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MISSILE SYSTEM SELECTION BASED ON THE FUZZY SETS THEORY*

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Abstract: This paper presents a possible military application of fuzzy sets. The multi-criteria decision-making approach in a fuzzy environment is applied in the selection of missile systems under conditions often present in military activities. The basic theoretical concepts of fuzzy sets, of interest for the decision making problem solution in the presence of uncertain information, are considered. The selection method is described. As an illustration, a numerical example of a missile system selection is given. The features of the method are given, as well as directions for further work.

Keywords: Operations research, military application, fuzzy sets, decision-making.

1. INTRODUCTION

A fuzzy multiple criteria decision making problem is considered. The performance evaluation of missile systems in an uncertain environment is of interest for inclusion of the qualitative requirements of the systems in the design process.

A traditional multiple criteria decision making (MCDM) problem is determined by a (finite) set of decision alternatives $\{A_1, A_2, \ldots, A_m\}$ and by a (finite) set of criteria $\{C_1, C_2, \ldots, C_n\}$, according to which the desirability of an alternative is to be evaluated (Fig. 1). The aim of the MCDM is to determine the optimal alternative

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A^*, with the highest degree of desirability with respect to all relevant criteria C_i, (Fig. 1).

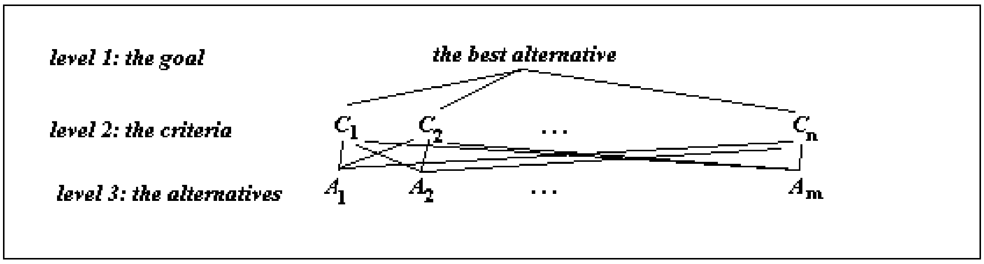

Figure I: Selecting an alternative

A traditional MCDM problem can be expressed in a matrix format, given by Table 1. A decision matrix is an $m \times n$ matrix whose element a_{ij} indicates the performance rating of the i-th alternative, A_i, with respect to the j-th criterion C_j. The higher the value of the element a_{ij}, the more the i-th altrernative A_i satisfies the j-th criterion, C_j. The problem of selecting the most suitable alternative is solved by aggregating the decision alternatives' evaluation with respect to all criteria, and then by ranking the alternatives according to the aggregated evaluation. The classic maximin method, [4], is used to select an alternative A^*, such that:

$$A^* = \{A_k \mid \min_{1 \le j \le n} a_{kj} = \max_{1 \le j \le m} \min_{1 \le j \le n} a_{ij}\} \tag{1}$$

Table 1: A decision matrix for a MCDM problem

alternatives	**criteria**			
	C_1	C_2	**...**	C_n
A_1	a_{11}	a_{12}	...	a_{1n}
A_2	a_{21}	a_{22}	...	a_{2n}
...	...			
A_n	a_{m1}	a_{m2}	...	a_{mn}

The traditional MCDM model is not able to encompass efficiently uncertain data, or linguistically expressed human experience and qualitative requirements.

In the situation when criteria C_i, $i=1,\ldots,n$, have different importance in a decision-making process, criterion importance, relative to some other criterion, is given by weight ω_i. Weight ω_i, assigned to the criterion C_i, gives the level of importance of criterion C_i in the aggregation process, Fig.1. In this situation, the decision making problem with weighted aggregation is formulated by a table similar to Table 1, but with a row of weights added. In this case, in the aggregation procedure (1), instead of the element a_{ij}, the element a'_{ij} is used, where $a'_{ij} = g(\omega_i, a_i)$, and g(.) is a function which satisfies the following properties, [4]:

- if $a > b$ then $g(\omega, a) \geq g(\omega, b)$;
- $g(\omega, a)$ is monotone in ω;
- $g(0, a)$ is equal to the identity element, an element which doesn't change the aggregated value when it is added to aggregates;
- $g(1, a) = a$.

