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# MULTICRITERIA APPROACH TO THE PROBLEM OF THE INTEGRATION OF SUPPLIERS INTO A PRODUCTION-DISTRIBUTION SYSTEM

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**Abstract:** This paper presents a new mathematical model for choosing a supply strategy, with respect to many criteria, which integrates suppliers into a production-distribution system. The problem presents part of a purchasing plan and it is very current in contemporary business practice. The importance of reliable supply is increasing with global sourcing. The integration of suppliers and production-distribution systems means that both entities have the same or similar objectives. The criteria present supplier performance and are given either as cardinal values or by linguistic expressions. Uncertainties are modeled by discrete fuzzy numbers. The uncertainties are described by possibility measures and are based on the subjective judgement of experts. The algorithm for choosing the best supply strategy which integrates primarily into a production-distribution system is based on the application of a new fuzzified analytic hierarchy process.

**Keywords:** Supply strategy, integration, production-distribution system, fuzzy data, multicriteria optimization.

## 1. INTRODUCTION

The problem of the integration of suppliers is part of the purchasing problem of production-distribution systems (PDS). Technological, political, economic and other environmental changes in the business world, increasing competition with respect to quality, changes in prices, and the fast development of information technology (IT) whose implementation is simple and not expensive, require changes in purchasing management. The global position of supply is changing very fast so that the organizing and managing of a PDS are defined according to those changes. The problem of the best purchasing has recently emerged as an important strategic area of management decision making (Dooley, K., 1995). The importance of reliable supply is increasing with global sourcing.

The PDS can be supplied in two ways: (1) single sourcing and (2) multiple sourcing. The goal of JIT (Just-in-Time) purchasing is a single, reliable supplier. The JIT philosophy represents an aggressive approach to dealing with all sources of inefficiency by insisting on a reduction in the number of suppliers so that any sources of uncertainty are exposed and eliminated. However, single sourcing is not reliable, for example, when a reliable supplier is not available. Multiple sourcing offers a number of alternative-supply strategies which can be integrated into the PDS. Most USA firms do not purchase using a single source strategy. They prefer the purchasing safety of multiple sources. They are moving towards smaller suppliers. Even Honda, a pioneer in JIT purchasing, uses multiple sources (McMillan, 1990).

The literature contains many papers treating the multiple sourcing problem. The purchasing problem is treated as a concept (Dobler, 1990) or is stated as an optimization problem. The models are single optimization criterion types. Lau and Lau (Lau, and Lau, 1994) proposed a method to determine an optimal supply strategy for a situation where two competing suppliers offer different prices, quality levels and lead time performance. They give a solution procedure for obtaining a lowest cost ordering policy. This procedure can easily identify the optimal policy for any given combination of those parameters. Kelle and Silver (Kelle, and Silver, 1990) have investigated how the lead time demand and hence safety stock can be reduced by splitting an order between two or more suppliers. Kelle and Miller (Kelle, P., and Miller, A., 1998) considered the following problem: under which circumstances is single or dual sourcing preferable. This model allocates the order between the suppliers in order to minimize the stockout risk based on their lead time characteristics. They assume: (1) demand for the item is known, (2) both suppliers have random lead times with different characteristics and (3) the split rate between the two suppliers is $r$, $r \in [0,1]$. The research presented is considered to be a step towards better risk evaluation and decision support in ordering strategically important items.

This paper is organized in the following way. In Section 2 the problem of supplier integration is presented which exists on the raw market as an input to the PDS. In this paper, a new fuzzy approach to the treatment of uncertain and imprecise variables is developed. Suppliers are integrated into the PDS according to a highly ranked supply strategy. Section 3 presents the modeling of uncertainty and optimization criteria. Section 4 presents the fuzzy version of the Analytic Hierarchy Process that is developed and used. In Section 5 the application of the model developed is illustrated by an example.

## 2. PROBLEM STATEMENT

The problem of choosing the best supply strategy which is part of the PDS purchasing strategy management is further explained. It is assumed that the PDS operates in an uncertain environment in which the important variables are changed rapidly and continually. The managing of the PDS is based on taking these changes into account. It means that modern concepts of managing are used.

The problem treated is a subproblem of purchasing in the PDS which implies the decomposition of a complex system into subsystems. The analysis of the complex system and the definition of critical parameters which influence the problem are made according to Zrnic (Zrnic, 1996).

