

AN INVENTORY MODEL FOR DETERIORATING ITEMS UNDER THE CONDITION OF PERMISSIBLE DELAY IN PAYMENTS

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Abstract: In economic order quantity (EOQ) models, it is often assumed that the payment of an order is made on the receipt of items by the inventory system. However, such an assumption is not quite practical in the real world. Under most market behaviours, it can be easily found that a vendor provides a credit period for buyers to stimulate demand. In this paper, a varying rate of deterioration and the condition of permissible delay in payments used in conjunction with the economic order quantity model are the focus of discussion. Numerical examples are presented to illustrate the proposed models.

Keywords: Inventory, EOQ, deterioration.

1. INTRODUCTION

In the literature of inventory theory, deteriorating inventory models have been continually modified so as to accommodate the more practical features of real inventory systems. Ghare and Schrader [1] were the first to address problems with a constant demand and a deterioration rate. Since this introduction, a lot of studies such as Covert and Philip [2], Philip [3], Misra [4], Tadikamalla [5], Dave and Patel [6], Hariga [7], Chen [8], Chakrabarty et al. [9], Bhunia and Maiti [10], and Chang and Dye [11] have been made on deteriorating inventory control.

On the other hand, in the developed mathematical models, it is often assumed that payment will be made to the vendor for the goods immediately after receiving the consignment. As pointed out by Aggarwal and Jaggi [12], a permissible delay in

payments can be economically worthwhile for buyers. In such a case, it is possible for a vendor to allow a certain credit period for buyers to simulate demand so as maximize his own benefits and advantages. Recently, several researchers have developed analytical inventory models with the consideration of permissible delay in payments. Goyal [13] established a single-item inventory model under the condition of permissible delay in payments. Chung [14] presented the discounted cash-flow (DCF) approach for an analysis of the optimal inventory policy in the presence of trade credit. Later, Shinn et al. [15] extended Goyal's [13] model and considered quantity discounts for freight cost. Recently, Chung [16] presented a simple procedure to determine the optimal replenishment cycle to simplify the solution procedure described in Goyal [13].

More recently, in order to advance the practical inventory solution, Aggarwal and Jaggi [12] considered an inventory model with a constant deterioration rate under the condition of permissible delay in payments. Hwang and Shinn [17] were concerned with a combined price and lot size determination problem for an exponentially deteriorating product when the vendor permits delay in payments. Jamal et al. [18] extended Aggarwal and Jaggi's [12] model to allow for shortages. The purpose of this study is to propose a general deterioration rate including the condition of permissible delay in payments to extend the applications of developing mathematical inventory models and fit into more general inventory features.

This paper is organized as follows. In the next section, the assumptions and notations are presented. In Section 3, we present the mathematical model and develop the main result of this paper. In Section 4, numerical examples including two special cases are provided: first, when the deterioration rate is linear dependent on time, and second, when the distribution of time to deteriorate follows a two-parameter Weibull distribution. The method is illustrated by numerical examples, and a sensitivity analysis of the optimal solution with respect to parameters of the system is also carried out, which is followed by the concluding remarks.

2. ASSUMPTIONS AND NOTATIONS

The mathematical model in this paper is developed on the basis of the following assumptions and notations.

Assumptions

1. The inventory system involves only one item.
2. Replenishment occurs instantaneously at an infinite rate.
3. Let $\theta(t)$ be the deterioration rate of the on-hand inventory at time t , where $0 < \theta(t) < 1$ and $\theta'(t) \geq 0$.
4. Shortages are not allowed.
5. Before the replenishment account has to be settled, the buyer can use sales revenue to earn interest with an annual rate I_e . However, beyond the fixed credit period, the product still in stock is assumed to be financed with an annual rate I_r , where $I_r \geq I_e$.

