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NORMALIZATION OF ATTRIBUTE VALUES IN *MADM* VIOLATES THE CONDITIONS OF CONSISTENT CHOICE *IV*, *DI* AND α

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Abstract: In this paper we analyze the influence of various normalization methods (simple, Nijkamp's and vector) on the results of the well-known methods of M(ultiple) A(ttribute) D(ecision) M(aking): SAW, TOPSIS and ELECTRE. It is shown that the final choice recommended by the MADM methods depends on: 1) the type of normalization used; 2) the type of the Likert scale; and 3) the formulation of the attributes. This means that these MADM methods do not satisfy the conditions of consistent choice: independence of the value scale (IV), and descriptive invariability (DI). Also, it is shown that the MADM methods violate the contraction consistency condition α: If an alternative A is the best in a set S of alternatives, then it should be the best in any subset of S to which it belongs. We conclude that the normalization procedures analyzed here could cause inconsistent choices.

Key words: Normalization, benefit and cost attributes, Likert-type scale, conditions of consistent choice.

1. INTRODUCTION

Multiple Attribute Decision Making (MADM) deals with the problem of choosing one alternative from a set of m alternatives A_i $(i = 1,\ldots,m)$ on the basis of n relevant attributes X_j $(j = 1,\ldots,n)$. Alternatives are represented by vectors $A_i = (x_{i1}, x_{i2},\ldots,x_{ij},\ldots,x_{in})$, where x_{ij} is the value of the ith alternative on the jth attribute. The attributes may be quantitative (when they are measurable on a cardinal scale) or qualitative (when they are measurable on an ordinal scale). In order to calculate the choice criterion in the application of most of MADM methods, it is necessary to make the attribute values comparable on a common scale. Therefore, numerical values are first associated to qualitative attributes (by using a Likert-type scale), and then the values of each attribute are *normalized* separately. Normalization is the mapping of empirical attribute values (measured on different scales) to the scale [0, 1]. Afterwards, different procedures are applied in order to evaluate each alternative by a single value and choose the best according to the set criterion.

It is our intention to point out the deformations of the empirical values x_{ij} caused by the normalization, which raise doubts about the adequacy of the application of normalized values as a basis for decision making. We shall first make a list of various types of attributes that are important in the normalization, and then make a list of formulae we use in the simple, Nijkamp's and vector normalization of these attributes. We shall show that: 1) the rankings of the alternatives change with the change of the normalization procedure used; 2) with simple or vector normalization, the final decision depends on the Likert-type scale (chosen to measure qualitative attributes), as well as on the attribute framing (whether the attribute is positively or negatively framed); 3) the MADM methods based on normalized ratings violate A. Sen's contraction consistency condition α.

2. ATTRIBUTES AND THEIR NORMALIZATION PROCEDURES

2.1. Types of attributes

In addition to the division of attributes into quantitative and qualitative ones, in the data normalization it is necessary to establish which of the following two[1] groups the attribute X_j belongs ([7], p. 15-16.):

a) *Benefit* attributes (denoted by X_j^+) are positively correlated with utility or the preferences of the decision maker (DM); as the attribute values increase, so does the utility of DM.

b) *Cost* attributes (X_j^-) are negatively correlated with the utility of the DM; as the attribute values increase, the utility of the DM decreases.

In order to carry out the normalization properly, the attributes must be grouped to types (a) and (b). Namely, within each of the normalization procedures, different formulae are used for different types of attributes. Let us note that this demarcation is unimportant for qualitative attributes. The descriptive modalities of the qualitative attribute are formulated in such a way that with an increase in the

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[1] We shall ignore the group of so-called *nonmonotonic* attributes because they occur rarely and are not important in our analysis.

intensity of the phenomenon, the utility of the DM increases as well. Hence, these attributes belong to type (a).