The problem treated exists in the subsystem SUPPLIERS which is identified by the set of indices, so that $\text{SUPPLIERS} = \{1,\dots,s,\dots S\}$ and $S$ is the total number of suppliers. Following the literature, we shall assume $S < 5$ (McMillan, 1990, and Galovic and etc., 1997). The subsystem SUPPLIERS operates in an uncertain environment which is denoted as the global supply market.

Each supplier is described by attributes such as: unit price of material, quality of material, form of payment, reliability, lead time, transportation time, etc. Supply managers determine for each particular item the performance which describes each supplier, i.e. each item. We shall assume that the performance of a supplier is an uncertain and imprecise value that can be described by linguistic descriptors. The modeling of linguistic descriptors is based on the theory of fuzzy set and the rules of fuzzy of algebra (Zimmermann, 1992).

The optimization criteria to select the supply strategy are based on supplier performance. In general, we shall consider $K$ criteria, simultaneously. They are denoted by $K = \{1,\dots,k,\dots,K\}$ , so that $K$ is the set of indices of the criteria. Since these supplier performances are uncertain and imprecise variables, it means that the optimization criteria are uncertain and imprecise variables, too. The procedure of optimization criteria modeling is presented in Section 3.

In selecting the best criteria it is necessary to take into account the relative importance of each criterion. Generally, all considered criteria do not have the same importance. In this paper, we shall assume that the relative importance of each pair of criteria which belong to set $K$ is given. The relative importance is given by linguistic descriptors. The procedure of calculating the relative importance of each particular criterion is obtained. Modeling the relative importance of each pair criteria is presented in Section 4.

In this paper, the optimization criteria considered are: unit price of the material, lead time and form of payment. It should be mentioned that some other criteria can be considered to solve a concrete problem. The authors think that these three criteria provide the possibility of using different approaches in calculating criteria values.

Supply strategies are defined by supply managers. In general, $I$ possible supply strategies are defined. They are presented by $I = \{1,\dots,i,\dots,I\}$, so that $I$ is the set of indices of supply strategies.

### 2.1. Assumptions in the model development

1. The assortment of the PDS involves $P$ different kinds of end products. They are presented by $P = \{1,\dots,p,\dots,P\}$, so that $P$ is the set of indices of end products. Each end product is made of a single or several kinds of materials. The material for consumption in each kind of product is determined by constructive and technological documentation.

2. In this paper, only one kind of material is taken into consideration.

3. It is assumed that $N$ customers exist on market.

4. The time horizon is defined and is marked as $\tau$. The time horizon is discretized, so that $\tau = T\Delta t$, where $T$ is the total number of subintervals in time horizon $\tau$ and $\Delta t$ is a discrete time step.

### 2.2. Formal Statement of the Problem

The problem of choosing the best supply strategy is presented as a task of multicriteria optimization. The treated problem is presented by matrix $F$, and $F = [f_{ik}]_{I \times K}$

The matrix element, $f_{ik}$, presents the value of criterion $k$ for alternative $i$. Since all the criteria are uncertain and imprecise variables it means that all the elements of matrix $F$ are also uncertain and imprecise. The value of each element $f_{ik}$ is obtained in the procedure which is based on rules of fuzzy algebra.

$w_1,\dots,w_k,\dots,w_K$ are the absolute importances of the considered criteria. In general, these values are different. In this paper, the relative importance of each pair of considered criteria is treated and it influences the choice of the best alternative.

The problem is solved by fuzzified AHP- FAHP. The fuzzification of AHP is explained in Section 4.

## 3. MODEL DEVELOPMENT

The following criteria are associated:

**1. Unit price of the material**

Unit price of the material depends on: consumption and agreement between the supplier and the PDS. It should be noticed that this variable is different for each supplier.

The total order quantity is

$$Q = \sum_{t=1}^{T} \sum_{n=1}^{N} \sum_{p=1}^{P} a_p D_{tn}^p - Q_1 \tag{1}$$

where

$N$- number of customers

$P$- number of end products

$a_p$ - index of considered kind of raw materials for end products

$D_{tn}^p$- demand for product $p$ $(p = 1, \ldots, p, \ldots P)$ which is placed by customer $n$ $(n = 1, \ldots, n, \ldots N)$ during $\Delta t$

$Q_1$ - total quantities of considered kind of material stocked at the beginning period $\tau$.