Notations

- R = annual demand (demand rate being constant)
- A = ordering cost per order
- $I(t)$ = the inventory level at time t
- P = unit purchase cost, \$/per unit
- h = holding cost excluding interest charges, \$/unit/year
- I_e = interest which can be earned, \$/year
- I_r = interest charges which are invested in inventory, \$/year, $I_r \geq I_e$
- M = permissible delay in settling the account
- T = the length of replenishment cycle
- $C(T)$ = the total relevant inventory cost
- $C_1(T)$ = the total relevant inventory cost for $T > M$ in Case 1
- $C_2(T)$ = the total relevant inventory cost for $T \leq M$ in Case 2
- $V(T)$ = the average total inventory cost per unit time
- $V_1(T)$ = the average total inventory cost per unit time for $T > M$ in Case 1
- $V_2(T)$ = the average total inventory cost per unit time for $T \leq M$ in Case 2

3. MODEL FORMULATION

With the assumptions and notations, the behavior of the inventory system at any time t can be depicted in Fig. 1.

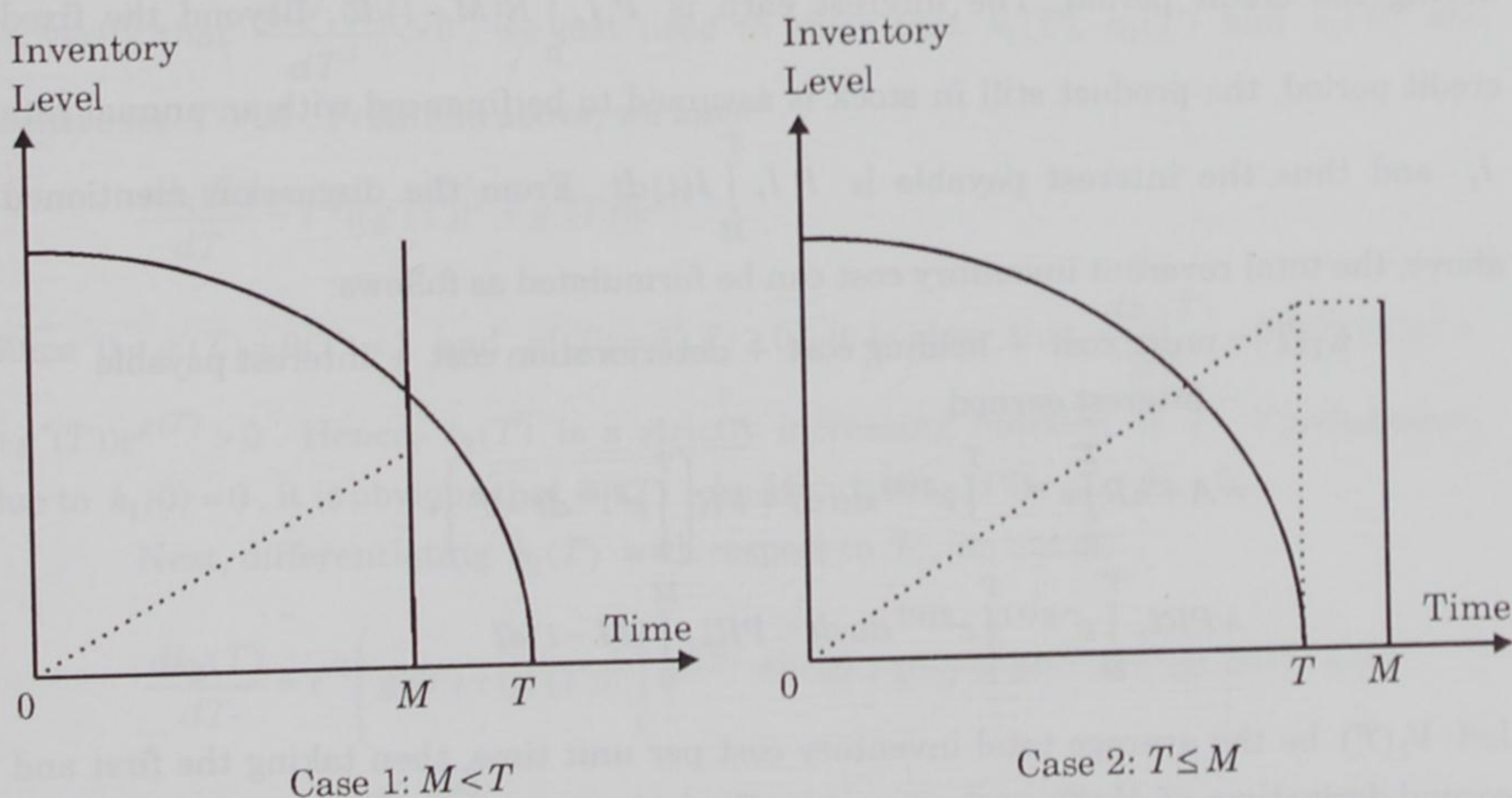


Figure 1: Credit period vs. replenishment cycle

Case 1: $T > M$

In this case, it is assumed that the replenishment cycle is larger than the credit period. Considering the inventory level at time t , depletion of the inventory occurs due to the effects of demand and deterioration during the replenishment cycle. Hence, the variation of inventory level, $I(t)$, with respect to time can be described by the following differential equation:

$$\frac{dI(t)}{dt} = -R - \theta(t)I(t), \quad 0 \leq t \leq T, \quad (1)$$

with boundary condition $I(T) = 0$.

The solution of (1) may be represented by

$$I(t) = Re^{-\int_0^t \theta(u) du} \int_t^T e^{\int_0^t \theta(u) du} dt, \quad 0 \leq t \leq T. \quad (2)$$

