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ANALYSIS OF AN INTEGRATED PRODUCTION-SALES PROBLEM FOR ONE-MACHINE, TWO-PRODUCT SYSTEM

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Abstract: In the present study, we are concerned with the modeling and analysis of one-machine, two-product, integrated production-sales system, in which three decision-makers that take different responsibilities are considered. Through the constructed mathematical model, the relations that show the optimal solutions of decision variables are derived. It is found that the ratio of the optimal production horizons of two products is equal to the product of the ratio of the sales rates of the two products to the ratio of unit costs within their production horizons. Through the sensitivity analysis, direct or indirect influences on the optimal solution induced by the variation of parameters are discussed.

Keywords: Production planning, pricing, optimization, decision analysis.

1. INTRODUCTION

Today, market competition has become more and more internationalized and aggressive. It has become more difficult for a company to increase profits simply by controlling selling prices. Thus some American companies have proposed the new management idea of ZWC (Zero Working Capital) and put it into practice. Under the condition of not increasing a company's fixed assets, employees and liability, the purpose of ZWC management is to compress the company's current assets, decrease working capital and lower production costs in order to increase profits [6].

Nowadays, in Taiwan, small and medium enterprises are facing increasingly aggressive market competition, unexpected economic cycles and the limitation of production capacity. In order to ensure a company's growth and long term prosperity, ZWC provides new opportunities to improve existing difficulties. ZWC management tries to develop new products to satisfy or increase market demand by utilizing existing technologies and facilities. It is through ZWC that a company can fulfill the purpose of developing new products and satisfying customers' needs by way of minimizing development costs.

Among the published articles, most of the mathematical models that utilize the same manufacturing facility to produce various products (or one product with various sizes or types) only focus on scheduling, production lot sizing and selling price control. These mathematical models were made simply based on one-way decision making. The problems that have been raised are as follows:

1. Problems concerned with scheduling: Boctor [1] dealt with the two-product, single-machine, static demand, infinite horizon lot scheduling problem and derived the necessary and sufficient conditions for the feasibility of two-product schedules without adding any preliminary requirements. Hodgson and Ge [5] proposed the modeling and analysis of the optimal dynamic lot size and sequencing policies in a single-machine, multi-product, integrated production-inventory system.

2. Problems concerned with production lot sizing on inventory: Schwarz [11] proposed a model for the infinite horizon, continuous review, and deterministic planning problem. Taylor and Bradley [12] developed the optimal ordering strategies for situations where the price increase becomes effective at any future specified time. Chen and Lai [3] considered a monopoly agent's optimal control of inventory and prices over a given selling planning horizon. Chen and Chen [4] studied the optimal production rates of a basic assembly system under random demand and found that the optimal production rates of semi-finished or finished goods were the same in some circumstances.

3. Problems concerned with controlling the sales price of inventory: Rakesh and Steinberg [9] considered the relationship between dynamic pricing and ordering decisions for a monopolistic retailer facing known demand. Raman and Chatterjee [10] proposed a stochastic optimization model to study the optimal pricing policy under demand uncertainty in dynamic markets; they found that the degree of impact of demand uncertainty on the optimal pricing policy is determined by the interaction among uncertainty, demand and/or cost dynamics, and the firm's discount rate.

From the above review of the literature, it can be seen that a model which can deal with the control of manufacturing and selling for small and medium enterprises, especially for those that produce various products in the same facility, does not exist. The purpose of this study is to explore the possibility of putting the ZWC management concept into practice, for small and medium enterprises, in order to fully utilize an existing facility and reach maximum efficiency.

2. NOTATIONS AND ASSUMPTIONS

The following notations are used in this paper:

q_1 : The production rate of Product 1

q_2 : The production rate of Product 2

h_1 : The unit inventory holding cost of Product 1 per unit time

h_2 : The unit inventory holding cost of Product 2 per unit time

p_1 : The unit price of Product 1

p_2 : The unit price of Product 2

$-a_1 p_1 + b_1$: The sales rate of Product 1, where $a_1 > 0$, $b_1 > 0$, $0 \le p_1 \le \frac{b_1}{a_1}$

$-a_2 p_2 + b_2$: The sales rate of Product 2, where $a_2 > 0$, $b_2 > 0$, $0 \le p_2 \le \frac{b_2}{a_2}$

$[0, T]$: The cycle of selling time to sell Product 1 and Product 2, T is the length of sales time