2.2. Various normalization procedures

Among the normalization procedures proposed in the literature, the following three are most frequently used (Table 1.):

1. Simple normalization (SN),
2. Nijkamp's normalization (NN), and
3. Vector normalization (VN).

Table 1: Formulae for calculating the normalization ratings of SN, NN and VN type, for benefit and cost attributes

Type of normalization	Type of attribute	
	Benefit attribute, X_j^* (r_{ij}^*)	Cost attribute, X_j^- (r_{ij}^-)
Simple (SN)	$r_{ij}^S = \frac{x_{ij}}{x_j^*}$	$r_{ij}^S = \frac{x_j^-}{x_{ij}}$, $x_{ij} > 0$
Nijkamp's (NN)	$r_{ij}^N = 1 - \frac{x_j^* - x_{ij}}{x_j^* - x_j^-}$	$r_{ij}^N = 1 - \frac{x_{ij} - x_j^-}{x_j^* - x_j^-}$
Vector (VN)	$r_{ij}^V = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$	$r_{ij}^V = \frac{\frac{1}{x_{ij}}}{\sqrt{\sum_{i=1}^{m}\left(\frac{1}{x_{ij}}\right)^2}}$, $x_{ij} > 0$

where $x_j^* = \max_i x_{ij}$, and $x_j^- = \min_i x_{ij}$.

(SN is usually called *linear*, but we changed the name because its cost attribute transformation is not linear. On the other hand, NN is a linear transformation, but we named it after the author (see [4]) to avoid possible confusion with SN). In order to make a distinction between the normalized ratings obtained by the three different procedures, we denoted them as r_{ij}^S, r_{ij}^N and r_{ij}^V, respectively. For the same purpose we denoted the normalized ratings of benefit and cost attributes as r_{ij}^* and r_{ij}^-, respectively.

Using these (or any other) normalization procedures, the values of all attributes (measured on different measurement scales) are mapped to a unique scale [0,1], or to one segment of it ($0 \le r_{ij} \le 1$), in order to obtain their comparability. The purpose of normalization is to make possible some form of aggregation of the ratings of all attributes, that is, the calculation of a unique indicator, enabling either a rankings of alternatives or determination of the best one. We shall show that in the stage of the aggregation of normalized ratings, these normalization procedures could essentially affect the final choice.

3. EFFECTS OF THE NORMALIZATION PROCEDURE ON THE RESULTS OF THE *MADM* METHODS

Using the Simple Additive Weighting Method (SAW) as an example, we shall point out the consequences that normalized ratings have on the final outcome of the MADM methods. We shall show that:

1. the type of normalization used could determine the final outcome, which means that on the basis of the same data (normalized by different procedures), when the same MADM method is used, we could choose different alternatives;
2. the ultimate decision might depend on the arbitrarily chosen type of Likert scale (because positive affine transformations of 'emirical' values affect their normalized ratings), and
3. the final choice is affected by the framing of the attribute, namely, whether the property is given as a benefit or as a cost attribute, when both forms are possible (as normalized ratings of cost and benefit attributes behave differently).

3.1. The results of SAW depend on the type of normalization

Example 1. Let us consider the choice between five nondominated alternatives on the basis of four benefit attributes (Table 2, columns 1-5).

Table 2: Empirical values, their normalized ratings of SN type, and values of the SAW criterion for the decision problem shown in example 1

Alt.	Attributes X_1	X_2	X_3	X_4	SN r_{i1}^S	r_{i2}^S	r_{i3}^S	r_{i4}^S	$V(A_i)=\sum r_{ij}^S$
A_1	2	7	180	100	0.2222	0.7778	1	1	3.0000
A_2	3	8	140	98	0.3333	0.8889	0.7778	0.98	2.9800
A_3	5	5	155	100	0.5555	0.5555	0.8611	1	2.9721
A_4	1	9	155	95	0.1111	1	0.8611	0.95	2.9222
A_5	9	1	160	90	1	0.1111	0.8889	0.90	2.9000

In the first step, the empirical values x_{ij} are normalized by one of the normalization procedures. (In this example we shall apply all three procedures in the following order: SN, NN and VN.)

In the second step, the importance of each attribute is evaluated, i.e. an attribute weight, w_j, is associated to each attribute X_j (where $\sum w_j = 1$). In order to point out the effects of the change in normalized ratings, r_{ij}, on the value of the criterion, $V(A_i)$, in all examples we shall assume that all attributes are equally important, i.e. that all weights are equal ($w_j = 1/n = 1/4,\ j = 1,2,3,4$).