First, let $g'(x) = \theta x$ and from (2), the cost of holding $I(t)$ in stock for a small period of time dt is simply $hI(t)dt$. Therefore, the inventory holding cost over the

period $[0, T]$ is $h \int_0^T I(t) dt$. In addition, the deterioration cost during the same period is

proportional to $R \left(\int_0^T e^{g(t)} dt - T \right)$. However, before the replenishment account has to be

settled the buyer can use the sales revenue to earn interest with an annual rate I_e

during the credit period. The interest earned is $P I_e \int_0^M R(M-t) dt$. Beyond the fixed

credit period, the product still in stock is assumed to be financed with an annual rate

I_r and thus the interest payable is $P I_r \int_M^T I(t) dt$. From the discussion mentioned

above, the total relevant inventory cost can be formulated as follows:

$$C_1(T) = \text{order cost} + \text{holding cost} + \text{deterioration cost} + \text{interest payable} \\ - \text{interest earned}$$

$$= A + hR \int_0^T e^{-g(t)} \int_t^T e^{g(u)} du dt + PR \left(\int_0^T e^{g(t)} dt - T \right) + \\ + P I_r \int_M^T e^{-g(t)} \int_t^T e^{g(u)} du dt - P I_e \int_0^M (M-t) dt$$

Let $V_1(T)$ be the average total inventory cost per unit time, then taking the first and second derivatives of $V_1(T)$ with respect to T yields

$$\begin{aligned} \frac{dV_1(T)}{dT} = & -\frac{A}{T^2} + \frac{hR \left(T \int_0^T e^{g(T)-g(t)} dt - \int_0^T e^{-g(t)} \int_t^T e^{g(u)} du dt \right)}{T^2} + \\ & + \frac{PR \left(Te^{g(T)} - \int_0^T e^{g(t)} dt \right)}{T^2} + \\ & + \frac{PR \left(I_e M^2 + 2I_r \int_M^T e^{g(T)-g(t)} dt - 2I_r \int_M^T e^{-g(t)} \int_t^T e^{g(u)} du dt \right)}{2T^2} \end{aligned}$$