Then the weighted aggregation is defined as $A_i = \mathrm{Agg}(a'_{i1}, a'_{i2}, \ldots, a'_{in})$, where Agg is an aggregation operator. The information aggregation is considered, not only in MCDM, but also in other problems related to the development of intelligent systems: in pattern recognition, expert systems, neural networks, fuzzy controllers, and in machine vision systems.

The relative importance of the criteria could be approximated by Saaty's priority theory [6], which was developed to weight the significant factors in decisions making problems using pairwise comparisons. Saaty's Analytic Hierarchy Process (AHP), is a systematic process to represent the element of a problem hierarchically and includes procedures and principles which are used to synthesize the various ratings to derive priorities among criteria and subsequently select an alternative. The method of approximating the weights is based on Saaty's scale of relative importance, where, if we are comparing the importance of criterion C_i with the importance of criterion C_j, it holds that $a_{ij} = 1/a_{ji}$, for all i, j. According to the respective expert statement, one of the statements from the "Definitions" column in Table 2, an element a_{ij} takes one of the values of relative importance, the so-called "intensity of the relative importance".

However, traditional AHP has the following shortcomings:

1) AHP are mainly used in nearly crisp decision applications.
2) Saaty's AHP creates and deals with a very unbalanced scale of estimations.
3) The AHP does not take into account the uncertainty associated with the mapping of one's perception (or rating) to a number.
4) Ranking of the AHP is rather imprecise.
5) The subjective judgement, selection and preference of decision makers have a large influence on the AHP.

Table 2: Saaty's scale of relative importance

Intensity of relative importance	Definition
1	equal importance
3	weak importance (of one over the other)
5	strong importance
7	demonstrated importance over the other
9	absolute importance
2, 4, 6, 8	intermediate values between

There are situations in the real world when only incomplete or uncertain information is available about a decision making problem, which does not allow a more structured decision approach. Then it is natural to handle uncertainty by the fuzzy set theory, [2] a methodology for dealing with phenomena that are too complex or too ill-defined to be susceptible to analysis by conventional means. In MCDM problems these situations can appear when criteria C_i , or their attainment by alternatives A_i cannot be defined or judged crisply, but only as fuzzy sets, [11]. Then fuzzy models should be used. Also, fuzzy sets theory is provided with aggregation connectives, which can be used for integrating membership values, representing uncertain information about elements in the considered MCDM problem.

Among the first papers and almost a classic on fuzzy rating and ranking in the context of the fuzzy MCDM problem is [1]. It gives a method for computing fuzzy weighted averages and the evaluation procedure for the choice of alternative. Yager in [8] introduces an aggregation technique based on the ordered weighted averaging (OWA, "orand") operators. In many cases of MCDM the type of aggregation implicitly desired by a decision-maker may be, not logical, but some kind of trade-off aggregation. In OWA mean-like operations the membership grades are rank-ordered decreasingly and then weighted. OWA operators, which range between min (the largest conjunction) and max (the smallest disjunction), offer a large class of possibilities for aggregating grades of satisfaction in fuzzy MCDM where compensatory effects occur. In [7], Sugeno explains the idea of fuzzy measure that provides a unifying framework for representing measures of uncertainty and discusses the fuzzy integral, similar to the Lebesque integral. The fuzzy integral provides an important tool for the aggregation of fuzzy information.