In the third step we apply the choice criterion:

$$V(A^*) = \max_i V(A_i) = \max_i \sum_j w_j \cdot r_{ij} \tag{3.1}$$

Since $\sum w_j \cdot r_{ij} = \sum (1/n) \cdot r_{ij} = 1/n \sum r_{ij}$, the weights will be ignored in the calculation of the value $V(A_i)$. Thus, instead of using (3.1), we shall choose the alternative (A^*) with the greatest sum of normalized ratings:

$$V(A^*) = \max_i \sum_j r_{ij} \tag{3.2}$$

a) Let us *apply SAW to the normalized ratings of SN type*. Table 2 contains the normalized ratings r_{ij}^S (columns 6-9), and the values $V(A_i)$ (column 10), providing the following rankings of alternatives: A_1, A_2, A_3, A_4, A_5. On the basis of the criterion (3.2), the SAW method suggests the choice of A_1 as the best.

b) If we *apply the procedure NN* to the same empirical data (Table 3, columns 2-5), and then use the same procedure of evaluation (column 6), we obtain a different rankings: A_1, A_3, A_2, A_4, A_5.

c) Finally, if the empirical data are *normalized by VN* (Table 3, columns 7-10), and then the choice criterion SAW is applied (column 11), the rankings of alternatives will be: A_5, A_3, A_2, A_1, A_4.

Table 3: Normalized ratings of NN and VN type, and values of the SAW criterion based on them, for the decision problem shown in example 1

Alt.	NN					VN				
	r_{i1}^N	r_{i2}^N	r_{i3}^N	r_{i4}^N	$V(A_i)=\sum r_{ij}^N$	r_{i1}^V	r_{i2}^V	r_{i3}^V	r_{i4}^V	$V(A_i)=\sum r_{ij}^V$
A_1	0.125	0.75	1	1	2.875	0.1826	0.4719	0.5078	0.4626	1.6249
A_2	0.25	0.875	0	0.8	1.925	0.2739	0.5394	0.3949	0.4533	1.6615
A_3	0.5	0.5	0.375	1	2.375	0.4564	0.3371	0.4373	0.4626	1.6934
A_4	0	1	0.375	0.5	1.875	0.0913	0.6068	0.4373	0.4395	1.5749
A_5	1	0	0.5	0	1.500	0.8216	0.0673	0.4514	0.4163	1.7567

Thus, *applying SAW to the same empirical data, normalized by different procedures (SN, NN and VN), we obtained three different rankings* (Table 4).

Table 4: Different rankings obtained by applying SAW on normalized data of SN, NN, and VN type in example 1

Rank	1	2	3	4	5
SAW (SN)	A_1	A_2	A_3	A_4	A_5
SAW (NN)	A_1	A_3	A_2	A_4	A_5
SAW (VN)	A_5	A_3	A_2	A_1	A_4

3.2. The effect of Likert-type scales on the results of SAW

In MADM, Likert-type scales are used for measuring qualitative attributes. Five-point and nine-point scales are most frequently used, and they are marked with numbers from 1 (for the lowest attribute level) to 5 or 9 (for the highest level). For example, if attribute *reliability* is measured on the five-point scale, then numbers 1, 2, 3, 4, 5, correspond to the following levels: very low, low, average, high and very high. Likert-type scales are often treated as interval scales, with the explanation that the same differences between successive numerical values correspond to the same differences in the levels of the qualitative attribute (see [7], pp. 14-15.). Being treated as interval scales, they can be mapped onto each other by positive affine transformations ($y = ax + b, a > 0$). Hence, if instead of the scale 1, 2, 3, 4, 5, we use its transformation $y = 2x - 1$, we evaluate qualitative attributes on another five-point scale: 1, 3, 5, 7, 9, where the numbers correspond to the same levels of qualitative attribute. Therefore, it should be irrelevant which of these scales is used for the purpose of evaluating alternatives on the basis of qualitative attributes. One of the consistency choice conditions in normative decision theory - *Independence of the value scale, IV,* demands that 'the choice suggested by a decision rule is independent of the scale on which we measure value.' ([2], p. 42.) Within the MADM field, condition *IV* might be interpreted as a requirement that the final choice suggested by the MADM method should not depend on the type of Likert scale used.