Expression (1) can be transformed into expression

$$Q = \sum_{p=1}^{P} a_p \sum_{n=1}^{N} \sum_{t=1}^{T} D_{tn}^p - Q_1 \tag{1.1}$$

$\sum_{t=1}^{T} D_{tn}^p$ - total demand for end product $p$ which comes from customer $n\tau$. It is presented by linguistic descriptor "demand is about $d$", where $d \in N$. In the mathematical approach, this linguistic descriptor is modeled by a discrete fuzzy number, $\overline{D}_n^p$. It is presented by expression

$$\overline{D}_n^p = \{d_r^n, \mu_{\overline{D}_n^p}(d_r^n)\} \tag{2}$$

where

* $d_r^n \in D_1$ and $D_1$ is the domain of fuzzy number $\overline{D}_n^p$ which consists of discrete values of demand which come from n customers for period $\tau$.

* $\mu_{\overline{D}_n^p}(d_r^n)$ is the membership function of fuzzy number, $\overline{D}_n^p$.

The expression (1.1) can be written as:

$$Q = \sum_{p=1}^{P} a_p \sum_{n=1}^{N} \overline{D}_n^p - Q_1 \tag{1.2}$$

$\sum_{n=1}^{N} \overline{D}_n^p$ is the total number for all end products in period $\tau$. In the mathematical approach, it is the sum of $N$ discrete fuzzy numbers and corresponding fuzzy numbers according to the rules of fuzzy algebra.

$$\overline{D}^p = \{d_x, \mu_{\overline{D}^p}(d_x)\} \tag{3}$$

where

* $d_x = \sum_{n=1}^{N} d_r^n$ , $d_x \in D_2$ . $D_2$ is the domain of fuzzy number $\overline{D}^p$

*The values of the membership function of fuzzy $\overline{D}^p$ are calculated according to the extension principle (Zimmermann,1985):

$$\mu_{\overline{D}^p}(d_x) = \sup_{d_x} \min\, (\mu_{\overline{D}_1^p}(d^n), \ldots, \mu_{\overline{D}_N^p}(d^n)) \tag{4}$$

$$Q = \sum_{p=1}^{P} a_p \overline{D}^p - Q_1 \tag{1.3}$$

$a_p \overline{D}^p$ is presented the quantity of the considered kind of material which is used for the production of product $p$ in period $\tau$. In the mathematical approach, this expression represents a discrete fuzzy number.

$$a_p \overline{D}^p = \{a_p d_x, \mu_{\underline{D}^p}(d_x)\} \tag{5}$$

The total consumption of material in the fuzzy approach is $\overline{D}$. It is calculated as the sum of $P$ fuzzy numbers and is a fuzzy number, too. It is presented by expression

$$\overline{D} = \{d_y, \mu_{\overline{D}}(d_y)\} \tag{6}$$

where:

* $d_y = \sum_{p=1}^{P} a_p d_x$ , $d_y \in D_3$ and $D_3$ is the domain of fuzzy number $\overline{D}$.

* $\mu_{\overline{D}}(d_y)$ is the membership function of fuzzy number $\overline{D}$ and it is calculated according to the extension principle

$$\mu_{\overline{D}}(d_y) = \sup_{\sum_{r=1}^{R} \overline{D}^r} \min \left(\mu_{\overline{D}^1}(d_x), \dots, \mu_{\overline{D}^R}(d_x)\right) \tag{7}$$

In the fuzzy approach, the initial stock level at beginning period $\tau$ is presented by expression

$$\overline{Q_1} = (Q,1) \tag{8}$$

The analytic expression which determines the total order quantity is given by replacing expressions (6) and (8) into expression (1). This variable is the difference of two fuzzy numbers:

$$\overline{Q} = \overline{D} - \overline{Q}_1 \tag{9}$$

It is shown by expression

$$\overline{Q} = \{d_y - Q, \mu_{\overline{Q}}(d_y - Q)\} \tag{10}$$

where

* $d_y - Q \in D_4$ and $D_4$ is the domain of fuzzy number $\overline{Q}$

* $\mu_{\overline{D}}(d_y - Q)$ is the membership function of fuzzy number $\overline{Q}$ which is calculated by the extension principle

$$\mu_{\overline{Q}}\ (d_y - Q) = \sup_{\overline{D} - \overline{Q}_1} \min\ (\mu_{\overline{D}}(d_y), 1) \tag{11}$$

The unit price of the material depends on the agreement between the supplier and the PDS:

$$c = f(Q) \tag{12}$$

Expression (12) is an empiric expression and is different for each considered supplier. The shape of function $f$ is determined according to the agreement between the supplier and the PDS.

