ON A GRAPH TRANSFORMATION THAT PRESERVES THE STABILITY NUMBER

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Abstract: We derive from Boolean methods a transformation which, when applicable, builds from a given graph a new graph with the same stability number and with the number of vertices decreased by one. We next describe classes of graphs for which such a transformation leads to a polynomial algorithm for computing the stability number.

Keywords: Boolean methods, stability number, polynomial algorithms.

1. INTRODUCTION

In the present paper all graphs will be assumed simple (no loops and no multiple edges are allowed). A set $S$ of vertices in a graph $G = (V,E)$ is stable if no two vertices in $S$ are linked by an edge. The maximum size of a stable set in graph $G$ is denoted by $\alpha(G)$ and is called the stability number of $G$. For a weighted graph $G$, the maximum weight of a stable set in $G$ is denoted $\alpha_W(G)$.

Given a positive integer $k$, finding whether an arbitrary graph contains a stable set with at least $k$ vertices is NP-complete [6]. However, there are special classes of graphs for which $\alpha(G)$ can be computed in polynomial time [e.g. 1, 3, 4, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 19, 20].

In some cases, Boolean methods can suggest graph theoretical procedures. Ebenegger, Hammer and de Werra [5] have described the relationship between the maximization of a pseudo-Boolean function and the determination of a stable set.

* The Boolean transformation studied in this paper as well as the definition of a magnet are due to Peter L. Hammer. I would like to thank him for having encouraged me to demonstrate the potentiality of such a transformation.
having maximum weight in a graph. This relation is summarized in Section 2. In the
same paper, Ebenegger et al. consider the computation of the stability number \( \alpha(G) \) of
a graph \( G = (V, E) \) (unweighted case) and describe the transformation of the

\[ G' = \text{construction of another graph with } \alpha(G') = \alpha(G) - 1. \]

By repeatedly applying this construction, one may compute \( \alpha(G) \) (in at most \( \alpha(G) - 1 \)
steps). Unfortunately, the number of vertices is generally increasing when the transformation is applied. However, specialized versions of this construction have provided polynomial algorithms for some classes of graphs [7, 9, 10, 12]. Recently, a different Boolean transformation has been studied in [14].

In order to compute the stability number \( \alpha(G) \) of a graph \( G = (V, E) \), we study
in Section 3 a simplification of the corresponding pseudo-Boolean function; the
transformation, when applicable, amounts to constructing another graph \( G'' \)
with \( |V''| \leq |V| - 1 \) and \( \alpha(G'') = \alpha(G') \). It is based on the Boolean equality \( xy + x\overline{y} = x \).

In Section 4, we describe classes of graphs for which the stability number can
be computed in polynomial time by using the above transformation.

A graph \( G = (V, E) \) is bipartite if the vertex set \( V \) can be partitioned into two
sets \( V_1 \) and \( V_2 \) such that each edge of \( E \) has one endpoint in \( V_1 \) and the other in \( V_2 \); we shall denote such a graph by \( G = (V_1, V_2, E) \). A bipartite graph \( G = (V_1, V_2, E) \) is
complete if each vertex in \( V_1 \) is adjacent to all vertices of \( V_2 \).

A chordless cycle, or chain, on \( k \) vertices is denoted \( C_k \) or \( P_k \) for short.

For a vertex \( x \) in graph \( G \), we denote by \( N_G(x) \) the set of vertices which are
adjacent to \( x \) in \( G \). Graph \( G \) is called \( H \)-free if none of its induced subgraphs are
isomorphic to \( H \).

For two sets \( A \) and \( B \), \( A \setminus B \) denotes the set of elements which are in \( A \) but not
in \( B \). The weight of a set of vertices is the total weight of its elements.

The graph theoretical terms not defined here are borrowed from [2] while for
pseudo-Boolean definitions, the reader is referred to [11].

