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MULTI-ITEM INVENTORY MODEL WITH PROBABILISTIC PRICE DEPENDENT DEMAND AND IMPRECISE GOAL AND CONSTRAINTS

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Abstract: A multi-item single-period stochastic inventory model is formulated in the fuzzy environment with budgetary, floor-space constraints. In this model, probabilistic demand is related to the unit selling price. Here fuzziness is introduced in both the objective function and constraint goals. The problem is solved using the fuzzy non-linear technique for linear membership functions. The model is illustrated numerically and sensitivity analysis is presented. The numerical results are compared with those of the crisp model.

Keywords: Inventory analysis, inventory control, multi-item model, fuzzy non-linear programming.

1. INTRODUCTION

Multi-item classical inventory models under resource constraints such as capital investment, limited storage area, number of orders and available set up tunes, etc. are presented in well-known books [Naddor (1966), Silver and Peterson (1985), Churchmann, et al. (1957), Hadley and Whitin (1963)]. Recently, Ben-daya and Raouf (1993)] presented a multi-item inventory model with stochastic demand under two constraints.

While modelling an inventory problem, it is assumed that constraints, objectives, etc. are defined with certainty. However, in real life, constraints like total capital investment, storage area, objective goal, etc. are not exactly known, i.e. somewhat vague in nature. In this situation, fuzzy set theory can be used in the formulation of inventory models. Inventory models are developed in a fuzzy environment expressing the goals and parameters by fuzzy functions and/or fuzzy numbers and thus inventory problems are reduced to fuzzy decision making problems which are solved by different fuzzy programming methods.

Bellman and Zadeh (1970) first introduced fuzzy set theory in decision making processes. Later, Tanaka, et al. (1974) applied the concepts of fuzzy set decision problems by considering the objectives as fuzzy goals over the α-cuts of a fuzzy constraint set and Zimmermann (1976) showed that classical algorithms can be used to solve multi-objective fuzzy linear programming problems. Fuzzy mathematical programming has been applied to several fields, for instance, project networking, reliability optimization, transportation problems, media selection for advertising, air pollution regulation, etc. (Ref. Lai and Hwang [1992, 1994]). However, it has not been much used in inventory models. Sommer (1981) applied fuzzy dynamic programming to an inventory and production scheduling problem. Kacprzyk and Staniewski (1982) considered a fuzzy inventory problem in which, instead of minimizing the total average cost, they reduced it to a multi-stage fuzzy decision making problem and solved by a branch and bound algorithm. Park (1987) examined the EOQ formula in the fuzzy set theory perspective associating the fuzziness with cost data. Recently, Roy and Maiti (1995) solved the classical EOQ model in a fuzzy environment with a fuzzy goal, fuzzy inventory cost and storage area by a fuzzy non-linear programming method using different types of membership functions for inventory parameters. They (1997) also examined the fuzzy EOQ model with demand-dependent unit price and imprecise storage area using a fuzzy non-linear programming method.

In most of the probabilistic inventory models, the demand distribution is assumed to be independent of selling price. However, in practice the demand distribution is a function of selling price. Here we consider a multi-item inventory model with stochastic price-dependent demand whose probability distribution depends on selling price as a parameter.

In some real life problems, some uncertain parameters are imprecise in nature due to their flexibility in marketing situations and some others may be randomised by their past observational data. So in an inventory system, mixed environment is an important area in which some parameters of the objective function and/or constraints are random, some others are imprecise and the rest are crisp. The basic idea used in solving a stochastic or fuzzy or fuzzy stochastic programming problem is to convert the specific problem into an equivalent deterministic/crisp problem which is then solved by different programming methods.

In this paper, we consider a multi-item inventory model with stochastic price-dependent demand whose probability distribution depends on selling price as a parameter. Here, expected profit rate, available storage area and total budget are imprecise, i.e. fuzzy in nature. They may take values within a specified interval. So we formulate a multi-item fuzzy stochastic inventory model with probabilistic price, dependent demand under imprecise profit goal, storage space and budget constraints and solve it using probability distribution and a fuzzy non-linear programming technique. The model is illustrated with a numerical example and a sensitivity analysis is presented with variation in tolerance limits for storage area, budget and expected profit goal.

2. MATHEMATICAL MODEL

Let n products stocked up to satisfy random external demand during a single period. For each item, an order quantity $Q_i, (i = 1,2,\dots,n)$ can be made for delivery prior to the beginning of the period. No subsequent orders can be made during the period. Unsatisfied demand results in a penalty cost representing lost sales or loss of goodwill. Excess demand is disposed of at a lower price.