The problem considered in this paper is the problem of evaluating three tactical missile system alternatives, A, B and C described by their technical specifications and expert (linguistically expressed) opinions about the systems. In [3], Cheng et al. propose an algorithm for evaluating missile systems by fuzzy AHP based on a fuzzy weighted order matrix. In the algorithm the scores of the alternatives concerning the tactical specifications are determined on the base of membership functions and practical data about the systems, while those scores concerning expert opinions are 1, if they are the best, or 0.5, if they are general. The specification data and characteristics are divided into five groups, which form the five criteria for the evaluation. The order of criteria is assessed to those criteria, and is represented by fuzzy triangular numbers. The fuzzy triangular numbers are used to indicate the relative strength of the alternatives in the hierarchy. To overcome problems with traditional AHP for priority derivation, a fuzzy AHP based on entropy weight is used. The priority among alternatives is derived by the entropy weight and through the use of interval arithmetic, α-cuts and an index of optimism to estimate the degree of satisfaction of the item. In this way, a series of pairwise comparisons required by the traditional AHP method is not needed in that fuzzy AHP.

In this paper each specification or characteristic of a system is treated as a criterion, so that each criterion (specification or characteristic) has its relative importance, assigned by the decision-maker. The decision-maker also assigns fuzzy scores to the system's numerical specification or linguistically expressed characteristics. The weights and scores are expressed by fuzzy triangular numbers. The final fuzzy scores are derived by fuzzy arithmetic. The priority among the alternatives is derived on the base of final fuzzy scores and through a kind of defuzzification. There is no need for entropy weight calculations so computer implementation is less complicated.

The advancements over [3] are: 1) priority among the alternatives is derived in a more effective way, at least for the case of the fuzzy arithmetic of fuzzy triangular numbers, and, 2) the influence of some specification or characteristic is not masked by grouping it with others under one criterion.

In Section 2 of this paper we briefly review some basic concepts of fuzzy sets theory from [9], [5]. In Section 3 these concepts are applied in an evaluation of tactical missile systems, described by different kinds of information, numerical and linguistic, of different importance. Section 4 is devoted to the numerical illustration of the evaluation procedure. In Section 5 the conclusions are given, as well as guidelines for further work.

2. SOME BASIC CONCEPTS OF FUZZY SETS THEORY

Fuzzy sets were introduced [9] as a means of modeling problems and manipulating data that are not precise, in which the source of imprecision is the absence of sharply defined criteria of class membership. Let X be a nonempty set, an universe of discourse. A fuzzy set A of a set X is a set of ordered pairs $\{(x_1, \mu_A(x_1)), \ldots, (x_n, \mu_A(x_n))\}$, where μ_A is a membership function, $\mu_A : X \to [0,1]$, in usual mathematical notation. A membership function $\mu_A(x)$ is interpreted as the degree of membership of element x (from X) in fuzzy set A, for each $x \in X$. A fuzzy set A is normal if a point $x \in X$ can always be found, such that $\mu_A(x) = 1$. A fuzzy set A is convex if and only if for any $x_1, x_2 \in X$ and any $\lambda \in [0,1]$,

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$$

A fuzzy number $\tilde{A}$ is a fuzzy set $\tilde{A}$ of the real line ($X \equiv R$, set of real numbers as the universe of discourse) that satisfies the conditions for normality and convexity. A fuzzy number $\tilde{A}$ is called a fuzzy triangular number with peak (or center) a, left width $l > 0$ and right width $r > 0$, if its membership function has the form given by Figure 2 and by the following expression:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1-(a-x)/l, & a-l \leq x \leq a, \\ 1-(x-a)/r, & a \leq x \leq a+r, \\ 0 & \text{otherwise}. \end{cases} \tag{2}$$