2. PSEUDO-BOOLEAN FUNCTIONS AND CONFLICT GRAPHS

It is known that a pseudo-Boolean function \( f \) can always be written in a
polynomial form, i.e.,

\[ f(x_1, \ldots, x_n) = K + \sum_{i=1}^{p} w_i T_i \]

where \( T_i = \prod_{j \in A_i} x_j \prod_{k \in B_i} x_k \) with \( A_i, B_i \subseteq \{1, \ldots, n\} \) and \( A_i \cap B_i = \emptyset \).
If all \( w_i (1 \leq i \leq p) \) are strictly positive and \( K = 0 \), we say that \( f \) is a posiform. To a posiform \( f \) we associate a weighted conflict graph \( G = (V, E) \) defined as follows:

\[
V = \{1, \ldots, p\} \text{ and each vertex } i \text{ has a weight } w_i, \\
E = \{(i, j) \mid \exists k \in ((A_i \cap B_j) \cup (A_j \cap B_i))\}.
\]

Hence, two vertices \( i \) and \( j \) of \( G \) are linked by an edge if \( x_k \) appears in \( T_i \) (or \( T_j \)) while \( x_k \) appears in \( T_j \) (or \( T_i \)). It is clear from the definition of \( G \) that the maximum value of \( f \) is equal to the maximum weight \( \alpha(G) \) of a stable set in \( G \).

Conversely, for each graph \( G \) with positive weights \( w_u \) associated with each vertex \( u \) of \( G \), there exist posiforms \( f \) such that \( G \) is the conflict graph of \( f \) [5]. Indeed, consider an arbitrary covering of the edge set of \( G \) by complete bipartite partial subgraphs \( G_i((V_i, V_j), E_i) \) of \( G \), \( i = 1, \ldots, q \). Notice that \( G_1, \ldots, G_q \) are partial, but not necessarily induced subgraphs of \( G \). Then set:

\[
f = \sum_{u \in V} w_u T_u
\]

where \( f \) is with 

Let \( u \) and \( v \) be two terms of the posiform \( f \) such that \( u \) appears in \( T_i \) and \( v \) appears in \( T_j \). Then \( u \) and \( v \) are adjacent in \( G \). Hence, \( u \) is adjacent to \( v \) in \( G \), showing that \( G \) is the conflict graph associated with \( f \).

### 3. MAGNETS IN GRAPHS

A magnet in a graph \( G \) is defined as a pair \((a, b)\) of adjacent vertices with the same weight and such that each vertex in \( G \) is adjacent to each vertex in \( G \). In other words, the two endpoints of an edge induce a magnet in a graph \( G \) if and only if this edge is not the middle edge of any path in \( G \).

Given a magnet \( (a, b) \) in a graph \( G \), we consider a new graph, denoted \( T \), and obtained from \( G \) by replacing vertices \( a \) and \( b \) by a new vertex having the same weight as \( a \) and \( b \) and linked to every common neighbor of \( a \) and \( b \) in \( G \). This transformation is illustrated in Fig. 1.

The following theorem states that the maximum weight of a stable set in \( G \) is not modified by transformation \( T \).
Theorem 1. Let $a$ be a magnet in a weighted graph $G$. Then $\alpha(G) \geq \alpha(T(G))$.

Proof: We shall give two proofs of this theorem, a Boolean and a graph theoretical one.

Boolean proof:
The edges incident to $a$ or $b$ can be covered by the two following complete bipartite partial subgraphs $G_1$ and $G_2$ of $G$:

$$\begin{align*}
G_1 &= \{(v, w) \in V \times V : v \text{ is adjacent to } a \text{ or } b \} \\
G_2 &= \{(v, w) \in V \times V : v \text{ is adjacent to no vertex of } \{a, b\} \}
\end{align*}$$

Consider now any covering of the edges in $E$ by complete bipartite partial subgraphs $G_1$ and $G_2$. The graphs $G_1$ and $G_2$ cover all the edges of $E$ and the associated posiform satisfies $\sum_{v \in V} T_v f(v) \leq \alpha(G_1) + \alpha(G_2)$. Hence, it follows that $f$ has the same maximum value as the posiform $\sum_{v \in V} T_v g(v)$. This means that the conflict graph associated with $g$ satisfies $\alpha(G) \leq \alpha(T(G))$.