Example 2a. Imagine a DM who is to choose between four alternatives on the basis of three criteria (Table 5, columns 1-4.). The attributes X_1 and X_2 are qualitative, while X_3 is a quantitative benefit attribute. (Here we disregard the fact that in cases with prevailing qualitative attributes one should apply some of the methods based upon qualitative data.) Suppose the DM has decided to measure X_1 and X_2 on the five-point Likert-type scale. This means that he can arbitrarily apply either type (1) - 1, 2, 3, 4, 5 or type (2) - 1, 3, 5, 7, 9. In this example we shall show that by applying SAW to differently numbered Likert-type scales, the DM makes different choices. We shall use *normalization of the SN type*, first.

Table 5: Empirical values, their normalized ratings of the SN type, and values of the SAW criterion for the decision problem shown in example 2a

Alt.	X_1 Likert scale type (1)	X_2	X_3	SN r_{i1}^S	r_{i2}^S	r_{i3}^S	$V(A_i)=\sum r_{ij}^S$
A_1	2	3	120	0.4	0.6	1	2.2000
A_2	3	4	75	0.6	0.8	0.6250	2.0250
A_3	1	5	100	0.2	1	0.8333	2.0333
A_4	5	1	100	1	0.2	0.8333	2.0333
	type (2)						
A_1	3	5	120	0.3333	0.5556	1	1.8889
A_2	5	7	75	0.5556	0.7778	0.6250	1.9584
A_3	1	9	100	0.1111	1	0.8333	1.9444
A_4	9	1	100	1	0.1111	0.8333	1.9444

Applying the five-point Likert *scale of type (1)* (Table 5, rows 1-5) the rankings of the alternatives is as follows: A_1, $A_3 \approx A_4$, A_2 (the symbol $\approx$ means that the alternatives have the same value $V(A_i)$ and that the DM should be indifferent while choosing between them). However, if instead of the Likert scales of type (1) the DM uses its transformation of the form $y = 2x - 1$, he will obtain *the scale (2)* (Table 5, rows 6-10), and the rankings of alternatives becomes: A_2, $A_3 \approx A_4$, A_1.

Example 2b. The change of numerical values on the Likert-type scale can also affect the final choice if *SAW is applied to the normalized ratings of the VN type*. Take the decision-making problem shown in Table 6 (columns 1-4).

Table 6: Empirical values, their normalized ratings of the VN type, and values of the SAW criterion for the decision problem shown in example 2b

Alt.	X_1 Likert scale type (1)	X_2	X_3	VN r_{i1}^V	r_{i2}^V	r_{i3}^V	$V(A_i)=\sum r_{ij}^V$
A_1	2	4	102	0.3203	0.5601	0.4947	1.3751
A_2	3	3	110	0.4804	0.4201	0.5335	1.4340
A_3	1	5	100	0.1601	0.7001	0.4850	1.3452
A_4	5	1	100	0.8006	0.1400	0.4850	1.4256
	type (2)						
A_1	3	7	102	0.2785	0.5604	0.4947	1.3336
A_2	5	5	110	0.4642	0.4003	0.5335	1.3980
A_3	1	9	100	0.0928	0.7206	0.4850	1.2984
A_4	9	1	100	0.8356	0.0801	0.4850	1.4007

If the DM uses the Likert scale of type (1), he obtains the rankings: A_2, A_4, A_1, A_3. If he uses its transformation (2), he gets the rankings: A_4, A_2, A_1, A_3.

We conclude that by *applying SAW to data of either SN or VN type different results are obtained, depending on the type of Likert scale used. An exception is the NN scale* where different numbers on the Likert-type scales, corresponding to the same levels of the qualitative attribute, have the same ratings r_{ij}^{N}.

As can be seen from Tables 5 and 6, the positive affine transformation of numerical values on the Likert-type scale affected their ratings of SN and VN type, and subsequently caused changes in criterion values. This result shows that *MADM methods, which are based on SN or VN data, do not satisfy the condition IV.*

This disturbing result might even be applied to all attributes that are measurable up to the interval scale. It means that if a *quantitative* attribute is measurable by the interval scale, then the very choice of its measurement unit could determine the final decision. (For example, if *temperature* is considered as an attribute, the final choice could depend on whether it is measured in Fahrenheit degrees or in Centigrade degrees.)