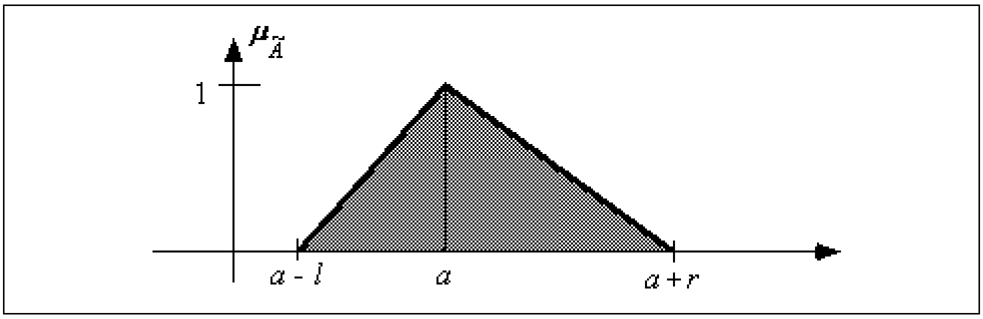

Figure 2. A triangular fuzzy number

A triangular fuzzy number with a center a (a is a real number) may be interpreted as a fuzzy quantity: "x is approximately equal to a", and the notation $\tilde{A} = (a, l, r)$ can be used.

A α-cut (α- level set) of a fuzzy set A of X is a non-fuzzy set, denoted by $[A]^\alpha$, and defined as a subset of all elements x of X, such that their membership function $\mu_A(x)$ is greater than or equal to a real number $\alpha \in [0,1]$, i.e.:

$$[A]^\alpha = \{x, x \in X \mid \mu_A(x) \geq \alpha\}, \quad \alpha \in [0,1] \tag{3}$$

It holds that:

$$\mu_A(x) = \sup_{\alpha \in [0,1]} \min\{\alpha, \mu_{[A]^\alpha}(x)\},$$

where the notation "sup" is the notation of the minimum of the upper bounds of the considered set, and $\mu_{[A]^\alpha}(x)$ is the so-called characteristic function of a α-cut (the α-

cut is a crisp set), so the values of all membership degrees $\mu_{[A]^\alpha}$ for all $x \in [A]^\alpha$ are equal to 1. So, a fuzzy set A can be represented by the union of crisp sets, its α-cuts:

$$A = \bigcup_{\alpha \in [0,1]} \alpha [A]^\alpha .$$

Let us introduce the notation $a_L(\alpha) = \min [A]^\alpha$, and $a_R(\alpha) = \max [A]^\alpha$. Then the notation

$$[A]^\alpha = [a_L(\alpha), a_R(\alpha)] \tag{4}$$

can be used. For a triangular fuzzy number $\tilde{A} = (a, l, r)$ it holds, from (4) that:

$$[\tilde{A}]^\alpha = [a - (1-\alpha)l, a + (1-\alpha)r], \quad \forall \alpha \in [0,1]. \tag{5}$$

Also the triangular fuzzy number $\tilde{A}$ can be given in the form

$$\tilde{A} = (a_L, a, a_R), \tag{6}$$

where, (Fig. 2), $a_L = a - l, a_R = a + r$.

Let $\tilde{A}$ and $\tilde{B}$ be two triangular fuzzy numbers: $\tilde{A} = (a_L, a, a_R)$, $\tilde{B} = (b_L, b, b_R)$. The extended arithmetic operations of positive fuzzy numbers can be as follows:

$$\tilde{A} \oplus \tilde{B} = (a_L, a, a_R) \oplus (b_L, b, b_R) = (a_L + b_L, a + b, a_R + b_R) \tag{7}$$

$$\tilde{A} \ \Theta \ \tilde{B} = (a_L, a, a_R) \ \Theta \ (b_L, b, b_R) = (a_L - b_R, a - b, a_R - b_L) \tag{8}$$

$$\tilde{A} \otimes \tilde{B} = (a_L, a, a_R) \otimes (b_L, b, b_R) = (a_L b_L, ab, a_R b_R) \tag{9}$$

$$\tilde{A} \oslash \tilde{B} = (a_L, a, a_R) \oslash (b_L, b, b_R) = (a_L / b_R, a/b, a_R / b_L) \tag{10}$$

These concepts are applied in Section 3, in treating the MCDM problem of evaluating three tactical missile systems using the fuzzy sets theory. In a case characterized by a lack of precise and reliable information about the considered problem, elements of the decision matrix can be given as degrees of "how an alternative satisfies a specific criterion". For each alternative A_i according to criterion C_j a membership function μ_{ij} is given as a fuzzy triangular number, enabling the expression of the degree to which the i-th alternative satisfies the j-th criterion.