Graph theoretical proof:
Denote $S$ and consider any stable set $S$ in $G$. If $a$ or $b$ belongs to $S$ then $S$ is stable in $G$ and the weight of $S$ is equal to the weight of $S$. Otherwise $S$ is stable in $T(G)$. This proves that $\alpha(T(G)) \geq \alpha(G)$.
In order to show that $G$ has stability number $\alpha$, consider any stable set $S$ in $G$ and define $bN(a)$ and $aN(b)$ and $S \cap NC$:

- if $a$ does not belong to $S$, then $S$ is stable in $G$;
- if $a$ belongs to $S$ while $b \not\in AS$, then both $bN(a)$ and $C(S \cap NC)$ are empty.

Hence, $S$ is stable in $G$ and $\alpha(G) = \alpha(S)$.

In a graph $G$, we say that some vertex $a$ dominates some vertex $b$ if $aN(b) \subseteq G$. It is well known that if a graph $G$ contains two adjacent vertices $a$ and $b$ such that $a$ dominates $b$, then the graph obtained from $G$ by removing vertex $a$ has a stability number equal to $\alpha(G)$. This is in fact a corollary of Theorem 1. Indeed, in this case, all weights are equal to one (unweighted case) and $G$ contains an induced subgraph of $S \cap NC$, which means that $a$ and $b$ are adjacent vertices of $G$ that are not adjacent in $S$.

Let $a$ and $b$ be two adjacent vertices in a graph $G$. If $a$ dominates $b$, we say that $(a, b)$ is a $d$-magnet in $G$ and the neighbors of $a$ and $b$ are exactly those of $b$ in $G$. Therefore, $(a, b)$ is the graph obtained from $G$ by removing vertex $a$.

Property 1. Let $(a, b)$ be a magnet in a graph $G$, let $H$ be an induced subgraph of $G$, and assume $G$ is $H$-free. Then $(a, b)$ is a vertex in $H$, and there are at least two adjacent vertices $c$ and $d$ in $H$ that are not adjacent to $a$.

Proof: Let $W$ denote the vertex set of $H$. If $a$ does not belong to $W$, then $a$ induces an $H$ in $G$, a contradiction. So assume that $a$ belongs to $W$. Define $aW$ and consider the subgraph $H$ induced by $aW$. If $a$ dominates $b$, or $b$ dominates $a$ in $H$, then $H$ or $H$ induces a graph isomorphic to $H$ in $G$, a contradiction. Hence $H$ contains an induced $H$, which means that $c$ and $d$ are adjacent vertices of $a$ that are not adjacent to $a$.

Definition 1. A graph $H$ is a demagnetization (or $d$-demagnetization) of a graph $G$ if

- $H$ does not contain any magnet (or $d$-magnet), and
- there exists a sequence of graphs such that for a magnet (or $d$-magnet) in $G$. 


It follows from Theorem 1 that if $H$ is a demagnetization of $G$ then
\[ \alpha(H) = \alpha(G). \]
From now on, we will only consider the unweighted case. In the next
section, we study classes of graphs $G$ such that the stability number
\( \alpha \) can be computed in polynomial time by finding the maximum stable set in a demagnetization $H$ of $G$.

4. TWO POLYNOMIALLY SOLVABLE CASES

Given a graph $G$, we distinguish among three kinds of vertices; and are the extreme vertices of are the interior ones, and all other vertices in $G$ are said to be exterior to .

A vertex $v$ in a graph $G$ is called special if each induced in $G$ contains $v$ as extreme vertex, and each induced in contains $v$ as interior vertex. In particular, if neither $G$, nor contains an induced , then each vertex in $G$ is special. Section 4.1 considers -free graphs that contain a special vertex; we show that a demagnetization $H$ of such graphs $G$ can be obtained so that the edge set of $H$ is empty (hence, equals the number of vertices in $H$).

A flag is a graph obtained from a by adding a vertex adjacent to exactly one vertex of the . A gem is the graph obtained from a by adding a vertex adjacent to all four vertices of the . A diamond is a complete graph on four vertices minus one edge. A is the graph obtained from a complete graph on six vertices by removing a perfect matching in it. All these graphs are represented in Fig. 2. Let $G$ be a -free, flag-free, gem-free and -free graph, let $H$ be any $d$-demagnetization of $G$, and let $L$ be any demagnetization of $H$; we prove in Section 4.2 that each connected component of $L$ is either an isolated vertex, or else a.

