Yugoslav Journal of Operations Research
11 (2001), Number 2, 171-177

OPTIMIZATION OF INVESTMENTS IN RAIL-HIGHWAY CROSSINGS

Milan MARKOVI]
Public Railway Transportation Enterprise "Beograd"
Belgrade, Yugoslavia,

Slobodan GUBERINI]
Faculty of Transport and Traffic Engineering
University of Belgrade, Belgrade, Yugoslavia

Tatjana JANKOVI]
Public Railway Transportation Enterprise "Beograd"
Belgrade, Yugoslavia

Abstract: The paper presents the problem of optimizing investments in rail-highway crossings. The optimization problem is to allocate limited available funds to minimize the expected number of traffic accidents. This problem is treated as an optimal resource allocation problem and is solved by the Dynamic Programming method. The method is used to solve a real problem of investment optimization in the Yugoslav railway system.

Keywords: Transport, optimization, railway, traffic safety.

1. INTRODUCTION

Rail-highway grade crossings, as special types of intersections of the rail and road traffic, are points where traffic accidents happen very often, and with severe consequences. An illustration of poor traffic safety on rail-highway crossings is the information that in Yugoslavia 60% of the total number of persons injured in rail traffic accidents is injured on rail-highway crossings.

Although the factors contributing to accidents are numerous, in the vast majority of cases the number of traffic accidents can be reduced merely by adequate traffic control - the choice of safety measures. Changing other factors is either very expensive (demolition of facilities or rectification of curves in order to improve visibility), or it gives results only after long periods of time (increase in the education level and the awareness level of traffic participants). Therefore, all over the world great attention is given to the choice of safety measures on rail-highway crossings.

The choice of safety measures is a very important decision because it represents an investment that requires a significant amount of money (the cost of some safety measures for a rail-highway crossing is between 100,000 and 200,000 DM).

The problem a decision maker has to solve is how to allocate limited available funds so as to minimize the number of traffic accidents. This paper presents a method developed for solving this problem.

2. PROBLEM STATEMENT

The choice of safety measures for a rail-highway crossing directly influences the number of traffic accidents on the crossing. The mathematical models described in references [1, 2, 3] give the relationships between the number of traffic accidents and the types of safety measures, including some other relevant parameters (such as traffic volumes - rail and road traffic, vehicle speeds, visibility, etc.).

J. Coleman and R. Stewart [1] propose a regression relation that gives the number of accidents as a function of the safety measure type, the number of road vehicles, the number of trains and rail vehicles per day, the number of tracks and crossing type (rural or urban).

E. Farr and H.B. Tustin [2] describe a regression relation that gives the number of accidents as a function of the safety measure type, the number of road vehicles and trains per day, the number of road traffic lanes, the number of tracks, maximum permitted speeds, the type of road on the crossing and crossing type.

The regression model presented in reference [3] calculates the expected number of accidents per year, $M(U)$, using the following expression:

$$M(U) = 365 n_p P(S) P(G/S),$$

where:

U	–	number of accidents per year,
$M(U)$	–	mathematical expectation of the number of accidents per year,
n_p	–	number of vehicles per day,
$P(S)$	–	probability of the event that a road vehicle and a train meet on the crossing in the same time interval (event S). This probability can be calculated by the method presented in [3],
$P(G/S)$	–	probability that in the case when event S happened, the driver of the road vehicle fails to avoid the collision. A regression model is

developed to calculate this probability. The model determines the number of accidents as the function of the safety measure type, the number and speed of road vehicles, visibility and the angle between the road and tracks [3].

The choice of safety measures for rail-highway crossings determines the necessary investment costs. Since the available financial means are always limited and, as a rule, are less than the quantity needed to attain the maximum safety level (deleveling all grade-crossings), it is necessary to allocate the available financial resources so as to minimize the number of traffic accidents. If a model is used to predict the number of traffic accidents for each of the road-highway crossings considered, and for each type of safety measure adequate for the crossing, and the investment costs are known for each crossing and safety measure used, then the available financial resources can be allocated to the crossings so as to minimize the mathematical expectation of the total predicted number of accidents.

E. Farr and B. Tutsin developed a model [2] to predict the number of traffic accidents, and they mentioned that the developed model could be used for optimum resource allocation so as to maximize the number of avoided accidents. However, they did not describe the allocation method. The method for the optimization of investments presented in this paper is based on the well-known Dynamic Programming method [4], and it solves the problem of minimizing the mathematical expectation of the predicted number of traffic accidents under limited financial resources.