E : Production horizon within $[0, T]$ for Product 1, where $0 \le E \le T$

$\overline{E} = T - E$: Production horizon within $[0, T]$ for Product 2

For the coordination between production and sales, instant production behavior (manufacturing supply instant needs) can reduce the cost of holding stock, however the cost of production will be relatively high. This means that under the situation of limited manufacturing capacity, the cost of production per unit will increase followed by an increase in the production rate. Therefore, we predict the cost for unit production as follows:

$c_1 q_1$: Production cost per unit for Product 1, where $c_1 > 0$

$c_2 q_2$: Production cost per unit for Product 2, where $c_2 > 0$

where $a_1, b_1, c_1, h_1, T, a_2, b_2, c_2, h_2$ are parameters of decision-makers I, II and III, simultaneously, and q_1, p_1 are decision variables of decision-maker I, E is a parameter of decision-maker I, II, but is a decision variable of decision-maker III, q_2 and p_2 are decision variables of decision-maker II. In the present study, two assumptions are made:

(1) Utilize one machine to produce two kinds of products (these two kinds of products can be two different types of one product), namely, Product 1 and Product 2. The choice of production interval must match the sales decisions of the two types of products.

(2) There are three decision-makers that take different responsibilities. Under a given production horizon of Product 1, the problem for decision-maker I is how to decide the production rate and the selling price of this product in order to attain the maximum profit. Under a given production horizon of Product 2,the problem for decision-maker II is how to decide the production rate and the selling price of this product in order to attain the maximum profit. The problem for decision-maker III is how to arrange (or distribute) the production horizon for Product 1 and Product 2, in order to attain the maximum profits.

Figure 1 shows the behavior of this two-product one-machine production-sales system. Consider a given cycle of selling horizon $[0, T]$; as shown in Fig. 1 Product 1 is online with production rate q_1 in the time interval $[0, E]$, and is sold with a sales rate of $-a_1P_1 + b_1$ from $t = 0$. Product 2 is offline in the time interval $[0, E]$, but is online with production rate q_2 right after Product 1 is offline, and is sold with a sales rate of $-a_2P_2 + b_2$. Product 1 is supposed to be sold off at time T and Product 2 is supposed to be offline at time T.

It is obvious that, under optimal conditions, the offline timing for one product equals the sold off timing (the timing without stock) for the other product. i.e.

E = Product 1 offline timing = Product 2 online timing

T = Product 2 offline timing = Product 1 online timing

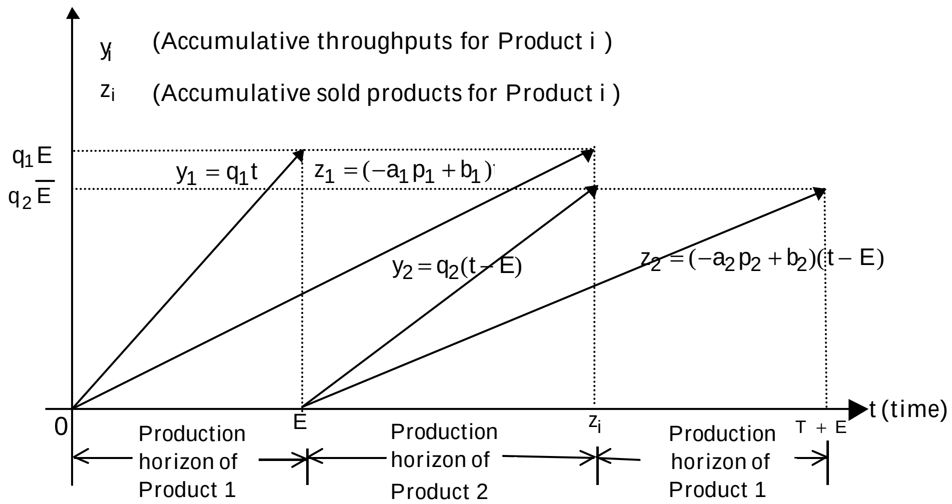

Figure 1: The relationship between accumulative throughputs and sold amounts of two products

It is noted that Product 2 is offline within the interval $[0, E]$ and Product 1 is offline within the interval $[E, T]$, so that the quantity of production for Product 1 within $[0, T]$ is equal to the quantity of sales for Product 1 at $t = T$. i.e.

$$q_1 E = (-a_1 p_1 + b_1) T$$

Similarly, the quantity of production for Product 2 within $[E, T + E]$ is equal to the quantity of sales for Product 2 within $[E, T + E]$ i.e.

$$q_2 \overline{E} = (-a_2 p_2 + b_2) T$$

Applying the above relationships, we have

The unit cost for manufacturing Product 1 $= c_1 q_1 = c_1 (\frac{-a_1 p_1 + b_1}{E}) T$