![Figure 2](image)

\text{flag (a,b,c,d,e)   gem (a,b,c,d,e)   diamond (a,b,c,d)   $3K_2(a,b,c,d,e,f)$}
4.1. On $C_8$-free graphs that contain a special vertex

**Lemma 1.** Let $a$ be a special vertex in a $C_8$-free graph $G$, and let $b$ be a vertex adjacent to $a$ that minimizes $d(a,b)$. Then $(a,b)$ is a magnet in $G$.

**Proof:** Argue by contradiction: assume $(a,b)$ is not a magnet in $G$. Then $G$ contains an induced $C_8$. Since $d(a,b)$ is smaller than or equal to the diameter of $G$, there exists a vertex $c$ adjacent to $b$ but not to $a$ and $b$ in $G$. Vertex $c$ cannot be adjacent to $d$, else $G$ contains an induced $C_8$. Hence, $a$ is not an extreme vertex of $G$, a contradiction. Such a magnet is called an $s$-magnet. We now prove that if is an $s$-magnet in a $C_8$-free graph $G$, then $G$ is also $C_8$-free and contains a special vertex.

**Lemma 2.** Let $a$ be an $s$-magnet in a $C_8$-free graph $G$. Then $G$ is $C_8$-free.

**Proof:** Argue by contradiction: assume $G$ contains an induced $C_8$. We know by Property 1 that $a$ belongs to $G$; we may assume $a$ is not an extreme vertex of $G$, a contradiction. It follows that $a$ is not an interior vertex of $G$, a contradiction.

**Lemma 3.** Let $a$ be an $s$-magnet in a $C_8$-free graph $G$. Then $a$ is a special vertex in $G$.

**Proof:** Argue by contradiction: assume first $a$ is not an extreme vertex of an induced $C_8$ in $G$. Then $a$ must belong to $G$, else $a$ is an exterior vertex of $G$. But $a$ cannot be equal to $a$, else $G$ contains an induced $C_8$ in $G$, a contradiction. It follows that either $a$ is not an extreme vertex of an induced $C_8$ in $G$, or else $a$ is not an interior vertex of an induced $C_8$ in $G$, a contradiction.

Assume now $a$ is not an interior vertex of an induced $C_8$ in $G$. Then $a$ must belong to $G$, else $a$ is exterior to $G$; we may assume $a$. It follows that contains an induced $C_8$ or in which $a$ is not an interior vertex, a contradiction.
A demagnetization of a graph is called an s-demagnetization if only special magnets are used when applying transformation T. The following theorem is a direct consequence of Lemma 2 and Lemma 3.

**Theorem 2.** Let G be a -free graph that contains a special vertex, and let H be an s-demagnetization of G. Then H has an empty edge set.

Notice that if a special vertex in a graph G is isolated, then both G and are -free (since, otherwise, a would be an exterior vertex of any in G or ). The following algorithm can therefore be used for computing the stability number of a -free graph that contains a special vertex.

<table>
<thead>
<tr>
<th>Input.</th>
<th>A -free graph G that contains a special vertex.</th>
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<tbody>
<tr>
<td>Output.</td>
<td>The stability number of G.</td>
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</table>

1. Determine a special vertex a in G. Set H := .
2. While the edge set of H is not empty do
   - If a is an isolated vertex, then set a equal to any nonisolated vertex in H.
   - Determine a vertex b adjacent to a that minimizes |
   - Set .
3. Set equal to the number of vertices in H.

Finding a special vertex (if any) in a graph can be performed in time. Determining a vertex b adjacent to a that minimizes takes time. Given a magnet in H, the construction of takes time. Since the main loop is performed times, it follows that the above algorithm runs in time.

A graph G is called Meyniel [18] if each odd cycle in G with at least five vertices contains at least two chords. It is easy to observe that a graph G is -free, -free and -free if and only if both G and are Meyniel. Notice that each vertex in such a graph is special; hence, the first step of the above algorithm can be simplified. It therefore follows that if both G and are Meyniel, then the stability number of G can be computed in time by means of the above algorithm.