The problem considered in this paper is to allocate limited funds, *X*, to *N* rail-highway crossings. The predicted number of accidents on the i-th crossing depends on the existing safety level and the amount of money, u_i, invested in safety improvements. Thus, the constraint imposed by the limited funds is:

$$\sum_{i=1}^{N} u_i \le X .$$

The amount of money invested in a crossing can be either 0 or the amount needed to raise the safety level. Let us assume that there are four safety levels and that crossing i is on safety level 1. Thus, the amount invested in crossing i has to be an element of set V_i, defined as follows:

$$V_i \subseteq \{0, u_{i1}, u_{i2}, u_{i3}\} ,$$

where:

u_{i1} – the amount needed to raise the safety level of crossing i to the first level,
u_{i2} – the amount needed to raise the safety level of crossing i to the second level,
u_{i3} – the amount needed to raise the safety level of crossing i to the third level,

If crossing i is already on safety level 1 then,

$$V_i = \{0, u_{i2}, u_{i3}\} ,$$

and if crossing i is on safety level 2 then,

$$V_i = \{0, u_{i3}\} .$$

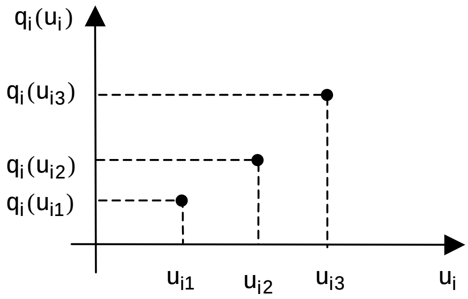

Fig. 1.

Figure 1 presents the form of the function $q_i(u_i)$. This is a discrete function and it is defined only for the elements of the set V_i.

If $q_i(u_i)$ is the predicted number of accidents on crossing i, as a function of the invested amount, then the total predicted number of accidents can be determined as:

$$T = \sum_{i=1}^{N} q_i(u_i) .$$

Now, the problem of the optimum allocation of funds available for safety level improvement on rail-highway crossings can be stated as follows:

Find the allocation of funds *X* to *N* rail-highway crossings, $u = (u_1, u_2, \ldots, u_N)$, so as to minimize the total number of expected traffic accidents

$$T = \sum_{i=1}^{N} q_i(u_i) \tag{1}$$

subject to the constraints:

$$\sum_{i=1}^{N} u_i \le X , \tag{2}$$

$$u_i \in V_i, \quad (i = 1, 2, \ldots, N) , \tag{3}$$

where

$$V_i = \{0, u_{i1}, u_{i2}, u_{i3}\}, \quad (i = 1, 2, \ldots, N) .$$

3. SOLUTION METHOD

Objective function (1) is minimized using the Dynamic Programming method. Application of the method is based on forming the following recurrence relationship:

$$f_N(X) = \min_{u_N}\{q_N(u_N) + f_{N-1}(X - u_N)\} \quad ,$$

where $N = 1,2,\ldots,N$, and $u_N \in V_N$.

Function $f_1(X)$ is defined as follows:

$$f_1(X) = \begin{cases} q_1(u_{1,k-1}) & \text{for} \quad u_{1,k-1} \le X < u_{1k} \\ q_1(u_{1,k}) & \text{for} \quad X \ge u_{1k} \end{cases}$$

where

$$k \in \{1,2,3\}$$

The optimum investment in the N-th crossing is the element of set V_N , for which $f_N(X)$ is obtained. If we denote this value by u_N^{op}, then the optimum investment in the $(N-1)$-st crossing is the element of set V_{N-1}, for which $f_{N-1}(X - u_N^{op})$ is obtained. Going this way backwards, the optimum investment in each of the *N* crossings is obtained [5].

4. AN EXAMPLE

The developed method is illustrated by example in which 21 rail-highway crossings are considered (Table 1). The crossings under ordinal numbers from 1 to 13 have no safety measures applied (safety level I), the crossings under ordinal numbers 5 and 6 have an automatic light and audio signalization (safety level II), whereas other crossings (ordinal numbers 14, 15, 16, 17, 20, and 21) are equipped with an automatic light and audio signalization and half-barriers (safety level IIa). Safety level III assumes deleveling of the crossing.

The method presented in [3] was used to predict the number of accidents for each crossing and for every feasible safety measure type (it is not feasible to consider safety levels lower than the existing one). The number of predicted accidents and investment costs necessary to attain the considered safety levels are presented in Table 1 for each crossing. The total amount of money to be allocated is 2,700,000 DM. The results obtained by the method presented in this paper – the optimum financial resources allocation - are given in Table 2.