The example given in Table 6 (Likert-type scale (2)) is an illustration of another illogical result based on the VN data. Namely, under the assumption that all attributes are equally important, that is, they have equal weights, it is clear that the DM should be indifferent while choosing between alternatives A_3 and A_4. The values of the attribute X_3 equal 100, while by the attributes X_1 and X_2 the alternatives are evaluated once as very favourable - 9 and once as very unfavourable - 1. However, these alternatives are placed on different ends of the rankings (A_4 is evaluated the best and A_3 the worst). The reason is that the ratings of VN type depend on the domain of attribute X_j, as well as on the distribution of values x_{ij} inside the fixed extremes, $x_{\min}$ and $x_{\max}$. In our example, the attributes X_1 and X_2 have the same domain but different distributions inside the interval ($x_{\min} = 1$, $x_{\max} = 9$). This caused great differences in the ratings r_{ij}^{V} for the same values x_{ij} of two attributes, which severely affected the choice criterion values of these two alternatives.

3.3. The impact of attribute framing on the result of SAW

Depending on the type of attribute (benefit or cost), the same empirical value, x_{ij}, has different normalized ratings. If the attribute is X_j^{+}, the most favorable value for the DM is its maximum value ($\max_i x_{ij}^{+}$) which reachs $\max_i r_{ij}^{+} \leq 1$; with a decrease of benefit attribute values, x_{ij}^{+}, their normalized ratings, r_{ij}^{+}, decrease as well. If the attribute is of X_j^{-} type, then the most favorable value is its minimum ($\min_i x_{ij}^{-}$), with normalized rating $\max_i r_{ij}^{-} \leq 1$; an increase of cost attribute values, x_{ij}^{-}, is followed by a decrease of their normalized ratings, r_{ij}^{-}. The normalized ratings of benefit attributes and cost attributes behave differently; i.e. they have different "dynamics". We shall show that, as a consequence of different dynamics of the ratings r_{ij}^{+} and r_{ij} obtained by SN or VN procedures, the final decision depends on the formulation of the attribute.

Example 3. Suppose that we have to choose among 3 candidates in a competition by applying two criteria: X_1 - the score in the IQ test, and X_2 - the result of the knowledge test. The attribute X_2 can be framed in two ways: either positively - as a benefit attribute (percentage of correct answers), or negatively - as a cost attribute (percentage of incorrect answers) (Table 7, columns 1-3). Since these two formulations of the attribute X_2 are *normatively equivalent* (they represent different frames of the same statement, i.e. the percentage of incorrect answers = 100 - the percentage of correct answers), the final rankings should be the same in both cases. In other words, SAW should satisfy *the condition of descriptive invariability, DI,* defined by A. Tversky and D. Kahneman in behavioural decision theory (see, for instance [6]). This means that the method should be resistant to changes in the attribute description, if those descriptions are normatively equivalent. Our example shows that the contrary is the case.

a) If X_2 is formalised as a benefit attribute (percentage of correct answers) and if the *SN type* of normalization is applied (columns 4-5), on the basis of the $V(A_i)$ values (column 6, rows 2- 4), the rankings will be: A_2, A_1, A_3. If the cost attribute frame is used (percentage of incorrect answers), the final decision would be made on the basis of the rankings: A_3, A_2, A_1 (column 6, row 6-8).

Table 7: Empirical values, their normalized ratings of SN and VN type, and values of the SAW criterion for the decision problem shown in example 3