The following notations are used:

n = number of items.

W = floor-space or shelf-space available.

B = budget available for replenishment.

For item $i\ (i = 1,2,\dots,n)$, let

(i) Q_i = order quantity.

(ii) C_i = purchase cost per unit item. Here C_i is a function of purchase quantity Q_i, which satisfies the condition $dC_i(Q_i)/dQ_i \le 0$. This cost function includes the standard constant cost and discount cost. $C_i(Q_i)$ is assumed to have the form $C_i(Q_i) = C_{1i} - C_{2i}Q_i\ (i = 1,2,\dots,n)$ with C_{1i} and C_{2i} as constant and C_{2i} is compared to C_{1i} such that $C_i(Q_i) > 0$ for all Q_i and $dC_i(Q_i)/dQ_i = -C_{2i} < 0$ for all Q_i.

(iii) h_i = inventory holding cost per unit item.

(iv) p_i = shortage cost, i.e. penalty cost for unsatisfied demand.

(v) s_i = salvage value per unit.

(vi) p_i = selling price per unit which is determined by a variable markup rate q_i over the purchasing price C_i i.e. $p_i = q_i C_i$, $q_i > 1$.

The probability distribution of demand may be uniform, normal, exponential, gamma, etc. according to past observed data. In this paper, we assume that the demand for the i-th item is a random gamma $(a_i, p_i / k_i)$ variable with probability density function $f_i(x)$ given by:

i.e. $$f_i(x) = \begin{cases} \dfrac{x^{(a_i-1)}}{\Gamma(a_i)} e^{\frac{-xp_i}{k_i}}, & \text{for } 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

If TEP(Q) is the total expected profit obtained from the policy Q, where Q is an n-dimensional vector with components as the decision variables Q_i $(i = 1,2,...,n)$, then

TEP(Q) = (revenue from sale + salvage value) - (item cost + inventory carrying cost + shortage cost).

The expression for TEP(Q) is given in the Appendix.

The problem of finding the optimal policy $Q = (Q_1, Q_2, ..., Q_n)$ subject to the restrictions on available space and budget can be stated as follows:

$$\max \text{TEP(Q)} \tag{1}$$

subject to

$$\sum_{i=1}^{n} w_i Q_i \le W,$$
$$\sum_{i=1}^{n} C_i Q_i \le B,$$
$$Q_i \ge 0, \; i = 1,2,..., n.$$

When the above expected profit goal, storage area and budget constraint goals become fuzzy, the said crisp model is transformed to

$$\widetilde{\max} \text{ TEP(Q)} \tag{2}$$

subject to

$$\sum_{i=1}^{n} w_i Q_i \le \widetilde{W},$$
$$\sum_{i=1}^{n} C_i Q_i \le \widetilde{B},$$
$$Q_i \ge 0, \; i = 1,2,..., n.$$

(A wavy bar (~) represents fuzzification of the parameters.)

3. MATHEMATICAL ANALYSIS

Fuzzy non-linear programming (FNLP)

Let us consider a fuzzy non-linear programming problem

$$\max\ g_0(x) \tag{3}$$

subject to

$$g_i(x) \leq b_i, \quad (i = 1,2,3,\ldots,n).$$

In fuzzy set theory objective and fuzzy resources are represented by membership functions, which may be linear or non-linear. Here m_0 and m_1 $(i = 1,2,3,\ldots,n)$ are assumed to be non-decreasing or non-increasing linear membership functions, respectively, such as

$$m_0(g_0(x)) = \begin{cases} 0 & \text{for } g_0(x) < b_0 - P_0 \\ 1 - \dfrac{b_0 - g_0(x)}{P_0(x)} & \text{for } b_0 - P_0 \leq g_0(x) \leq b_0 \\ 1 & \text{for } g_0(x) > b_0 \end{cases}$$

$$m_i(g_i(x)) = \begin{cases} 1 & \text{for } g_i(x) < b_i \\ 1 - \dfrac{g_i(x) - b_i}{P_i} & \text{for } b_i \leq g_i(x) \leq b_i + P_i \\ 0 & \text{for } g_i(x) > b_i + P_i \end{cases}$$

In this formulation, the fuzzy objective goal is b_0 and its tolerance is P_0 and for the fuzzy constraints, the goals are b_i 's and their corresponding tolerances are P_i 's $(i = 1,2,3,\ldots,n)$.

To solve the above problem, we use the max-min operator of Bellman and Zadeh (1970) and the approach of Zimmerman (1976).