3. THE EVALUATION OF TACTICAL MISSILE SYSTEMS

Three tactical missile systems A, B and C are considered. In the data evaluation process, the best system should be selected. This means that the selected system should satisfy all the criteria with a maximal degree. The given data are: tactical specifications of the three systems, Table 3, and a descriptively expressed expert's opinion about the characteristics of the considered systems, based on experience, given in Table 4.

Table 3: The tactical data specification

Characteristics	System A	System B	System C
Range (km)	43	36	38
Flight height (m)	25	20	23
Flight velocity (M, Mach number)	0.72	0.8	0.75
Fire rate (round/min)	0.6	0.6	0.7
Reaction time (min)	1.2	1.5	1.3
Missile dimensions (cm) lxd-span	521x35 - 135	381x34 - 105	445x35 - 120
Firing accuracy (%)	67	70	63
Destruction rate (%)	84	88	86
Kill radius (m)	15	12	18
Anti-jam (%)	68	75	70
Reliability (%)	80	83	76
System cost (10000)	800	755	785
System life (years)	7	5	5

The characteristics from Tables 3 and 4 are interpreted as the criteria $C_1, C_2, \ldots, C_n$, from Table 1, and, in the case, $n = 23$. The alternatives are: $A_1 = A$, $A_2 = B$, $A_3 = C$, $m = 3$. Part of the criteria which expresses the expert's attitude to the considered missile systems, Table 4, compels the use of some fuzzy decision-making procedures [4]. In this paper the feasibility of the following procedure is considered: a fuzzy scale given by the set of triangular fuzzy numbers between 1 and 9, by analogy with Saaty's scale of relative importance, Table 2, is used in order to aggregate the criteria from Tables 3 and 4.

Fuzzy set theory formalism supports a modeling of uncertainty which may arise from linguistic imprecision. Fuzzy models manipulate linguistic variables. A linguistic variable, [10] is a variable whose values are words drawn from a natural or synthetic language. It is the representation of a fuzzy space. This fuzzy space is a fuzzy set derived from the evaluation of the linguistic variable. The simplest linguistic variable is the label of a fuzzy set directly representing a specific region in the underlying problem space. The value of the linguistic variable, the term (expert opinion from Table 4) is represented by fuzzy sets in the form of fuzzy triangular numbers. These fuzzy triangular numbers are of the form given by (6). The fuzzy scale T used to aggregate criteria from Tables 3 and 4 and defined by the set of triangular fuzzy numbers, each fuzzy number in pair with its corresponding membership function in the form given by (6), is as follows:

$$T = \{(\tilde{1},(1,1,2)),\{(\tilde{a},(a-1,a,a+1)), \text{ for } a=2,\ldots,8\},(\tilde{9},(8,9,9))\}. \tag{11}$$

Table 4: Expert's opinion about system characteristics

Characteristics	System A	System B	System C
Operation condition requirements	high	standard	standard
Safety	good	standard	standard
Defilade	standard	good	standard
Simplicity	standard	standard	standard
Assembility	standard	standard	poor
Combat capability	good	standard	standard
Material limitations	high	standard	high
Mobility	poor	good	standard
Modulization	standard	good	standard
Standardization	standard	standard	good

The schematic representation of Eq. (11) is given in Figure 3.