The unit cost for manufacturing Product 2 $= c_2 q_2 = c_2 (\frac{-a_2 p_2 + b_2}{\overline{E}}) T$

The inventory holding cost of Product 1 within $[0, T]$

$$= h_1 (\int_0^E ((\frac{-a_1 p_1 + b_1}{E}) T - (-a_1 p_1 + b_1)) t dt + \int_E^T (-a_1 p_1 + b_1) \cdot (T - t) dt)$$
$$= h_1 (\frac{T\overline{E}}{2})(-a_1 p_1 + b_1)$$

The inventory holding cost of Product 2 within $[0, T]$

$$= h_2 (\int_0^{\overline{E}} ((\frac{-a_2 p_2 + b_2}{\overline{E}}) T - (-a_2 p_2 + b_2)) t dt + \int_{\overline{E}}^T (-a_2 p_2 + b_2) \cdot (T - t) dt)$$
$$= h_2 (\frac{TE}{2})(-a_2 p_2 + b_2)$$

3. MATHEMATICAL MODEL

Using the notations and assumptions of the previous section, the mathematical model for the problem faced by decision-makers can be constructed as follows:

Problem faced by decision-maker I:

For decision-maker I, who is responsible for the sales of Product 1, the problem is, for a given E, how to decide the production rate q_1, and the sales price p_1, in order to attain the maximum profit from Product 1 within the time interval $[0, T]$. The mathematical model (Model I) is as follows:

$$\max\ f(p_1, q_1) = [(p_1 - c_1 \cdot q_1)](-a_1 p_1 + b_1)T \ - (-a_1 p_1 + b_1) h_1 (\frac{T\overline{E}}{2}) \tag{3.1}$$

$$\text{s.t.} \quad q_1 E = (-a_1 p_1 + b_1)T\ ,\ \ q_1 \ge 0\ ,\ \ 0 \le p_1 \le \frac{b_1}{a_1} \tag{3.2}$$

Problem faced by decision-maker II:

For decision-maker II, who is responsible for the sales of Product 2, the problem is, for a given $\overline{E}$ (Note that $\overline{E} = T - E$), how to decide the production rate q_2, and the sales price p_2, in order to attain the maximum profit from Product 2 within the time interval $[0, T]$. The mathematical model (Model II) is as follows:

$$\max\ g(p_2, q_2) = [(p_2 - c_2 \cdot q_2)](-a_2 p_2 + b_2)T \ - (-a_2 p_2 + b_2) h_2 (\frac{TE}{2}) \tag{3.3}$$

$$\text{s.t.} \quad q_2 \overline{E} = (-a_2 p_2 + b_2)T\ ,\ \ q_2 \ge 0\ ,\ \ 0 \le p_2 \le \frac{b_2}{a_2} \tag{3.4}$$

Problem faced by decision-maker III:

Let $(\overline{p}_1(E), \overline{q}_1(E))$, $(\overline{p}_2(E), \overline{q}_2(E))$ be the optimal solutions of Model I and Model II. Then the problem that decision-maker III will face is how to determine the online timing E for Product 1 and the online timing $\overline{E}$ for Product 2, in order to attain the maximum profit from these two products. The mathematical model (Model III) is as follows:

$$\max_{0 \le E \le T}\ L(E) = f(\overline{p}_1(E), \overline{q}_1(E)) + g(\overline{p}_2(E), \overline{q}_2(E)) \tag{3.5}$$

4. OPTIMAL SOLUTIONS

4.1. Optimal solution of decision-maker I

The notation $f(p_1)$ is used instead of $f(p_1, q_1(p_1))$, where $q_1(p_1)$ is determined by constraint (3.2), i.e. $q_1(p_1)E = (-a_1 p_1 + b_1)T$. Let $(\overline{p}_1, \overline{q}_1)$, where $\overline{q}_1 = q_1(\overline{p}_1)$, be the optimal solution of Model I (i.e. $\overline{p}_1$ is the optimal solution of $f(p_1)$). Note that $f(0) < 0$, $f(\frac{b_1}{a_1}) = 0$, $f'(\frac{b_1}{a_1}) = -b_1 T - h_1 \frac{\overline{E}T}{2} < 0$. Hence we have $\overline{p}_1 \ne 0, \overline{p}_1 \ne \frac{b_1}{a_1}$, and thus $0 < \overline{p}_1 < \frac{b_1}{a_1}$. Under the condition of $f'' < 0$, the necessary and sufficient condition for $\overline{p}_1$, which is the optimal solution of the problem for decision-maker I in Eq.(3.1), can be expressed as

$$2(1+\frac{a_1 c_1 T}{E})\bar{p}_1 = \frac{2b_1 c_1 T}{E} + \frac{h_1 \bar{E}}{2} + \frac{b_1}{a_1}$$

