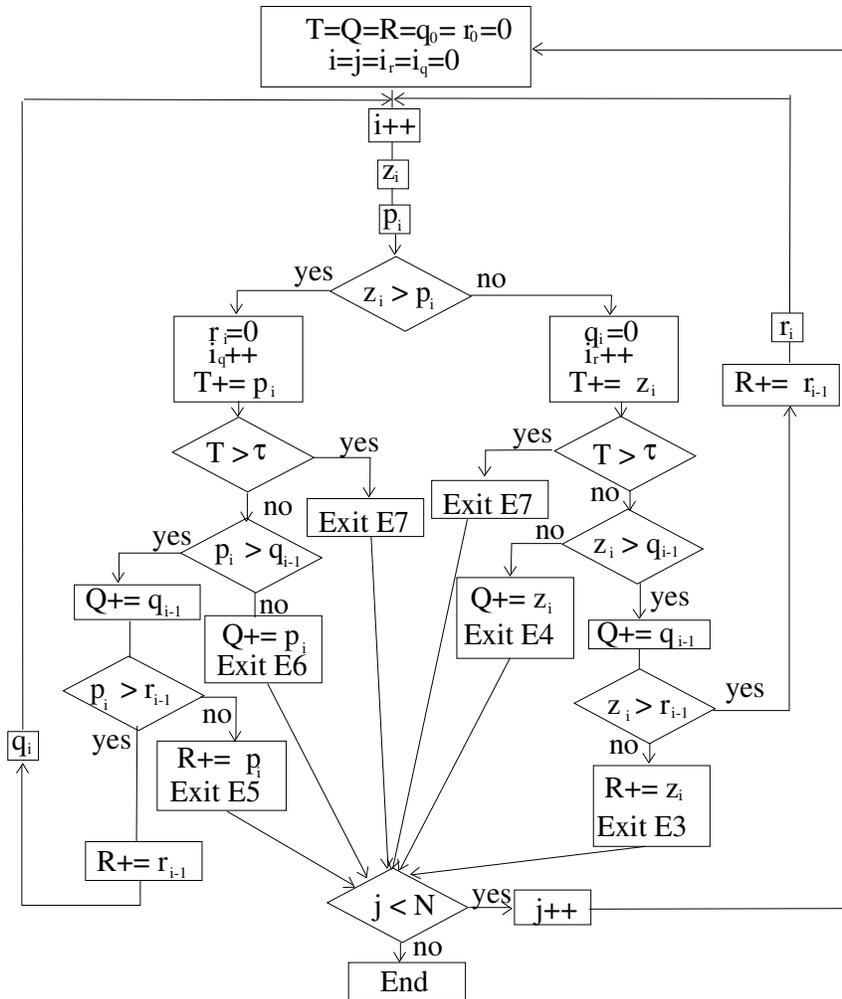

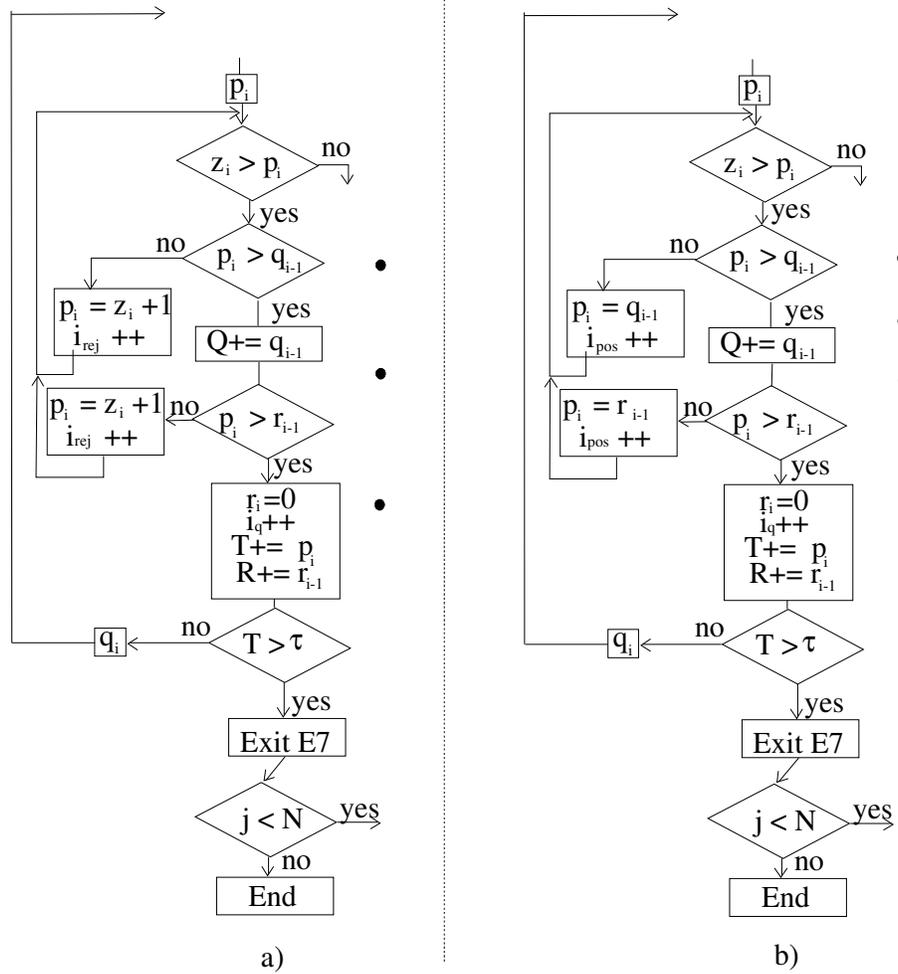**Figure 4:** Rigid preventive maintenance

Figure 5: a) economical; b) sliding preventive maintenance