and

$$\frac{d^2V_1(T)}{dT^2} = \frac{2A}{T^3} + \frac{PRk_1(T)}{T^3} + \frac{hRk_2(T)}{T^3} + \frac{PR(-I_e M^2 + I_r k_3(T))}{T^3},$$

where

$$\begin{aligned} k_1(T) &= 2 \int_0^T e^{g(t)} dt + (-2 + Tg'(T))Te^{g(T)}, \\ k_2(T) &= T^2 + 2 \int_0^T e^{-g(t)} \int_t^T e^{g(u)} du dt + T(-2 + Tg'(T)) \int_0^T e^{g(T)-g(t)} dt, \\ k_3(T) &= T^2 + 2 \int_M^T e^{-g(t)} \int_t^T e^{g(u)} du dt + T(-2 + Tg'(T)) \int_M^T e^{g(T)-g(t)} dt. \end{aligned}$$

To verify that $\frac{d^2V_1(T)}{dT^2} > 0$, we just need to show that $k_1(T)$, $k_2(T)$ and $k_3(T)$ are positive for $T > M$. From the above, we have

$$\frac{dk_1(T)}{dT} = T^2 ((g'(T))^2 + g''(T))e^{g(T)}.$$

Since $0 < g'(T) = \theta(T) < 1$ and $g''(T) = \theta'(T) \geq 0$, it is clear that $\frac{dk_1(T)}{dT} = T^2 ((g'(T))^2 + g''(T))e^{g(T)} > 0$. Hence, $k_1(T)$ is a strictly increasing function of T . Furthermore, due to $k_1(0) = 0$, it is obvious that $k_1(T) > k_1(M) > k_1(0) = 0$ for $T > M > 0$.

Next, differentiating $k_2(T)$ with respect to T , we obtain

$$\frac{dk_2(T)}{dT} = T^2 \left(g'(T) + (g'(T))^2 \int_0^T e^{g(T)-g(t)} dt + g''(T) \int_0^T e^{g(T)-g(t)} dt \right) e^{g(T)} > 0$$

for $T > M > 0$. Since $\frac{dk_2(T)}{dT} > 0$ and $k_2(0) = 0$, we also have $k_2(T) > k_2(M) > k_2(0) = 0$.

Finally, analogous to the discussion above,

$$\frac{dk_3(T)}{dT} = T^2 \left(g'(T) + (g'(T))^2 \int_M^T e^{g(T)-g(t)} dt + g''(T) \int_M^T e^{g(T)-g(t)} dt \right) e^{g(T)} > 0$$

and $k_3(M) = M^2$. Hence, it is easy to see that $k_3(T) > M^2$ for $T > M > 0$. Thus we have $-I_e M^2 + I_r k_3(T) > (I_r - I_e) M^2 \geq 0$ for $T > M > 0$.

From the analysis carried so far, we can conclude that $V_1(T)$ is a convex function of T and there exists a unique value of T that minimizes $V_1(T)$. Besides, by using L'Hospital's rule, it is not difficult to show that

$$\begin{aligned} \lim_{T \rightarrow \infty} \frac{dV_1(T)}{dT} &= \lim_{T \rightarrow \infty} \frac{1}{2} \left(PR e^{g(T)} g'(T) + hR \left(1 + \int_0^T e^{g(T)-g(t)} dt \right) g'(T) + \right. \\ &\quad \left. + PRI_r \left(1 + \int_M^T e^{g(T)-g(t)} dt \right) g'(T) \right) = \\ &= \infty \end{aligned}$$

Thus, the optimal value of T should be selected to satisfy

$$\frac{dV_1(T)}{dT} = 0, \text{ otherwise } T^* = M \text{ if } \left. \frac{dV_1(T)}{dT} \right|_{T=M} > 0. \quad (4)$$

Case 2: $T \leq M$

In this case, it is assumed that the length of the replenishment cycle is not larger than the credit period. The holding cost and deterioration are the same as in case 1. Since $T \leq M$, the buyer pays no interest and earns interest during the period $[0, M]$.