Can did.	X_1	X_2^+ (% of correct)	SN r_{i1}	SN r_{i2}^{+}	$V(A_i)=\sum r_{ij}^{S}$	VN r_{i1}	VN r_{i2}^{+}	$V(A_i)=\sum r_{ij}^{V}$
A_1	125	86	0.8333	0.9555	1.7888	0.5638	0.5705	1.1343
A_2	150	85	1	0.9444	1.9444	0.6766	0.5639	1.2405
A_3	105	90	0.7	1	1.7000	0.4736	0.5971	1.0707
		X_2 (% of incorrect)	SN r_{i1}	SN r_{i2}		VN r_{i1}	VN r_{i2}	
A_1	125	14	0.8333	0.7143	1.5476	0.5638	0.5109	1.0747
A_2	150	15	1	0.6667	1.6667	0.6766	0.4769	1.1535
A_3	105	10	0.7	1	1.7000	0.4736	0.7153	1.1889

b) Inconsistent results are obtained by applying SAW to the *VN data* as well. The rankings for the benefit attribute is: A_2, A_1, A_3 (column 9, rows 2-4) while the rankings for the cost attribute is completely reversed: A_3, A_2, A_1 (column 9, rows 6-8).

Thus, *the result of the SAW method (based either on SN or VN types of data) depends on whether the attribute is formulated as a benefit or as a cost attribute, which means that SAW violates the condition DI.*

4. *MADM* METHODS VIOLATE CONDITION α

Among the criteria for the stability of choices with changes in the size (m) of the set of observed alternatives, we have chosen A. Sen's *condition* α (originally defined in social choice theory, see [5], p. 17.) for being a very weak, and consequentially widely accepted *contraction consistency condition*.

Condition α: If alternative A is the best in the set of alternatives S $(A \in S)$, then it has to be the best in every subset E of S $(E \subset S)$ to which it belongs $(A \in E)$.

In other words, with the exclusion of some unimportant (irrelevant) alternatives from the observed set, the alternative previously named as the best, should remain in its leading position unless there is another dominating alternative entering the set.

Among the many MADM methods based on normalized ratings, r_{ij}, apart from SAW we have chosen TOPSIS and ELECTRE because they are very popular and commonly used. With no intention to describe these well known procedures (see, for instance [3], pp. 115-140.) we shall show that they, being based on normalized ratings, violate the condition α. With the SAW method we shall use all three normalization techniques (although SN and NN are usually encountered in practice). Analysing TOPSIS and ELECTRE methods we shall observe only the vector normalization which is usually used with these procedures.

4.1. SAW based on SN, NN or VN data violates condition α

a) Let us return to the example 1 shown in Table 2, in which the DM using SAW (on the *SN type data*) has formed the preference order: A_1, A_2, A_3, A_4, and A_5. Suppose that the DM has decided to exclude alternatives A_4 and A_5 from the analysis, for instance, because of their intolerably low values of attributes X_1 and X_2, respectively. As can be seen, these alternatives are ranked last, so one can expect that the previously defined order would remain unchanged, which means that the rankings should be: A_1, A_2, A_3. But, if the DM applies the SAW method to this subset of alternatives (Table 8), he will get just the opposite result: A_3, A_2, A_1. In this way we have shown that the SAW method, based on a normalized ratings of the SN type, violates the condition α.

Table 8: Empirical values, their normalized ratings of SN type, and values of the SAW criterion for the subset of alternatives (A_1, A_2, A_3) in the decision problem shown in example 1

Alt.	X_1	X_2	X_3	X_4	SN r_{i1}^S	r_{i2}^S	r_{i3}^S	r_{i4}^S	$V(A_i)=\sum r_{ij}^S$
A_1	2	7	180	100	0.4	0.875	1	1	3.2750
A_2	3	8	140	98	0.6	1	0.7778	0.98	3.3578
A_3	5	5	155	100	1	0.625	0.8611	1	3.4861

b) Transforming the same empirical data with *VN* and then applying the SAW method (Table 3) we have obtained the rankings: A_5, A_3, A_2, A_1, A_4. If we exclude alternatives A_1 and A_4 from the set, and make the preference order of the remaining subset of alternatives, instead of being: A_5, A_3, A_2, the rankings will be reversed: A_2, A_3, A_5 (Table 9).

Table 9: Empirical values, their normalized ratings of VN type, and values of the SAW criterion for the subset of alternatives (A_2, A_3, A_5) in the decision problem shown in example 1

Alt.	X_1	X_2	X_3	X_4	VN				$V(A_i)=\sum r_{ij}^V$
					r_{i1}^V	r_{i2}^V	r_{i3}^V	r_{i4}^V	
A_2	3	8	140	98	0.2683	0.8433	0.5321	0.5888	2.2325
A_3	5	5	155	100	0.5555	0.5270	0.5891	0.6008	2.1641
A_5	9	1	160	90	0.8050	0.1054	0.6081	0.5407	2.0592

This shows that the SAW method, when applied to the data transformed by VN, also violates condition α.

c) In the next example it will be shown that normalized ratings of the NN type are not an exception.