Hence, the optimal solution $(\bar{p}_1, \bar{q}_1)$ can be derived as

$$\bar{p}_1 = \frac{2b_1 c_1 T + \frac{h_1}{2} E\bar{E} + \frac{b_1}{a_1} E}{2(E + a_1 c_1 T)} \tag{4.1}$$

$$\bar{q}_1 = \frac{(-a_1 h_1 \bar{E} + 2b_1)T}{4(E + a_1 c_1 T)} \tag{4.2}$$

4.2. Optimal solution of decision-maker II

Similarly, the notation $g(p_2)$ is used instead of $g(p_2, q_2(p_2))$, where $q_2(p_2)$ is determined by constraint (3.4), i.e. $q_2(p_2)\bar{E} = (-a_2 p_2 + b_2)T$. Let $(\bar{p}_2, \bar{q}_2)$, where $\bar{q}_2 = q_2(\bar{p}_2)$, be the optimal solution of Model II (i.e. $\bar{p}_2$ is the optimal solution of $g(p_2)$). Note that $g(0) < 0$, $g(\frac{b_2}{a_2}) = 0$, $g'(\frac{b_2}{a_2}) = -b_2 T - h_2 \frac{ET}{2} < 0$. Hence we have $\bar{p}_2 \neq 0, \bar{p}_2 \neq \frac{b_2}{a_2}$, and thus $0 < \bar{p}_2 < \frac{b_2}{a_2}$. Under the condition of $g'' < 0$, the necessary and sufficient condition for $\bar{p}_2$, which is the optimal solution of the problem for decision-maker II in Eq.(3.3), can be expressed as

$$2(1+\frac{a_2 c_2 T}{\bar{E}})\bar{p}_2 = \frac{2b_2 c_2 T}{\bar{E}} + \frac{h_2 E}{2} + \frac{b_2}{a_2}$$

Therefore, the optimal solution $(\bar{p}_2, \bar{q}_2)$ can be derived as

$$\bar{p}_2 = \frac{2b_2 c_2 T + \frac{h_2}{2} E\bar{E} + \frac{b_2}{a_2} \bar{E}}{2(\bar{E} + a_2 c_2 T)} \tag{4.3}$$

$$\bar{q}_2 = \frac{(-a_2 h_2 E + 2b_2)T}{4(\bar{E} + a_2 c_2 T)} \tag{4.4}$$

4.3. Optimal solution of decision-maker III

Suppose E^* is the optimal solution of Model III and assume that $E^* \in (0, T)$ (since $E^* = 0$ or $E^* = T$ implies one of the two products will disappear from the current market). Hence E^* must satisfy the following two necessary conditions:

$$0=\left.\frac{dL}{dE}\right|_{E^*}$$

$$=\left[(-a_1\overline{p}_1+b_1)T\left(\frac{c_1T(-a_1\overline{p}_1+b_1)}{E^2}+\frac{h_1}{2}\right)-(-a_2\overline{p}_2+b_2)T\left(\frac{c_2T(-a_2\overline{p}_2+b_2)}{\overline{E}^2}+\frac{h_2}{2}\right)\right]\Bigg|_{E=E^*}$$

$$=T[A(E)-B(E)]\Big|_{E=E^*} \tag{4.5}$$

$$0\geq\left.\frac{d^2L}{dE^2}\right|_{E=E^*}=A'(E^*)-B'(E^*) \tag{4.6}$$

where

$$A(E)=(-a_1\overline{p}_1+b_1)\left[\frac{c_1T(-a_1\overline{p}_1+b_1)}{E^2}+\frac{h_1}{2}\right] \tag{4.7}$$

$$B(E)=(-a_2\overline{p}_2+b_2)\left[\frac{c_2T(-a_2\overline{p}_2+b_2)}{\overline{E}^2}+\frac{h_2}{2}\right] \tag{4.8}$$

Applying (4.1) and (4.3), $A(E)$ and $B(E)$ can be expressed as

$$A(E)=\frac{c_1T(-a_1h_1\overline{E}+2b_1)^2}{16(E+a_1c_1T)^2}+\frac{(-a_1h_1\overline{E}+2b_1)h_1E}{8(E+a_1c_1T)}$$

$$B(E)=\frac{c_2T(-a_2h_2E+2b_2)^2}{16(\overline{E}+a_2c_2T)^2}+\frac{(-a_2h_2E+2b_2)h_2\overline{E}}{8(\overline{E}+a_2c_2T)}$$