Note that the interest earned in this case is $PI_e \left(\int_0^T R(T-t) dt + RT(M-T) \right)$. From this,

the total relevant inventory cost can be formulated as

$$\begin{aligned} C_2(T) &= \text{order cost} + \text{holding cost} + \text{deterioration cost} - \text{interest earned} \\ &= A + hR \int_0^T e^{-g(t)} \int_t^T e^{g(u)} du dt + PR \left(\int_0^T e^{g(t)} dt - T \right) - \\ &\quad - PRI_e \left(\int_0^T (T-t) dt + T(M-T) \right). \end{aligned}$$

The first and second derivatives of average total cost, $V_2(T)$, with respect to T , result in

$$\frac{dV_2(T)}{dT} = -\frac{A}{T^2} + \frac{hR \left(T \int_0^T e^{g(T)-g(t)} dt - \int_0^T e^{-g(t)} \int_t^T e^{g(u)} du dt \right)}{T^2} + \frac{PR \left(Te^{g(T)} - \int_0^T e^{g(t)} dt \right)}{T^2} + \frac{PRI_e}{2}$$

and

$$\frac{d^2V_2(T)}{dT^2} = \frac{2A}{T^3} + \frac{hRk_2(T)}{T^3} + \frac{PRk_1(T)}{T^3}.$$

Using the fact that $k_1(x) > 0$ and $k_2(x) > 0$ for $0 < x \leq M$, it is easily shown that $V_2(T)$ is also a convex function of T and there exists a unique value of T that minimizes $V_2(T)$. Since $\lim_{T \rightarrow 0} \frac{dV_2(T)}{dT} = -\infty$, the optimal value of T should be selected to satisfy

$$\frac{dV_2(T)}{dT} = 0, \text{ otherwise } T^* = M \text{ if } \left. \frac{dV_2(T)}{dT} \right|_{T=M} < 0. \tag{6}$$

The objective of this problem is to determine the optimal value of T so that $V(T)$ is minimized. From the above discussions, we have $V(T) = \min\{V_1(T^*), V_2(T^*)\}$.

On the other hand, since $V_1(M) = V_2(M)$ and $\left. \frac{dV_2(T)}{dT} \right|_{T=M} = \frac{f(M)}{M^2} = \left. \frac{dV_1(T)}{dT} \right|_{T=M}$,

where

$$f(M) = -A + PR \left(Me^{g(M)} - \int_0^M e^{g(t)} dt \right) + \frac{1}{2}PRI_e M^2 + hR \left(Me^{g(M)} \int_0^M e^{-g(t)} dt - \int_0^M e^{-g(t)} \int_t^M e^{g(u)} du dt \right),$$

it is obvious to see that

$$V(T^*) = \begin{cases} V_1(T^*), & f(M) < 0, \\ V_2(T^*), & f(M) > 0, \\ V_1(T^*) = V_2(T^*), & f(M) = 0. \end{cases} \tag{7}$$

4. NUMERICAL EXAMPLES

In this section, the optimal solution procedure developed in the previous section is now illustrated with two special cases. In the first case, we assume that the deterioration rate is linear dependent on time and is in the following form:

$\theta(t) = a + bt$, $0 < a, b \ll 1$; $t > 0$. And second, the distribution of time to deteriorate follows a two-parameter Weibull distribution: $\theta(t) = \alpha\beta t^{\beta-1}$, $0 < \alpha \ll 1$, $\beta \geq 1$; $t > 0$, where α is the scale parameter and β is the shape parameter.