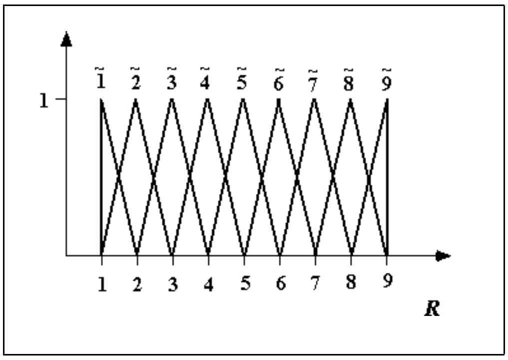

Figure 3: The schematic representation of the fuzzy scale T.

The decision-maker assigns a fuzzy score μ_{jS} from the set T of fuzzy triangular numbers (11), for each of the characteristics of the missile system alternative S, S = A, B, C. In that way values from Table 3 are fuzzified and can be aggregated with representations of the expert opinion from Table 4. The fuzzy score μ_{jS}, $j = 1,\ldots,23$, S = A, B, C, expresses the relative performance degree based on the value of the j-th system characteristic, degree of "how a value of the j-th characteristic of system S satisfies the criterion C_j for that characteristic". The fuzzy score μ_{jS} is determined by (numerical or linguistic) values of the considered j-th characteristic for all three systems. The greater the value of the fuzzy score μ_{jS}, the more the system matches the respective criterion. The decision-maker also gives different weights $\tilde{\omega}_j$ to each of the 23 criteria from Tables 3 and 4, according to his attitude about the importance of the criteria. The weights are also fuzzy numbers from the set T of fuzzy triangular numbers, (11). The decision matrix given in the general case by Table 1 for the considered case is given by Table 5. The rows (columns, respectively) from Table 1 are the columns (rows, respectively) in Table 5, for the sake of more convenient presentation.

Table 5: A decision matrix

Criteria	Weights	System A	System B	System C
1	$\tilde{w}_1$	μ_{1A}	μ_{1B}	μ_{1C}
2	$\tilde{w}_2$	μ_{2A}	μ_{2B}	μ_{2C}
...				
23	$\tilde{w}_{23}$	μ_{23A}	μ_{23B}	μ_{23C}

The numbers 1 - 23 given in Table 5 are labels of the criteria given in Table 6.

For a system S, S = A, B, C and for the values of the fuzzy scores μ_{jS} taken as the fuzzy numbers $\tilde{A}$ from the set T (11), the final fuzzy decision scores $F(S)$ can be calculated by the following expression using (7) - (10):

$$F(S) = \tilde{\omega}_1 \otimes \mu_{1S} \oplus \tilde{\omega}_2 \otimes \mu_{2S} \oplus \ldots \oplus \tilde{\omega}_{23} \otimes \mu_{23S}, \quad S = A, B, C. \tag{12}$$

The final fuzzy decision scores $F(A)$, $F(B)$ and $F(C)$ are membership functions which characterize in a fuzzy sense the final decision scores of the alternatives A, B and C, respectively. These membership functions have the form of fuzzy triangular numbers.

One way to evaluate a fuzzy decision (final fuzzy score) F is by splitting the fuzzy set into its α-level sets (3). By means of this concept one can construct a series of sets according to their truth (agreement, or confidence) levels. This might give some insight into fuzzy decisions but does not lead to one particular single decision score D. An easy way to look for that decision D is to use the mean of the α-cut of the fuzzy decision F, what can be used here in the case of fuzzy triangular numbers.