Thus, if the optimal solution E^* exists, then the following relations must be satisfied

$$\left(\frac{c_1T(-a_1h_1\overline{E}+2b_1)^2}{(E+a_1c_1T)^2}+\frac{2(-a_1h_1\overline{E}+2b_1)h_1E}{(E+a_1c_1T)}\right)_{E=E^*} =\left(\frac{c_2T(-a_2h_2E+2b_2)^2}{(\overline{E}+a_2c_2T)^2}+\frac{2(-a_2h_2E+2b_2)h_2\overline{E}}{(\overline{E}+a_2c_2T)}\right)_{E=E^*} \tag{4.9}$$

and we can also obtain

$$\frac{E^*}{\overline{E}^*}=\left[\frac{(-a_1\overline{p}_1+b_1)}{(-a_2\overline{p}_2+b_2)}\cdot\frac{\frac{c_1T(-a_1\overline{p}_1+b_1)}{E}+\frac{h_1}{2}E}{\frac{c_2T(-a_2\overline{p}_2+b_2)}{\overline{E}}+\frac{h_2}{2}\overline{E}}\right]\Bigg|_{E=E^*} \tag{4.10}$$

Eq.(4.10) indicates that the ratio of the optimal production horizons of two products equals the product of the ratio of the sales rates of the two products and the ratio of unit costs within their production horizons (the unit cost within the production horizon is defined as the production cost per unit plus the cost of the first product that is under manufacture and is sold in the middle of the processing period).

5. SENSITIVITY ANALYSIS

(A) Effects of a variation of parameters on the optimal solution E^*

(a) Effect on E^* due to a variation of parameter h_1

By differentiating Eq.(4.5) with respect to h_1, we obtain

$$0=\frac{d}{dh_1}[A(E^*,h_1)-B(E^*,h_1)]$$
$$=\left\{\left[\frac{\partial A(E^*,h_1)}{\partial E}\frac{\partial E^*}{\partial h_1}+\frac{\partial A(E^*,h_1)}{\partial h_1}\right]-\left[\frac{\partial B(E^*,h_1)}{\partial E}\frac{\partial E^*}{\partial h_1}+\frac{\partial B(E^*,h_1)}{\partial h_1}\right]\right\}$$

Hence, by using (4.6), (4.7) and (4.8), we have

$$\frac{\partial E^*}{\partial h_1}=\frac{\left(\frac{\partial A(E^*,h_1)}{\partial h_1}-\frac{\partial B(E^*,h_1)}{\partial h_1}\right)}{\left(\frac{\partial B(E^*,h_1)}{\partial E}-\frac{\partial A(E^*,h_1)}{\partial E}\right)}=\frac{\frac{1}{2}(-a_1\overline{p}_1+b_1)}{\left(\frac{\partial B(E^*,h_1)}{\partial E}-\frac{\partial A(E^*,h_1)}{\partial E}\right)}>0 \qquad (5.1)$$

(b) Effect on E^* due to a variation of parameter h_2

By differentiating Eq.(4.5) with respect to h_2, we obtain

$$0=\frac{d}{dh_2}\left[A(E^*,h_2)-B(E^*,h_2)\right]$$
$$=\left\{\left[\frac{\partial A(E^*,h_2)}{\partial E}\frac{\partial E^*}{\partial h_2}+\frac{\partial A(E^*,h_2)}{\partial h_2}\right]-\left[\frac{\partial B(E^*,h_2)}{\partial E}\frac{\partial E^*}{\partial h_2}+\frac{\partial B(E^*,h_2)}{\partial h_2}\right]\right\}$$

Hence, by using (4.6), (4.7) and (4.8), we have

$$\frac{\partial E^*}{\partial h_2}=\frac{\left(\frac{\partial B(E^*,h_2)}{\partial h_2}-\frac{\partial A(E^*,h_2)}{\partial h_2}\right)}{\left(\frac{\partial A(E^*,h_2)}{\partial E}-\frac{\partial B(E^*,h_2)}{\partial E}\right)}=\frac{\frac{1}{2}(-a_2\overline{p}_2+b_2)}{\left(\frac{\partial A(E^*,h_2)}{\partial E}-\frac{\partial B(E^*,h_2)}{\partial E}\right)}<0 \qquad (5.2)$$

(c) Effect on E^* due to a variation of other parameters

By using similar manipulations as in the cases mentioned above, the expressions concerning the effect on optimal solution E^* due to variations of $a_1, b_1, c_1, T, a_2, b_2, c_2$, respectively, can be obtained.