4.1. Linear deterioration rate

The exact solution procedure for the case of a linear deterioration rate can be deduced from the previous analysis by substituting $g(x) = ax + \frac{b}{2}x^2$ into the derived mathematical expressions. Using Taylor's series expansion, $V_1(T)$, $V_2(T)$ and $f(M)$ can be rewritten as follows:

$$V_1(T) = \frac{A}{T} + \frac{hR}{T} \int_0^T \left\{ \sum_{n=0}^{\infty} \frac{(-g(t))^n}{n!} \int_t^{\infty} \sum_{n=0}^{\infty} \frac{(g(u))^n}{n!} du \right\} dt + \frac{PR}{T} \left(\int_0^T \sum_{n=0}^{\infty} \frac{(g(t))^n}{n!} dt - T \right) + \frac{PRI_r}{T} \int_M^T \left\{ \sum_{n=0}^{\infty} \frac{(-g(t))^n}{n!} \int_t^{\infty} \sum_{n=0}^{\infty} \frac{(g(u))^n}{n!} du \right\} dt - \frac{PRI_e}{T} \int_0^M (M-t) dt, \quad (8)$$

$$V_2(T) = \frac{A}{T} + \frac{hR}{T} \int_0^T \left\{ \sum_{n=0}^{\infty} \frac{(-g(t))^n}{n!} \int_t^{\infty} \sum_{n=0}^{\infty} \frac{(g(u))^n}{n!} du \right\} dt + \frac{PR}{T} \left(\int_0^T \sum_{n=0}^{\infty} \frac{(g(t))^n}{n!} dt - T \right) - \frac{PRI_e}{T} \left(\int_0^T (T-t) dt + T(M-T) \right) \quad (9)$$

and

$$f(M) = -A + hR \left(M \sum_{n=0}^{\infty} \frac{(g(M))^n}{n!} \int_0^M \sum_{n=0}^{\infty} \frac{(-g(t))^n}{n!} dt - \int_0^M \left\{ \sum_{n=0}^{\infty} \frac{(-g(t))^n}{n!} \int_t^M \sum_{n=0}^{\infty} \frac{(g(u))^n}{n!} du \right\} dt \right) + PR \left(M \sum_{n=0}^{\infty} \frac{(g(M))^n}{n!} - \int_0^M \sum_{n=0}^{\infty} \frac{(g(t))^n}{n!} dt \right) + \frac{PRI_e M^2}{2}. \quad (10)$$

As a and b are very small, the approximation solution can be found by neglecting the second and higher terms of a , b and ab , so we have

$$V_1(T) \approx \frac{A}{T} + hR \left(\frac{T}{2} + \frac{aT^2}{6} + \frac{bT^3}{12} \right) + PR \left(\frac{aT}{2} + \frac{bT^2}{6} \right) - \frac{PRI_e M^2}{2T} + \frac{PRI_e}{T} \left(\frac{M^2}{2} - \frac{aM^3}{6} - \frac{bM^4}{12} - MT + \frac{aM^2T}{2} + \frac{bM^3T}{6} + \frac{T^2}{2} - \frac{aMT^2}{2} + \frac{aT^3}{6} - \frac{bMT^3}{6} + \frac{bT^4}{12} \right), \quad (11)$$

$$V_2(T) \approx \frac{A}{T} + hR \left(\frac{T}{2} + \frac{aT^2}{6} + \frac{bT^3}{12} \right) + PR \left(\frac{aT}{2} + \frac{bT^2}{6} \right) - PRI_e \left(M - \frac{T}{2} \right) \quad (12)$$

and

$$f(M) \approx -A + \frac{hRM^2(6 + 4a + 3bM^2)}{12} + \frac{PRM^2(3a + 2bM)}{6} + \frac{PRM^2I_e}{2} \quad (13)$$

The procedure for determining the approximate optimal value of T first computes $f(M)$ from (13). Then applying the above solution produced by $\frac{dV_i(T)}{dT} = 0$, $i = 1$ or 2 , is taken to be the approximate optimal value of T .