Expression (12) presents the initial set for determining the unit price of the material.

The unit price of the material is defined according to expressions (11) and (12). It is presented by expression

$$\overline{c}_s = \{c_y, \mu_{\overline{c}_s}(c_y)\} \tag{13}$$

* $c_y \in C$ and $c_y$ is the unit price of the material

* $\mu_{\overline{c}_s}(c_y)$ is the membership function of fuzzy number $\overline{c}_s$ which is calculated by the extension principle.

**2. Lead time**

The lead time of each supplier is the time from the moment the supplier receives an order to the moment the supplier is ready to make delivery. According to the definition of lead time, transportation time is not included in lead time. Lead time depends on the performance of suppliers.

It is assumed that the lead time is presented by linguistic descriptors: "small", "medium" and "large", for the example in a fixed period.

In the mathematical approach, these linguistic descriptors are modeled by discrete fuzzy numbers, $\overline{LT}_1, \overline{LT}_2, \overline{LT}_3$, respectively. The values of the domain of each fuzzy number are defined by data from experience and are expressed by a time unit, e.g., the time unit is day. Membership functions are defined by the subjective judgements of experts.

In this paper, the lead times as fuzzy numbers are:

"small"= $\overline{LT}_1 = \{(10,1),(12,0.6),(14,0.2)\}$

"medium"= $\overline{LT}_2 = \{(10,0.2),(14,0.4),(16,0.6),(18,0.8),(20,1),(22,0.8),(24,0.6),(26,0.4),(28,0.2)\}$

"large"= $\overline{LT}_3 = \{(26,0.2),(28,0.6),(28,1)\}$

**3. Form of payment**

The form of payment is one of the most important criteria for choosing the best supply strategy, especially when the PDS is operating in a business environment with a high degree of uncertainty.

The form of payment is different for each supplier and results from an agreement between the PDS, the supplier and the bank. There is no unique classification of this criterion. In this paper, five cases are considered: (1) credentials, (2) partial advanced payment and the rest in cash, (3) partial advanced payment and the rest on credit, (4) COD (cash on delivery) and (5) on credit.

Each form of payment is modeled by a scale of measures. This scale is defined according to the scale of measures of AHP. It is presented in Table 1.

**Table 1:** Scale of measures for form of payment

| Form of payment | Value |
|---|---|
| 1 | 1 |
| 2 | 3 |
| 3 | 5 |
| 4 | 7 |
| 5 | 9 |

In this scale of measures, number 1 means the worst form of payment and number 9 means the most suitable form of payment.

# 4. FUZZY ANALYTIC HIERARCHY PROCESS - FAHP

The problem of choosing the best supply strategy is presented as a multicriteria optimization problem and solved by the modified Analytic Hierarchy Process-AHP (Harker, 1988). The principles of fuzzification are presented in detail in the doctoral thesis by the author of this paper (Galovic, 1999). Further, the fuzzificated AHP method is presented by FAHP. Concisely speaking, the modification of this method consists of the relative importance of criterion $k$ to criterion $k'$ $(k, k' \in K)$ which is defined by a linguistic descriptor. It is important to mention that relative importance is defined independently of the supplier which exists on the raw market.

In this paper, it is assumed that the relative importance of a pair of criteria is defined by three linguistic descriptors: "less important" ($k$ is less important than $k'$), "important" ($k$ is not as important as $k'$) and "very important" ($k$ is much more important than $k'$) which is presented by $\overline{X}_1, \overline{X}_2, \overline{X}_3$, respectively. In the mathematical approach, $\overline{X}_1, \overline{X}_2, \overline{X}_3$ represent discrete fuzzy numbers. The domain of each discrete fuzzy number ($\overline{X}_1, \overline{X}_2, \overline{X}_3$) is an integer which belongs to the interval $[1-9]$. The scale $\{1,\ldots,9\}$ is defined in the prototype AHP method. The values of the membership functions of each discrete fuzzy number ($\overline{X}_1, \overline{X}_2, \overline{X}_3$) are given by the subjective judgement of experts. In general, the values of the membership functions of each treated discrete fuzzy number are different.