Table 1: The number of accidents per year on crossings that have no safety measures and investments in raising the safety level

Ord. no.	LEVEL I		LEVEL II		LEVEL IIa		LEVEL III	
	no. of accid.	Invest. (DM)	no. of accid.	Invest. (DM)	no. of accid.	Invest. (DM)	no. of accid.	Invest. (DM)
1	0.554	0	0.173	120,000	0.089	150,000	0	700,000
2	0.881	0	0.276	120,000	0.141	150,000	0	980,000
3	0.806	0	0.186	120,000	0.095	150,000	0	850,000
4	1.117	0	0.194	120,000	0.100	150,000	0	900,000
5	1.146	0	0.199	120,000	0.102	150,000	0	700,000
6	1.174	0	0.125	120,000	0.064	150,000	0	900,000
7	0.208	0	0.048	120,000	0.025	150,000	0	700,000
8	0.502	0	0.063	120,000	0.032	150,000	0	700,000
9	0.531	0	0.209	120,000	0.107	150,000	0	700,000
10	0.326	0	0.128	120,000	0.065	150,000	0	700,000
11	0.128	0	0.020	120,000	0.010	150,000	0	700,000
12	0.846	0	0.213	120,000	0.109	150,000	0	850,000
13	0.601	0	0.057	120,000	0.029	150,000	0	900,000
14					0.813	0	0	1,100,000
15					0.433	0	0	900,000
16					0.034	0	0	850,000
17					0.014	0	0	700,000
18			0.048	0	0.025	30,000	0	700,000
19			0.455	0	0.233	30,000	0	700,000
20					0.118	0	0	700,000
21					0.050	0	0	960,000

5. CONCLUSIONS

This paper presents a method to optimize investments in rail-highway crossings based on the Dynamic Programming method. The method enables the decision maker to allocate the available funds to the crossings so as to minimize the expected number of traffic accidents per year. This is very useful in determining how to use the funds that are set aside every year for crossings safety improvements. In order to apply this method, a computer program was developed and used to solve the numerical example in Section 4. The program is written in Visual Basic.

Table 2: The optimum financial resources allocation

Ord. no.	Existing safety level	Planned safety level	Planned invest.	Predict. no. of accid.
1	Unequipped	Light-audio & barr.	150,000	0.089
2	Unequipped	Light-audio & barr.	150,000	0.141
3	Unequipped	Light-audio & barr.	150,000	0.095
4	Unequipped	Light-audio & barr.	150,000	0.100
5	Unequipped	Light-audio & barr.	150,000	0.102
6	Unequipped	Light-audio & barr.	120,000	0.125
7	Unequipped	Unequipped	0	0.208
8	Unequipped	Light-audio & barr.	120,000	0.063
9	Unequipped	Light-audio & barr.	150,000	0.107
10	Unequipped	Light-audio & barr.	150,000	0.065
11	Unequipped	Unequipped	0	0.128
12	Unequipped	Light-audio & barr.	150,000	0.109
13	Unequipped	Light-audio	120,000	0.057
14	Light-audio & barr.	Deleveled	1,000,000	0.000
15	Light-audio & barr.	Light-audio & barr.	0	0.433
16	Light-audio & barr.	Light-audio & barr.	0	0.034
17	Light-audio & barr.	Light-audio & barr.	0	0.014
18	Light-audio	Light-audio	0	0.048
19	Light-audio	Light-audio & barr.	30,000	0.233
20	Light-audio & barr.	Light-audio & barr.	0	0.118
21	Light-audio & barr.	Light-audio & barr.	0	0.050
Σ			2,690,000	2.319

REFERENCES

[1] Coleman, J., and Stewart, R., "Investigation of accident data for railroad-highway grade crossings", Transportation Research Record, 611 (1976).

[2] Farr, E., and Tustin, H.B., "Optimizing resources at rail-highway crossings", ITE Journal, January, (1982).

[3] Markovi}, M, "Traffic process modeling on rail-highway crossings with a choice of traffic control measures", Ph.D. Thesis, Faculty of Transport and Traffic Engineering, University of Belgrade, Belgrade, 1992.

[4] Bellman, R., and Dreyfus, S., Applied Dynamic Programming, Princeton Univ. Press, Princeton, New Jersey, 1961.

[5] Petrovi}, R., Special Methods in Optimization, Tehni~ka knjiga, Belgrade, 1977 (in Serbian).