Example 1. In order to illustrate the above solution procedure, we consider an inventory system with the following data: $R = 1000$ units/year, $A = \$250$ per order, $P = \$100$ /unit/year, $h = \$20$ /unit/year, $I_e = 0.13$ /\$/year, $I_r = 0.15$ /\$/year, $M = 30/365$ year. For the linear deterioration rate case, we let $\theta(t) = 0.08 + 0.1t$. For this case, since $f(M) = -109.343 < 0$, from (7), we have the optimal value of $V(T) = V_1(T^*)$. Solving $\frac{dV_1(T)}{dT} = 0$ and then putting the obtained value into (11), we have the optimal values of T and $V(T)$, which are $T^* = 0.1082$ and $V(T^*) = 3489.28$.

4.2. Weibull deterioration rate

In this case, it is assumed that the deterioration rate is a two-parameter Weibull distribution: $\theta(t) = \alpha\beta t^{\beta-1}$, where $0 < \alpha \ll 1$, $\beta \geq 1$. Analogous to the discussion in the previous case, substituting $g(x) = \alpha x^\beta$ into (8), (9) and (10), the approximation solution can be found by neglecting the second and higher terms of α as α is very small, so we have

$$V_1(T) \approx \frac{A}{T} + hR \left(\frac{T}{2} + \frac{\alpha\beta T^{1+\beta}}{(1+\beta)(2+\beta)} \right) + \frac{\alpha PRT^\beta}{1+\beta} - \frac{PRI_e M^2}{2T} + \frac{PRI_r}{T} \left(\frac{M^2}{2} - \frac{\alpha\beta M^{2+\beta}}{(1+\beta)(2+\beta)} + \frac{\alpha M^{1+\beta} T}{1+\beta} - \frac{MT(1+\beta+\alpha T^\beta)}{1+\beta} \right) + \frac{T^2}{2} + \frac{\alpha\beta T^{2+\beta}}{(1+\beta)(2+\beta)} \quad (14)$$

$$V_2(T) \approx \frac{A}{T} + hR \left(\frac{T}{2} + \frac{\alpha\beta T^{1+\beta}}{(1+\beta)(2+\beta)} \right) + \frac{\alpha PRT^\beta}{1+\beta} - PRI_e \left(M - \frac{T}{2} \right) \quad (15)$$

and

$$f(M) \approx -A + hR \left(\frac{M^2}{2} + \frac{\alpha\beta M^{2+\beta}}{2+\beta} \right) + \frac{\alpha\beta PRM^{1+\beta}}{1+\beta} + \frac{PRI_e M^2}{2} \quad (16)$$

The solution procedure for determining the approximate optimal value of T in this case follows the same technique as in the previous case. We next illustrate the optimal solution procedure for this type of deterioration rate.

Example 2. In this example, the same parameters are used as in Example 1 except putting $\theta(t) = \alpha\beta t^{\beta-1}$, where $\alpha = 0.08$ and $\beta = 1.5$. Compute $f(M)$ firstly; since $f(M) = -132.293 < 0$, the optimal value of $V(T) = V_1(T^*)$ from (7). Solving $\frac{dV_1(T)}{dT} = 0$ and then putting the obtained value into (14), we have the optimal values of T and $V(T)$, which are $T^* = 0.1158$ and $V(T^*) = 3138.24$.

Next, as in the above examples, the effects of changes in $\theta(t)$ and M on the optimal T and the optimal $V(T)$ for Example 1 and Example 2 are examined. The computed results are shown in Table 1 and Table 2. The results obtained for the illustrative examples provide certain insights about the problems studied. Some of them are as follows:

Linear deterioration rate case

For fixed a and M , increasing the value of b will result in a decrease in the optimal T and an increase in the optimal $V(T)$.

For fixed a and b , increasing the value of M will result in a significant decrease in the optimal $V(T)$ but the optimal T increases.