Applying the derived expressions, a summary of the sensitivity analysis of decision variable E^* with respect to parameters is presented in Table 1.

Table 1: The sensitivity analysis of decision variable E^* with respect to parameters

Decision variable	Parameters								
	a_1	b_1	c_1	h_1	T	a_2	b_2	c_2	h_2
E^*	–	+	+	+	+	+	–	–	–

"+": Decision variable is an increasing function of the parameter.

"–": Decision variable is a decreasing function of the parameter.

(B) Effects of a variation of parameters on the optimal solution (p_1^*, q_1^*) and (p_2^*, q_2^*), respectively.

The optimal solutions of Model I and Model II under a given E^* are (p_1^*, q_1^*) and (p_2^*, q_2^*), respectively.

(a) Effects on (p_1^*, q_1^*) and (p_2^*, q_2^*) due to variations of h_1, a_1, b_1, c_1, T

From Eqs.(4.1), (4.2), (4.3) and (4.4), we obtain

$$p_1^* = \overline{p}_1(E^*(h_1), h_1) = \frac{2b_1c_1T + \frac{h_1}{2}E^*\overline{E}^* + \frac{b_1}{a_1}E^*}{2(E^* + a_1c_1T)} \tag{5.3}$$

$$q_1^* = \overline{q}_1(E^*(h_1), h_1) = \frac{(-a_1h_1\overline{E}^* + 2b_1)T}{4(E^* + a_1c_1T)} \tag{5.4}$$

$$p_2^* = \overline{p}_2(E^*(h_1), h_1) = \frac{2b_2c_2T + \frac{h_2}{2}E^*\overline{E}^* + \frac{b_2}{a_2}\overline{E}^*}{2(\overline{E}^* + a_2c_2T)} \tag{5.5}$$

$$q_2^* = \overline{q}_2(E^*(h_1), h_1) = \frac{(-a_2h_2E^* + 2b_2)T}{4(\overline{E}^* + a_2c_2T)} \tag{5.6}$$

Differentiating Eqs.(5.3),(5.4),(5.5) and (5.6) with respect to h_1, we obtain

$$\left.\frac{d\overline{p}_1}{dh_1}\right|_{E=E^*}=\left.\frac{\left(-\frac{h_1}{2}E^2+\frac{a_1c_1h_1}{2}T(\overline{E}-E)-b_1c_1T\right)\frac{\partial E}{\partial h_1}+\frac{1}{2}E^2\overline{E}+\frac{a_1c_1T}{2}E\overline{E}}{2(E+a_1c_1T)^2}\right|_{E=E^*} \tag{5.7}$$

$$\left.\frac{d\overline{q}_1}{dh_1}\right|_{E=E^*}=\left.\frac{(a_1^2c_1h_1T^2+a_1h_1T^2-2b_1T)\frac{\partial E}{\partial h_1}-a_1TE\overline{E}-a_1^2c_1T^2\overline{E}}{4(E+a_1c_1T)^2}\right|_{E=E^*} \tag{5.8}$$

$$\left.\frac{d\overline{p}_2}{dh_1}\right|_{E=E^*}=\left.\frac{\left(\frac{h_1}{2}\overline{E}^2+\frac{a_2c_2h_2}{2}T(\overline{E}-E)+b_2c_2T\right)\frac{\partial E}{\partial h_1}}{2(\overline{E}+a_2c_2T)^2}\right|_{E=E^*} \tag{5.9}$$

$$\left.\frac{d\overline{q}_2}{dh_1}\right|_{E=E^*}=\left.\frac{\left(-a_2^2c_2h_2T^2-a_2h_2T^2+2b_2T\right)\frac{\partial E}{\partial h_1}}{4(\overline{E}+a_2c_2T)^2}\right|_{E=E^*} \tag{5.10}$$

Similarly, we can obtain the rate of change of (p_1^*,q_1^*) and (p_2^*,q_2^*) with respect to a_1,b_1,c_1 and T. It has been mentioned that (5.7), (5.8) represent the direct effects of a variation of parameter h_1 and (5.9), (5.10) represent the indirect effects of a variation of parameter h_1. The indirect effects on optimal solution q_2^* due to variations of a_1,b_1,c_1,h_1 and T can be summarized as shown in Table 2.