Let the α-cuts in the form given by expression (4), i.e. (5) for $F(A)$, $F(B)$ and $F(C)$, respectively, be:

$$[a_L(\alpha), a_R(\alpha)], [b_L(\alpha), b_R(\alpha)], [c_L(\alpha), c_R(\alpha)], \quad \alpha \in [0,1] \tag{13}$$

The following decision scores can be found as the mean values for the corresponding sets, using expressions (13) in the α-cuts of the final fuzzy score (12) calculations:

$$D_A(\alpha) = (a_L(\alpha) + a_R(\alpha))/2 = p_1, \tag{14}$$

$$D_B(\alpha) = (b_L(\alpha) + b_R(\alpha))/2 = p_2, \tag{15}$$

$$D_C(\alpha) = (c_L(\alpha) + c_R(\alpha))/2 = p_3. \tag{16}$$

The degree to which the alternative selection matches the criteria for the specified value of α, is given by normalized decision scores:

$$N_A(\alpha) = p_1 / (p_1 + p_2 + p_3), \tag{17}$$

$$N_B(\alpha) = p_2 / (p_1 + p_2 + p_3), \tag{18}$$

$$N_C(\alpha) = p_3 / (p_1 + p_2 + p_3). \tag{19}$$

The greater the value of $N(\alpha)$, the more the selection matches the criteria.

4. NUMERICAL DATA

The decision-maker assigns the fuzzy scores to the specification data and characteristics of the considered systems with respect to the criteria given in Table 6. The weights of the criteria are also assigned. The scores and the weights are the triangular fuzzy numbers from set T (11), so the decision matrix is formed as in Table 6.

Table 6: The decision matrix

Criterion	Weights	System A	System B	System C
C_1 range	$\tilde{7}$	$\tilde{3}$	$\tilde{1}$	$\tilde{2}$
C_2 flight height	$\tilde{1}$	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
C_3 flight velocity	$\tilde{9}$	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
C_4 reliability	$\tilde{8}$	$\tilde{2}$	$\tilde{3}$	$\tilde{1}$
C_5 firing accuracy	$\tilde{9}$	$\tilde{2}$	$\tilde{3}$	$\tilde{1}$
C_6 destruction rate	$\tilde{7}$	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
C_7 kill radius	$\tilde{6}$	$\tilde{2}$	$\tilde{1}$	$\tilde{3}$
C_8 missile dimensions	$\tilde{4}$	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
C_9 reaction time	$\tilde{9}$	$\tilde{3}$	$\tilde{1}$	$\tilde{2}$
C_{10} fire rate	$\tilde{9}$	$\tilde{1}$	$\tilde{1}$	$\tilde{2}$
C_{12} combat capability	$\tilde{9}$	$\tilde{2}$	$\tilde{1}$	$\tilde{1}$
C_{13} operation condition requirements	$\tilde{5}$	$\tilde{2}$	$\tilde{1}$	$\tilde{1}$
C_{14} safety	$\tilde{6}$	$\tilde{2}$	$\tilde{1}$	$\tilde{1}$
C_{15} defilade	$\tilde{2}$	$\tilde{1}$	$\tilde{2}$	$\tilde{1}$
C_{16} simplicity	$\tilde{3}$	$\tilde{1}$	$\tilde{1}$	$\tilde{1}$
C_{17} assembility	$\tilde{3}$	$\tilde{2}$	$\tilde{2}$	$\tilde{1}$
C_{20} material limitations	$\tilde{5}$	$\tilde{2}$	$\tilde{1}$	$\tilde{2}$
C_{21} modulization	$\tilde{5}$	$\tilde{1}$	$\tilde{2}$	$\tilde{1}$
C_{22} mobility	$\tilde{7}$	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
C_{23} standardization	$\tilde{3}$	$\tilde{1}$	$\tilde{1}$	$\tilde{2}$

Using Borland C++ 4.5 we have written a program to select the best alternative in the considered problem. As the computer implementation of the solution algorithms has shown, an object-oriented paradigm is very suitable for implementations of this kind. For the decision matrix given in Table 6, we have the results for $F(A)$, $F(B)$ and $F(C)$, given in Fig. 4.