The membership function of the decision set, $m_D(x)$, is $m_D(x) = \min\{m_0(x), m_1(x), m_2(x),\ldots, m_n(x)\}$ for all $x \in X$.

The max operator is used here to model the intersection of the fuzzy sets of objective and constraints. Since the decision maker wants to have a crisp decision proposal, the maximizing decision will correspond to the value of x, $x_{\max}$ (say) that has the highest degree of membership in the decision set.

$$m_D(x_{\max}) = \max_{x \geq 0}[\min\{m_0(x), m_1(x), m_2(x),\ldots, m_n(x)\}].$$

It is equivalent to solving the following crisp non-linear programming problem:

$$\max a \tag{4}$$

subject to

$$m_0(x) \ge a,$$
$$m_i(x) \ge a, \quad (i = 1,2,3,\ldots,n).$$
$$x \ge 0, \quad a \in [0,1]$$

4. SOLUTION OF THE PROPOSED INVENTORY MODEL

By the FNLP method, the proposed inventory model depicted by eq. (2),

$$\tilde{\max}\ TEP(Q) = \sum_{i=1}^{n}\{ p_i[V_1(Q_i) + Q_iV_2(Q_i)] + s_iV_3(Q_i) - C_iQ_i - h_i[Q_i/2 + \\ + 0.5V_3(Q_i)] - p_iV_4(Q_i)\} \tag{5}$$

subject to

$$\sum_{i=1}^{n} w_iQ_i \le \tilde{W},$$
$$\sum_{i=1}^{n} C_iQ_i \le \tilde{B},$$
$$Q_i \ge 0, \ i = 1,2,\ldots,n.$$

reduced to

$$\max a \tag{6}$$

subject to

$$\sum_{i=1}^{n}\{ p_i[V_1(Q_i) + Q_iV_2(Q_i)] + s_iV_3(Q_i) - C_iQ_i - h_i[Q_i/2 + 0.5V_3(Q_i)] - p_iV_4(Q_i)\} \ge$$
$$\ge C_0 - (1-a)P_{TEP},$$
$$\sum_{i=1}^{n} w_iQ_i \le W + (1-\alpha)P_W,$$
$$\sum_{i=1}^{n} C_iQ_i \le B + (1-\alpha)P_B,$$
$$Q_i \ge 0, \ i = 1,2,\ldots,n$$
$$0 < \alpha < 1.$$

Here the profit goal is C_0 with tolerance P_{TEP}, the space constraint goal is W with tolerance P_W and the budget constraint goal is B with tolerance P_B. Now, the non-linear programming problem is solved by a computer program based on the gradient method algorithm.

5. NUMERICAL EXAMPLE

To illustrate the model (2), we assume the following numerical values of the inventory parameters as in Table 1.

Table 1.

Items	C_{1i} ($)	C_{2i} ($)	s_i ($)	h_i ($)	p_i ($)	k_i	a_i	w_i sq.ft.	q_i
1	15	0.01	12	2	10	100	1	2	1.6
2	20	0.02	13	2.5	12	120	2	3	1.5
3	18	0.01	11	2	10	110	3	4	1.6
$W = 55$ sq.ft., $B = 325(\$)$, $C_0 = 165$, $P_{TEP} = 30$, $P_W = 15$, $P_B = 50$									

So, solving the problem by FNLP, optimal results are shown in Table 2.

Table 2: Optimal values of the proposed model

model	Q_1	Q_2	Q_3	TEP ($)	α	W sq.ft.	B ($)
Crisp model	4.734	7.823	5.516	149.107	1	55	325
Fuzzy model	5.152	8.335	5.780	152.164	0.572	58.509	346.393

Here, the fuzzy model gives better result than the crisp one.

6. SENSITIVITY ANALYSIS

Here we study the effective tolerances in the proposed model with earlier numerical values and construct the following four tables.

Table 3: Effect of variations in P_{TEP}

P_{TEP}	a	TEP($)	Q_1	Q_2	Q_3	W (sq.ft.)	B($)
15	0.2494	153.741	5.52	8.73	5.96	61.08	362.53
20	0.4032	153.061	5.34	8.54	5.89	59.86	354.84
30	0.5720	152.164	5.15	8.34	5.78	58.51	346.39
50	0.7242	151.210	4.92	8.15	5.72	57.29	338.79
70	0.7960	150.710	4.90	8.06	5.68	56.72	335.20
100	0.8531	150.291	4.84	7.99	5.66	56.27	332.35
120	0.8762	150.121	4.81	7.96	5.64	56.08	331.20
150	0.8996	149.991	4.79	7.93	5.63	55.89	330.02
165	0.9018	149.872	4.78	7.92	5.63	55.82	329.58
170	0.9109	149.851	4.78	7.92	5.63	55.82	329.46

In Table 3 we see that for higher tolerances of P_0, the value of a does not achieve 1 though it is very near to 1 as expected. For higher acceptable variations of P_{TEP}, the optimal total expected profit is very nearer to thr profit goal. Here, Q_i^* $(i = 1,2,3)$ becomes invariant for large values of P_{TEP}.