Example 4. The DM is to choose among four alternatives on the basis of three benefit attributes. Table 10 contains empirical data (columns 1-4), their normalized ratings of NN type, as well as the values of the choice criterion made on the basis of the whole set of alternatives (columns 5-8), and the same values calculated for the subset of alternatives (A_1, A_2) (columns 9-12).

Table 10: Empirical values, their normalized ratings of NN type, and values of the SAW criterion for the whole set and for the subset of alternatives (A_1, A_2) for the decision problem shown in example 4

Alt.	X_1	X_2	X_3	NN			$V(A_i)=\sum r_{ij}^N$	NN			$V(A_i)=\sum r_{ij}^N$
				r_{i1}^N	r_{i2}^N	r_{i3}^N		r_{i1}^N	r_{i2}^N	r_{i3}^N	
A_1	40	200	100	0	1	1	2.00	0	1	1	2.0
A_2	48	190	98	0.8	0.9	0.96	2.66	1	0	0	1.0
A_3	49	100	95	0.9	0	0.90	1.80				
A_4	50	150	50	1	0.5	0	1.50				

Based on $V(A_i)$ values calculated for the whole set of four alternatives (column 8), the preference order is: A_2, A_1, A_3, A_4. But, with the exclusion of alternatives A_3 and A_4, the rankings changes in favour of alternative A_1 and is now: A_1, A_2 (column 12).

We conclude that *the SAW method, based on normalized ratings of SN, NN, or VN type, violates condition* α.

4.2. TOPSIS based on VN data violates condition α

Example 5. In this decision problem we have to choose one among five alternatives on the basis of three benefit criteria (Table 11., columns 1-4).

In the first step of the TOPSIS method, empirical data are transformed by vector normalization; after that, normalized ratings are multiplied by the attribute weights. Supposing that all attributes are equally important, the *attribute weight vector* is w = (0.333, 0.333, 0.333).

Table 11: Empirical values their weighted VN ratings for the decision problem shown in example 5, and TOPSIS measures and solutions

Alt.	X_1	X_2	X_3	$w_j r_{ij}^V$			
A_1	2	7	180	0.06086	0.15731	0.16800	A^+ = (0.2739, 0.2026, 0.1680)
A_2	3	8	140	0.09129	0.17979	0.13067	A^- = (0.3043, 0.0225, 0.1307).
A_3	5	5	155	0.15215	0.11237	0.14467	S^+ = (0.21178, 0.18774, 0.15329, 0.24407, 0.18109)
A_4	1	9	161	0.03043	0.20260	0.15027	S^- =(0.14318, 0.16868, 0.15196, 0.181192, 0.24414)
A_5	9	1	160	0.27386	0.02247	0.14934	C^+ = (0.39669, 0.47326, 0.49781, 0.42607, 0.57413).

Starting from the *weighted normalized ratings*, $w_j r_{ij}^V$, (columns 5-7) we identify Ideal Solutions (A^+) and Negative-Ideal Solutions (A^-) and calculate Separation Measures (S^+ and S^-). Finally, we calculate the values of the choice criterion - *Similarities to Ideal Solution* (C^+)(column 8) on the basis of which the rankings is: A_5, A_3, A_2, A_4, A_1. TOPSIS suggests selection of the alternative A^* with $C^+(A^*) = \max_i C^+(A_i)$, which means that A_5 should be chosen.

But, if alternatives A_1 and A_4 are eliminated from the set, and the choice is being made from the subset (A_2, A_3, A_5), on the basis of the new set of weighted normalized ratings (Table 12, columns 1-4) and calculated values of the choice criterion C^+ (column 5), the obtained rankings: A_2, A_3, A_5, differs from the previous one: A_5, A_3, A_2. We have shown that the *TOPSIS method, when based on normalized ratings of VN type, violates condition* α.