Table 2: The sensitivity analysis of decision variable q_2^* with respect to parameters

Conditions	Decision variable	Parameters				
		a_1	b_1	c_1	h_1	T
$-a_2^2c_2h_2T^2-a_2h_2T^2+2b_2T>0$	q_2^*	–	+	+	+	+
$-a_2^2c_2h_2T^2-a_2h_2T^2+2b_2T<0$	q_2^*	+	–	–	–	–

(b) Effects on (p_1^*,q_1^*) and (p_2^*,q_2^*) due to variations of h_2,a_2,b_2,c_2,T

From Eq.(4.1), (4.2), (4.3) and (4.4), we obtain

$$p_1^*=\overline{p}_1(E^*(h_2),h_2)=\frac{2b_1c_1T+\frac{h_1}{2}E^*\overline{E}^*+\frac{b_1}{a_1}E^*}{2(E^*+a_1c_1T)} \tag{5.11}$$

$$q_1^* = \overline{q}_1(E^*(h_2), h_2) = \frac{(-a_1 h_1 \overline{E}^* + 2b_1)T}{4(E^* + a_1 c_1 T)} \tag{5.12}$$

$$p_2^* = \overline{p}_2(E^*(h_2), h_2) = \frac{2b_2 c_2 T + \frac{h_2}{2} E^* \overline{E}^* + \frac{b_2}{a_2} \overline{E}^*}{2(\overline{E}^* + a_2 c_2 T)} \tag{5.13}$$

$$q_2^* = \overline{q}_2(E^*(h_2), h_2) = \frac{(-a_2 h_2 E^* + 2b_2)T}{4(\overline{E}^* + a_2 c_2 T)} \tag{5.14}$$

Differentiating Eqs. (5.11), (5.12), (5.13) and (5.14) with respect to h_2, we obtain

$$\left.\frac{d\overline{p}_1}{dh_2}\right|_{E=E^*} = \left.\frac{\left(-\frac{h_1}{2} E^2 + \frac{a_1 c_1 h_1}{2} T(\overline{E} - E) - b_1 c_1 T\right)\frac{\partial E}{\partial h_2}}{2(E + a_1 c_1 T)^2}\right|_{E=E^*} \tag{5.15}$$

$$\left.\frac{d\overline{q}_1}{dh_2}\right|_{E=E^*} = \left.\frac{(a_1^2 c_1 h_1 T^2 + a_1 h_1 T^2 - 2b_1 T)\frac{\partial E}{\partial h_2}}{4(E + a_1 c_1 T)^2}\right|_{E=E^*} \tag{5.16}$$

$$\left.\frac{d\overline{p}_2}{dh_2}\right|_{E=E^*} = \left.\frac{\left(\frac{h_2}{2} \overline{E}^2 + \frac{a_2 c_2 h_2}{2} T(\overline{E} - E) + b_2 c_2 T\right)\frac{\partial E}{\partial h_2} + \frac{1}{2} E\overline{E}^2 + \frac{a_2 c_2 T}{2} E\overline{E}}{2(\overline{E} + a_2 c_2 T)^2}\right|_{E=E^*} \tag{5.17}$$

$$\left.\frac{d\overline{q}_2}{dh_2}\right|_{E=E^*} = \left.\frac{(-a_2 h_2 T^2 - a_2^2 c_2 h_2 T^2 + 2b_2 T)\frac{\partial E}{\partial h_2} - a_2 T E\overline{E} - a_2^2 c_2 T^2 E}{4(\overline{E} + a_2 c_2 T)^2}\right|_{E=E^*} \tag{5.18}$$

Similarly, we can obtain the rate of change of (p_1^*, q_1^*) and (p_2^*, q_2^*) with respect to a_2, b_2, c_2 and T. It is also noted that (5.15) and (5.16) represent the indirect effects due to a variation of parameter h_2, (5.17) and (5.18) represent the direct effects due to a variation of parameter h_2. The indirect effects on optimal solution q_1^* due to variations of a_2, b_2, c_2, h_2 and T can be summarized as shown in Table 3.

Table 3: The sensitivity analysis of decision variable q_1^* with respect to parameters

Conditions	Decision variable	Parameters				
		a_2	b_2	c_2	h_2	T
$a_1^2 c_1 h_1 T^2 + a_1 h_1 T^2 - 2b_1 T > 0$	q_1^*	+	–	–	–	+
$a_1^2 c_1 h_1 T^2 + a_1 h_1 T^2 - 2b_1 T < 0$	q_1^*	–	+	+	+	–

6. DISCUSSION AND CONCLUSIONS

Small and medium enterprises have played an important role in Taiwan's economic development. However, due to changes in the domestic industrial environment and pressure from internationalization and deregulation, small and medium enterprises have to consolidate various sources in order to fit into the changing industrial climate and amplify business performance. To do so, these companies have to fully utilize the existing facilities to produce products that can cater to consumers that prefer unique, new and changeable types of products. At the same time, products have to be diversified within short production time periods. This is the reason why we considered a problem that utilizes a single machine to manufacture two products.