For fixed b and M , increasing the value of a will result in a significant increase in the optimal $V(T)$ but the optimal T increases.

Table 1: Effects of M , a and b for the linear deterioration rate case

a	b	M					
		15/365		30/365		45/365	
		T^*	$V(T^*)$	T^*	$V(T^*)$	T^*	$V(T^*)$
0	0.05	0.1192	3593.1	0.1204	3018.9	0.1223	2472.0
	0.10	0.1186	3605.0	0.1197	3031.0	0.1216	2484.5
	0.15	0.1179	3616.9	0.1191	3043.1	0.1209	2496.9
	0.20	0.1173	3628.6	0.1184	3055.0	0.1202	2509.2
0.04	0.05	0.1129	3827.2	0.1141	3255.1	0.1155	2711.5
	0.10	0.1124	3838.0	0.1135	3266.0	0.1149	2722.7
	0.15	0.1119	3848.6	0.1130	3276.8	0.1144	2733.8
	0.20	0.1113	3859.1	0.1125	3287.5	0.1138	2744.8
0.08	0.05	0.1075	4049.5	0.1087	3479.4	0.1098	2938.4
	0.10	0.1071	4059.2	0.1082	3489.3	0.1093	2948.5
	0.15	0.1067	4068.9	0.1078	3499.1	0.1088	2958.5
	0.20	0.1062	4078.4	0.1073	3508.8	0.1084	2968.5
0.12	0.05	0.1029	4261.5	0.1039	3693.4	0.1048	3154.4
	0.10	0.1025	4270.4	0.1036	3702.5	0.1044	3163.6
	0.15	0.1021	4279.3	0.1032	3711.5	0.1040	3172.8
	0.20	0.1018	4288.0	0.1028	3720.4	0.1036	3181.8

Table 2: Effects of M , α and β for the Weibull deterioration rate case

α	β	M					
		15/365		30/365		45/365	
		T^*	$V(T^*)$	T^*	$V(T^*)$	T^*	$V(T^*)$
0.04	1.00	0.1135	3816.4	0.1147	3244.1	0.1162	2700.2
	1.25	0.1156	3704.9	0.1168	3131.9	0.1184	2586.9
	1.50	0.1171	3647.1	0.1183	3073.5	0.1201	2527.8
	2.00	0.1188	3600.3	0.1200	3026.2	0.1219	2479.5
0.08	1.00	0.1080	4039.7	0.1091	3469.4	0.1103	2928.2
	1.25	0.1118	3823.6	0.1130	3251.8	0.1143	2708.8
	1.50	0.1146	3711.0	0.1158	3138.2	0.1173	2593.9
	2.00	0.1178	3619.2	0.1189	3045.5	0.1207	2499.4
0.12	1.00	0.1033	4252.6	0.1043	3684.3	0.1052	3145.1
	1.25	0.1084	3937.5	0.1095	3367.0	0.1107	2825.6
	1.50	0.1123	3772.8	0.1134	3200.9	0.1148	2657.7
	2.00	0.1168	3637.8	0.1179	3064.4	0.1196	2518.9

Weibull deterioration rate

For fixed α and M , increasing the value of β will result in a decrease in the optimal $V(T)$ and an increase in the optimal T .

For fixed α and β , increasing the value of M will result in a significant decrease in the optimal $V(T)$ but the optimal T increases.

For fixed β and M , increasing the value of α will result in a significant increase in the optimal $V(T)$ but the optimal T increases.

5. CONCLUDING REMARKS

This paper develops a varying rate of deteriorating inventory model with permissible delay in payments. The phenomena of the deterioration of physical goods and a vendor who may offer a fixed credit period to settle the account are very common in the market. The analytical formulations of the problem on the general framework described have been given. Furthermore, we also provided two special types of deterioration rate to illustrate the proposed models. The approximate optimal solutions in both cases of the problem have been derived. Future research work may consider the added effect of a more realistic demand rate in the model.

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