Table 12: Weighted VN ratings of empirical data from subset (A_2, A_3, A_5) for the decision problem shown in example 5, and TOPSIS measures and solutions

Alt.	$w_j r_{ij}^V$			
				A^+ = (0.27975, 0.281091, 0.202706)
A_2	0.093250	0.281091	0.177368	A^- = (0.093250, 0.035136, 0.177368)
A_3	0.155417	0.175682	0.196372	S^+ = (0.1882133, 0.1631253, 0.245955)
A_5	0.279751	0.035136	0.202706	S^- = (0.245955, 0.1548517, 0.1882133)
				C^+ = (0.5664969, 0.4869902, 0.433503)

4.3. *ELECTRE* based on VN data violates condition α

In order to show that the ELECTRE method also does not satisfy the same condition of consistent choice, let us return to example 5 (Table 11.). With no intention to describe the whole procedure, we shall only point to the starting and final results. Table 11 contains the values of the *weighted normalized decision matrix* (columns 5-7). If we start from it, in the final step we get the *aggregate dominance matrix:*

$$\begin{matrix} - & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 \\ 0 & 0 & 1 & 0 & - \end{matrix}$$

Table 13: Weighted VN ratings of empirical data from subset (A_3, A_4, A_5) for the decision problem shown in example 5

Alt.	$w_j r_{ij}^V$		
A_3	0.161123	0.161123	0.187977
A_4	0.032225	0.290021	0.195254
A_5	0.290021	0.032225	0.194041

On the basis of the matrix values, we conclude that alternative A_5 is the only one left in the *kernel*, which makes A_5 the optimal choice by ELECTRE. However, by eliminating alternatives A_1 and A_2 from the previous set, and using ELECTRE on the subset (A_3, A_4, A_5), the new aggregate dominance matrix (calculated from the weighted normalized ratings, Table 13, columns 2-4) is obtained:

$$\begin{matrix} - & 0 & 0 \\ 1 & - & 1 \\ 1 & 0 & - \end{matrix}$$

This matrix renders the following overranking relationships: $A_4 \rightarrow A_3$, $A_4 \rightarrow A_5$, $A_5 \rightarrow A_3$, on the basis of which we obtain the rankings: A_4, A_5, A_3. ELECTRE now suggests alternative A_4 as the best. We conclude that *the ELECTRE method, based on normalized ratings of VN type, violates condition* α.

5. CONCLUDING REMARKS

After analysing the effects of the three most commonly used normalizations (SN, NN and VN) on the results of some MADM methods, we made the following conclusions:

The type of normalization applied could determine the final decision (the rankings of alternatives). Every MADM methods uses, as a rule, only one type of normalization, but there are some methods for which more than one transformation is proposed as equally suitable (e.g.: with the SAW method, both SN and NN are suggested). Although different MADM methods lead to different rankings of the same set of alternatives (reflecting the different logical foundations of the various choice criteria), it does not seem rational for one MADM method to propose different choices, when based on different but equally acceptable normalization procedures.

MADM methods, which are based on SN or VN data, violate *the condition of independence of the value scale* (one of the main consistency conditions of normative decision theory). This means that positive affine transformations of empirical values could change the final rankings. It was shown that the final choice changed when for qualitative attributes, instead of the scale 1, 2, 3, 4, 5, its affine transformation, i.e. 1, 3, 5, 7, 9, was used. The same applies to those quantitative attributes that are measurable with an interval scale, where the very choice of their measurement units could determine the final result.

MADM methods, based on SN or VN data, *violate the condition of descriptive invariability*. This means that the framing of attributes (whether an attribute is represented in its benefit form or in its cost form, if both forms are possible) could affect the final result.

MADM methods, based on SN or VN data, violate *the contraction consistency condition, α*. The violation of this condition implies that the exclusion of some irrelevant alternatives (those which are not candidates for the final choice) could change the rankings of the remaining ones, changing thus the final decision.

This paper emphasises the need for detailed analysis of the normalization procedures. Their effects on the results of MADM methods have been extremely underestimated, particularly in comparison to the effects of the attribute weighting procedures. These results also create the need for defining a set of rationality conditions on the basis of which MADM methods should be evaluated and mutually compared.

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