### 4.2. On -free, flag-free, gem-free and -free graphs

**Lemma 4.** Let G be a -free, flag-free, gem-free and -free graph that contains no d-magnet. Then G is diamond-free.
Proof: Argue by contradiction: assume $G$ contains an induced diamond $\Delta$. Since $b$ does not dominate $a$, there is a vertex $e$ in $G$ adjacent to $a$ but not to $b$. Vertex $e$ cannot be adjacent to exactly one vertex among $c$ and $d$ else induces a gem in $G$.

We first show that $e$ cannot be adjacent to both $c$ and $d$. If this is the case, then there exists a vertex $f$ adjacent to $e$ but not to $a$ in $G$ (else $e$ dominates $a$). Now $f$ is adjacent to $b$ else $G$ contains an induced flag $F$, gem $G$ or gem $G$. Also, $f$ is neither adjacent to $c$, nor to $d$, else $G$ contains an induced gem $G$, gem $G$ or gem $G$. Since $a$ does not dominate $c$, there exists a vertex $g$ adjacent to $c$ but not to $a$. Vertex $g$ is not adjacent to $b$, else $G$ contains an induced gem $G$, gem $G$ or gem $G$. It follows that $G$ contains an induced gem $G$, flag $F$, flag $F$ or flag $F$, a contradiction.

So $e$ is adjacent neither to $a$, nor to $b$. Up to this point, we have proved that any vertex that is adjacent to exactly one vertex among $a$ and $b$ is adjacent neither to $c$, nor to $d$. Now, since $a$ does not dominate $e$, there exists a vertex $f$ adjacent to $e$ but not to $a$. Vertex $f$ cannot be adjacent to $c$ or $d$ else $f$ would not be adjacent to $b$ (by the above observation) and $G$ would contain an induced flag $F$, flag $F$ or flag $F$. Since $a$ does not dominate $c$, there exists a vertex $g$ adjacent to $c$ but not to $a$, and this implies that $g$ is not adjacent to $b$ (by the above observation). So, $g$ is adjacent to $e$ and $f$ else $G$ contains an induced $G$, flag $F$ or flag $F$. Finally, $G$ contains an induced flag $F$ or flag $F$, a contradiction.

Corollary 1. Let $G$ be a $d$-free, flag-free, gem-free and $d$-free graph. Let $H$ be a $d$-demagnetization of $G$. Then $G$ is $d$-free, flag-free and diamond-free.

Proof: $H$ is an induced subgraph of $G$. Hence, this corollary directly follows from Lemma 4 and from the fact that a diamond is an induced subgraph of a gem and of a $d$-free graph.

Lemma 5. Let $A$ be a magnet in a $d$-free, flag-free and diamond-free graph $G$. Then $A$ is also $d$-free, flag-free and diamond-free.

Proof: The fact that $A$ is diamond-free follows from Property 1. Notice also that if two vertices $c$ and $d$ are adjacent to $A$ in $G$, then $c$ is adjacent to $d$, else $G$ contains an induced diamond $\Delta$. It now remains to prove that $H$ is $d$-free and flag-free. We argue by contradiction and assume first that $H$ contains an induced $G$. According to Property 1 and the above observation,
only be equal to $x$ or $u$. We may assume $x$. Now, neither $a$, nor $b$ is adjacent to $z$ in $G$, else $G$ contains an induced diamond or diamond. So $G$ contains an induced flag or flag, a contradiction.

So, we assume now $H$ contains an induced flag. Again, according to Property 1 and to the above observation, can only be equal to $x$. Now, neither $a$ nor $b$ can be adjacent to $z$ or $u$, else it induces a diamond in $G$. It follows that $G$ contains an induced flag or flag, a contradiction.

Lemma 6. Let $G$ be a $P_5$-free, flag-free and diamond-free connected graph containing an induced $P_5$. Then $G$ is.