Table 4: Effect of variations in P_W

P_W	a	TEP	Q_1	Q_2	Q_3	W	B
1	0.4912	149.742	5.01	7.99	5.38	55.51	329.97
4	0.5330	150.99	5.17	8.19	5.49	56.87	338.23
8	0.5719	152.160	5.20	8.26	5.74	58.42	346.41
10	0.5720	152.164	5.15	8.34	5.78	58.51	346.39
12	0.5721	152.164	5.15	8.33	5.80	58.51	346.39
30	0.5721	152.164	5.15	8.33	5.80	58.51	346.39
50	0.5721	152.164	5.15	8.33	5.80	58.51	346.39

In Table 4, as P_W decreases below 10, the total expected profit decreases and the total expected profit becomes invariant for large values of P_W.

Table 5: Effect of variations in P_B

P_B	a	TEP($)	Q_1	Q_2	Q_3	W (sq.ft.)	B($)
10	0.4951	149.742	4.79	7.93	5.63	55.90	330.02
30	0.5403	150.991	4.98	8.15	5.72	57.29	338.79
50	0.5720	152.160	5.15	8.34	5.78	58.51	346.39
70	0.5892	152.164	5.15	8.51	5.67	59.11	351.86
100	0.5892	152.164	5.15	8.51	5.67	59.11	351.86
1000	0.5892	152.164	5.15	8.51	5.67	59.11	351.86

In Table 5, as P_B decreases below 50, the total expected profit decreases but when $P_B \geq 70$ total expected profit remains invariant.

7. DISCUSSION

In a realistic inventory system, some inventory parameters are crisp, some probabilistic, and others imprecise. Here, we have for the first time formulated a real-life stochastic inventory model for multi-items in a fuzzy environment by the FNLP technique. Till now, nobody has solved an inventory problem in a mixed environment. Some sensitivity analysis on the tolerance limits along with a numerical example have been presented. For further research, this method can be extended to multi-objective inventory problems in mixed environments.

REFERENCES

[1] Naddor, E., Inventory Systems, John Wiley, New York, 1966.
[2] Silver, E.A., and Peterson, R., Decision System for Inventory Management and Research, John Wiley, New York, 1985.
[3] Churchman, C.W., Ackoff, R.L., and Arnoff, F.L., Introduction to Operations Research, John Wiley, New York, 1957.
[4] Hadley, G., and Whitin, T.M., Analysis of Inventory Systems, Prentice Hall, Ingleword Cliffs, N J, 1963.
[5] Ben-Daya, M., and Raouf, A., "On the constrained multi-item single-period inventory problem", Int. J. Prod. Mgmt., 13 (1993) 104-112.
[6] Bellman, R.E., and Zadeh, L.A., "Decision-making in a fuzzy environment", Management Sci., 17 (4) (1970), 141-164.
[7] Tanaka, H., Okuda, T., and Asai, K., "On fuzzy mathematical programming", J. Cybernetics, 3 (4) (1974) 37-46.

[8] Zimmermann, H.-J., "Description and optimization of fuzzy systems", Int. J. Gen. Sys., 2 (4) (1976) 209-215.

[9] Sommer, G., "Fuzzy inventory scheduling", in: G. Lasker (ed.), Applied Systems and Cybernetics, VI, Academic Press, New York, 1981.

[10] Kacprzyk, J., and Staniewski, P., "Long term inventory policy-making through fuzzy decision making models", Fuzzy Sets and Systems, 8 (1982) 117-132.

[11] Park, K.S., "Fuzzy set theoretic interpretation of economic order quantity", IEEE Transaction on Systems, Man. and Cybernetics, 17(16) (1987) 1082-1084.

[12] Lai, Y.J., and Hwang, C.L., Fuzzy Mathematical Programming: Methods and Applications, Springer-Velag, Heidelberg, 1992.

[13] Lai, Y.J., and Hwang, C.L., Fuzzy Multiple Objective Decision Making, Springer-Verlag, Heidelberg, 1994.