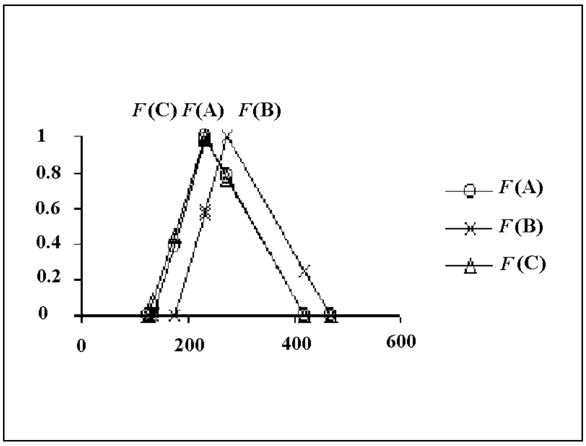

Figure 4: The membership functions for the numbers $F(A)$, $F(B)$, $F(C)$.

The calculated values of $N_A(\alpha)$, $N_B(\alpha)$, and $N_C(\alpha)$, the alternative matching degrees to the given criteria for different values of α are given in Fig. 5.

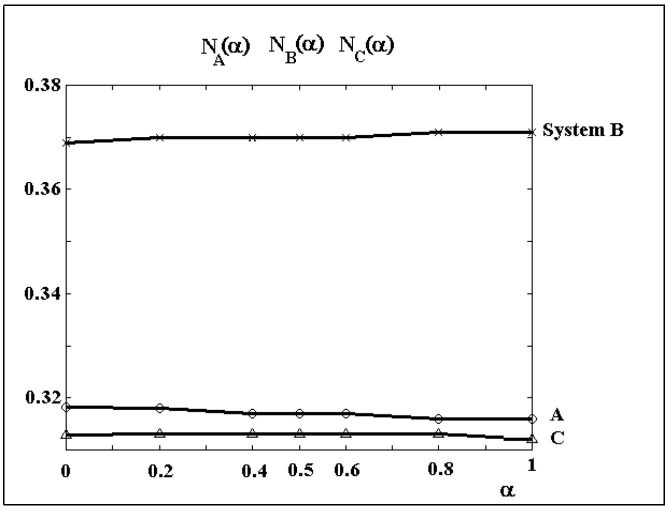

Figure 5: The values for $N_A(\alpha)$, $N_B(\alpha)$, $N_C(\alpha)$.

System B is the best choice. Whatever the value of the measure α that establishes the minimum degree of membership in a fuzzy set is assumed or requested to be, the decision is that system B is the best alternative. The degree to which it satisfies all the criteria is the highest.

A direct comparison of the obtained result with the result obtained in [3] is not possible, because in the method proposed in [3] the fuzzy weight vector (subjectively introduced) is used to represent the relative importance of the five groups into which the specification data and characteristics are divided. Here each specification or characteristic is treated as a criterion to which a weight is subjectively assigned. That is more refined. The difference in weights may cause different decisions.

5. CONCLUSIONS

This paper describes a possible application of fuzzy sets theory in multiple-criteria decision-making in the presence of uncertainty, in the selection problem of the most suitable system among three tactical missile systems. The presented method is based on the weighted aggregation of numerical and linguistic data. In this approach each specification or characteristic of a system is treated as a criterion, so that each specification or characteristic has its relative importance, assigned by the decision-maker. The decision-maker also assigns fuzzy scores to the system's numerical data or linguistically expressed characteristics. The weights and scores are expressed by fuzzy triangular numbers chosen from the scale of fuzzy numbers, which enables the formation of priority structure of decisions. Priority among the alternatives is derived on the basis of fuzzy arithmetic for fuzzy triangular numbers and through a kind of defuzzification. There is no need for complicated entropy weight calculations, so the computer implementation and calculations are less complicated and the approach is more suitable for application. The influence of some specification or characteristic is not masked by grouping it with others under one criterion.

The successful performance evaluation of missile systems in an uncertain environment based on the theory of fuzzy sets enables qualitative requirements about systems expressed linguistically to be included in the design process of such a system.

It would be of interest to investigate the possibilities of generalizing this procedure. Also, the issue of applying the results in soft computing systems deserves further investigations.

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