Proof: Argue by contradiction: let be an induced subgraph of and suppose . Since $G$ is connected, we may assume that there is a vertex $a$ in adjacent to $z$. Vertex $a$ cannot be adjacent to both and $G$ contains an induced diamond. We may assume that $a$ is not adjacent to $z$. Now $a$ is adjacent to else $G$ contains an induced or flag. Hence, $G$ contains an induced diamond or flag, a contradiction.

Theorem 3. Let $G$ be a $P_5$-free, flag-free, gem-free and $K_2$-free graph, let $H$ be a demagnetization of $G$ and let $L$ be a demagnetization of $H$. Then each connected component of $L$ is either

(i) an isolated vertex, or
(ii) $P_5$

Proof: We know from Corollary 1 that $H$ is $P_5$-free, flag-free and diamond-free. Hence, it follows from Lemma 5 that $L$ is also $P_5$-free, flag-free and diamond-free.

Consider any connected component of $L$. We may assume that has a nonempty edge set, else is an isolated vertex and nothing has to be proved. So let $a$ and $b$ be two adjacent vertices in $L$. Since is not a magnet in there is an induced in . Now, since is not a magnet in, there is an induced in (vertex $e$ is possibly equal to $b$). It follows that $f$ is not adjacent to $b$ in , else it contains an induced flag, flag, or diamond. Also, $f$ is adjacent to $d$ in, else it contains an induced . It follows that contains an induced , which means that , by Lemma 6.
It follows from the above theorem that the stability number of \( P \)-free, flag-free, gem-free and \( K \)-free graphs can be computed in polynomial time by means of the following algorithm.

**Input.** A \( P \)-free, flag-free, gem-free and \( K \)-free graph.

**Output.** The stability number \( \alpha(G) \) of \( G \).

1. Set \( H \):
   While \( H \) contains a \( d \)-magnet do
   Choose any \( d \)-magnet \( (a,b) \) in \( H \) and set \( (a,b) \) in \( H \).

2. Set \( L \):
   While \( L \) contains a magnet do
   Choose any magnet \( (a,b) \) in \( L \) and set \( (a,b) \) in \( L \).

3. Let \( n \) and \( c \) be the number of vertices and the number of induced \( 5 \)-cliques in \( L \), respectively. Set \( \alpha(G) \) equal to \( cn \).

Finding a magnet or a \( d \)-magnet (if any) in a graph can be performed in \( O(|V(G)|) \) time. Since at most \( |V(G)| \) magnets are determined in Steps 1 and 2, the above algorithm runs in \( O(|V(G)|^2) \) time.

Let \( G \) be an arbitrary graph, and let \( L \) be the graph resulting from the application on \( G \) of Steps 1 and 2 of the above algorithm. If each connected component of \( L \) is either an isolated vertex, or a \( 5 \)-clique, then Step 3 can be applied on \( L \) in order to compute the stability number of \( G \). According to Theorem 3, such a situation necessarily occurs if \( G \) is \( P \)-free, flag-free, gem-free and \( K \)-free. It can, however, also occur for other kinds of graphs. For example, if \( G \) is a flag, then \( L \) contains exactly three isolated vertices, which means that the stability number of a flag is three. There is therefore no need to design a recognition algorithm for \( P \)-free, flag-free, gem-free and \( K \)-free graphs. It is more interesting to apply Steps 1 and 2 of the above algorithm to any given graph \( G \), and to check whether the reduced graph \( L \) has the desired structure.

### 5. CONCLUDING REMARKS

One of the aims of this paper was to prove that Boolean methods can suggest graph theoretical procedures. We have studied a simplification on posiforms which, when applicable, amounts to reducing the size of the corresponding conflict graph while preserving its stability number. We have described in section 4 classes of graphs \( G \) for which such a transformation leads to a polynomial algorithm for the computation of \( \alpha(G) \).
Let $CC$ be a class of graphs for which the stability number can be determined in polynomial time. Future research in the use of transformation $T$ would be to characterize those graphs $G$ that admit a demagnetization $H$ with $CC \in H$. We could for example choose $CC$ as being the class of claw-free graphs \cite{19, 20}. Notice that given any magnet in a claw-free graph $G$, the graph is also claw-free, by Property 1.

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