[14] Roy, T.K., and Maiti, M., "A fuzzy inventory model with constraints", Opsearch, 32 (4) (1995) 287-298.

[15] Roy, T.K. and Maiti, M., "A fuzzy EOQ model with demand-dependent unit cost under limited storage capacity", European Journal of Operational Research, 99 (1997) 425-432.

APPENDIX

Expression for expected profit

Recall that:

$$TEP(Q_1, Q_2, ..., Q_n) = \sum_{i=1}^{n} \{ p_i \int_0^{Q_i} x f_i(x) dx + p_i Q_i \int_{Q_i}^{\infty} f_i(x) dx + s_i \int_0^{Q_i} (Q_i - x) f_i(x) dx -$$

$$- C_i Q_i - h_i [Q_i / 2 + 0.5 \int_0^{Q_i} (Q_i - x) f_i(x) dx] - \pi_i \int_{Q_i}^{\infty} (x - Q_i) f_i(x) dx \} =$$

$$= \sum_{i=1}^{n} \{ p_i [V_1(Q_i) + Q_i V_2(Q_i)] + s_i V_3(Q_i) - C_i Q_i - h_i [Q_i / 2 + 0.5 V_3(Q_i)] - \pi_i V_4(Q_i) \}$$

where

$$V_1(Q_i) = \int_0^{Q_i} x f_i(x) dx =$$

$$= \frac{1}{\Gamma(a_i)} \int_0^{Q_i} x^{a_i} e^{-\frac{p_i}{k_i} x} \left(\frac{p_i}{k_i}\right)^{a_i} dx =$$

$$= \frac{1}{\Gamma(a_i)} \left(\frac{p_i}{k_i}\right)^{a_i} [-\{ \frac{k_i}{p_i} Q_i^{a_i} + \left(\frac{k_i}{p_i}\right)^2 a_i Q_i^{(a_i - 1)} + \left(\frac{k_i}{p_i}\right)^3 a_i (a_i - 1) Q_i^{(a_i - 2)} + ...$$

$$... + \left(\frac{k_i}{p_i}\right)^{(a_i + 1)} (a_i)! \} e^{-\frac{p_i Q_i}{k_i}} + \left(\frac{k_i}{p_i}\right)^{(a_i + 1)} (a_i)!]$$

$$V_2(Q_i) = \int_{Q_i}^{\infty} f_i(x)dx =$$

$$= \frac{1}{\Gamma(a_i)} \int_{Q_i}^{\infty} x^{a_i - 1} e^{-\frac{p_i}{k_i}x} (\frac{p_i}{k_i})^{a_i} dx =$$

$$= \frac{1}{\Gamma(a_i)} (\frac{p_i}{k_i})^{a_i} [\frac{k_i}{p_i} Q_i^{(a_i-1)} + (\frac{k_i}{p_i})^2 (a_i - 1) Q_i^{(a_i-2)} + \ldots + (\frac{k_i}{p_i})^{a_i} (a_i - 1)!] e^{-\frac{p_i Q_i}{k_i}}$$

$$V_3(Q_i) = \int_0^{Q_i} (Q_i - x) f_i(x) dx =$$

$$= \frac{1}{\Gamma(a_i)} (\frac{p_i}{k_i})^{a_i} [(\frac{k_i}{p_i})^2 Q_i^{(a_i-1)} + (\frac{k_i}{p_i})^3 2(a_i - 1) Q_i^{(a_i-2)} + \ldots$$

$$\ldots + (\frac{k_i}{p_i})^{a_i} \{a_i! - (a_i - 1)!\} Q_i + (\frac{k_i}{p_i})^{(a_i+1)} (a_i)!] + \{Q_i - a_i \frac{k_i}{p_i}\} \frac{(a_i - 1)!}{\Gamma(a_i)}$$

$$V_4(Q_i) = \int_{Q_i}^{\infty} (x - Q_i) f_i(x) dx =$$

$$= \frac{1}{\Gamma(a_i)} (\frac{p_i}{k_i})^{a_i} [(\frac{k_i}{p_i})^2 Q_i^{(a_i-1)} + (\frac{k_i}{p_i})^3 2(a_i - 1) Q_i^{(a_i-2)} + \ldots$$

$$\ldots + (\frac{k_i}{p_i})^{a_i} \{a_i! - (a_i - 1)!\} Q_i + (\frac{k_i}{p_i})^{(a_i+1)} (a_i)!] e^{-\frac{p_i}{k_i} Q_i}$$