In recent years, computerization has become a trend in Taiwan. Small and medium enterprises realized that they have to computerize their management in order to improve their business and increase their competitiveness. Due to the prevalence of a global computer network system, small and medium enterprises can promote the sales of their products and engage in sales through this system. As a consequence, the integration between manufacturing and sales becomes easy even for those firms that are facing insufficient manpower. That is the main reason why this study focused on the integration of sales and manufacturing.

With a combined production and sales operations, the objective is not purely to reduce production costs or control pricing. This is why we have bundled production cost and sales price together. As to practical business management, due to limited human resources, the decision-makers of small and medium enterprises do not want and are unable to collect highly analytical information technology. They can only make decisions based on some simple signals. This is why the variation of parameters in this model is presumed to be linear.

In this study, we presented a mathematical model that focused on an integrated production-sale system for a one-machine, two-product problem. It is through this model that we can probe problems of this type and find out the optimal solutions that are the optimal production rate for each type of product, the optimal selling price, and the optimal production horizon. Under the circumstance of an optimal production horizon for each type of product, it is shown that the ratio of the optimal production horizons of the two products equals the ratio of the sales rates of the two products multiplied by the ratio of unit costs within the production horizon of the two products.

Among the parameters related to the production of Product 1, it is found that a variation of a_1 has a direct and negative influence on its production horizon E^* and the variations of b_1, c_1, h_1 and T have direct and positive influences on its production horizon E^*. On the other hand, among parameters related to the production of Product 2, it is found that a variation of a_2 has a direct and positive influence on production horizon E^* and variations of b_2, c_2, h_2 and T have direct and negative influences on production horizon E^*. In other words, this means that the optimal production horizon of Product 1 will increase with an increase of its unit inventory holding cost (h_1), but will decrease with an increase of unit inventory holding cost (h_2) of Product 2 during production and sales. It is also obvious that the optimal production horizon for each type of product will increase with an increase of unit inventory holding cost.

Under a given optimal production horizon E^*, the rate of change of (p_1^*, q_1^*) and (p_2^*, q_2^*) with respect to the corresponding parameters are derived ((5.7)~(5.10), (5.15)~(5.18) etc.). When the parameters of Product 2 satisfy the given inequalities the indirect effects on the optimal production rate of Product 2, due to variations of parameters of Product 1, are summarized in Table 2. When the parameters of Product 1 satisfy the given inequalities, the indirect effects on the optimal production rate of Product 1, due to variations of the parameters of Product 2, are summarized in Table 3. The present study can be extended to the problem of one machine and more than two products, and that will be our future work.

REFERENCES

[1] Boctor, F.F., "The two-product, single-machine, static demand, infinite horizon lot size scheduling", Management Science, 28 (1982) 798-807.

[2] Chase, R.B., and Aquilano, N.J., Production and Operations Management: Manufacturing and Services, IRWIN Publishing Company, Reading, Chicago, 1995.

[3] Chen, M.S., and Lai, K.K., "A monopoly agent's optimal control on inventory and prices over a given selling planning horizon", International Journal of Information and Management Sciences, 3 (1992) 69-77.

[4] Chen, M.S., and Chen, Y.C., "On the optimal production rates of a basic assembly system under random demand", International Journal of Information and Management Sciences, 6 (1995) 1-25.

[5] Hodgon, T.J., Ge, G., King, R.E., and Said, H., "Dynamic lot size/sequencing policies in a multi-product, single-machine system", IIE Transactions, 29 (1997) 127-137.

[6] Huang-Wei, A.W., "Zero working capital - An American new business idea", Policy-Marking Reference, 5 (1996) 22.

[7] Kotler, P., Marketing Management: Analysis, Planning, Implementation, and Control, Prentice Hall International, Inc., New Jersey, 1995.

[8] Lev, B., and Weiss, H.J., "Inventory models with cost changes", Operations Research, 38 (1990) 53-63.

[9] Rakesh, A.R., and Steinberg, R., "Dynamic pricing and ordering decisions by a monopolist", Management Science, 38 (1992) 240-262.

[10] Raman, K., and Chatterjee, R., "Optimal monopolist pricing under demand uncertainty in dynamic markets", Management Science, 41 (1995) 144-161.

[11] Schwarz, L.B., "A note on the near optimality of "5-EOQ's worth" forecast horizons", Operations Research, 25 (1977) 533-536.

[12] Taylor, S.G., and Bradley, C.E., "Optimal ordering strategies for announced price increases", Operations Research, 33 (1985) 312-325.