In this paper, each discrete fuzzy number considered has the discretization step $\Delta = 1$. The values of membership functions are equal and are: $\alpha_1 = 0$, $\alpha_2 = 0.25$, $\alpha_3 = 0.5$, $\alpha_4 = 0.75$ and $\alpha_5 = 1$. According to these assumptions, it means that:

$$\overline{X}_1 = \{(1,1),(2,0.75),(3,0.5),(4,0.25),(5,0)\}$$

$$\overline{X}_2 = \{(1,0),(2,0.25),(3,0.5),(4,0.75),(5,1),(6,0.75),(7,0.5),(8,0.25),(9,0)\}$$

$$\overline{X}_3 = \{(5,0),(6,0.25),(7,0.5),(8,0.75),(9,1)\}$$

It should be mentioned that the number of fuzzy numbers ($\overline{X}_1, \overline{X}_2, \overline{X}_3$) is not strictly determined and depends on the nature of the problem considered and on the judgements of experts.

The procedure to find the best alternative is realized by the FAHP method which is performed in two steps: (1) first, relative criteria importance is defined and (2) the elements of the comparison pair matrix of alternative preference under each criterion are defined.

The elements of the comparison pair matrix of relative criteria importance are defined as the importance criterion $k$ to $k'$ $(k, k' \in K)$.

The next step in the procedure to find the best alternative is defining the comparison pair matrix of alternative performance for each criterion and for each value of the membership function of discrete fuzzy numbers ($\overline{X}_1, \overline{X}_2, \overline{X}_3$), which are in general, different. In this paper, we shall assume that the comparison pair matrices of alternative preference under criterion $k$ are equal for all values of the membership function.

The elements of these matrices are defined as preference alternative $i$ to alternative $i'$. $(i, i' \in I)$. The values of these elements are generated from matrix $F$ which is described in Section 2.3. The value of the element is calculated according to the expression $f_{ik}/f_{i'k}$. Since all the elements of matrix $F$ are discrete fuzzy numbers, this means that the values of the elements of the comparison pair matrix of alternative preference are obtained in the procedure of comparing two fuzzy numbers.

Two fuzzy numbers can be compared in different ways. One of them is to transform fuzzy numbers $f_{ik}$ into discrete stochastic variables. This transformation is performed by an inverse procedure developed by Dubois and Prade (Dubois, Prade, 1986).

The probability $p(v_{ik} > v_{i'k})$ is defined by the theory of probability. These probabilities have deterministic values. $p(v_{ik} > v_{i'k})$ is the degree of probability that the stochastic variable $v_{ik}$ is strictly bigger than stochastic variable $v_{ik}$ and presents the preference alternative $i$ to alternative $i'$.

When the comparison pair matrix of relative criteria importance and the comparison pair matrix of preference alternative are defined, the FAHP method starts to operate. The procedure to find the best alternative is based on the concept of equal possibilities.

The result obtained by the FAHP method is a fuzzy number. It has to be defuzzified. In this paper, deffuzification is obtained by the method of maximum possibility (Graham, 1991). It is presented as:

| Values of membership functions | Values of alternatives | | | | |
|---|---|---|---|---|---|
| | 1 | .......... | $i$ | .......... | $I$ |
| $\alpha_1$=0 | $n_1^1$ | .......... | $n_i^1$ | .......... | $n_I^1$ |
| $\alpha_2$=0.25 | $n_1^2$ | .......... | $n_i^2$ | .......... | $n_I^2$ |
| $\alpha_3$=0.5 | $n_1^3$ | .......... | $n_i^3$ | .......... | $n_I^3$ |
| $\alpha_4$=0.75 | $n_1^4$ | .......... | $n_i^4$ | .......... | $n_I^4$ |
| $\alpha_5$=1 | $n_1^5$ | .......... | $n_i^5$ | .......... | $n_I^5$ |

The maximum possibility of each alternative is calculated by $\max(n_1^i, n_2^i, n_3^i, n_4^i, n_5^i) = n_i^*$ The best alternative, $i^*$, under all criteria is obtained by relation $\max(n_1^*, n_2^*, n_3^*, n_4^*, n_5^*) = n^*$. The best alternative is the alternative whose possibility is the biggest.

# 5. AN ILLUSTRATIVE EXAMPLE

We shall assume that the following assumptions are necessary to choose the best supply strategy:

1. We shall consider three products, so that $P = \{1,2,3\}$.

2. There are two customers in the PDS.

3. The demand that occurs in the PDS is presented by the following discrete fuzzy numbers:

$$D_1^1 = \{(20,0.25),(60,0.5),(100,0.75),(140,1)\}$$

$$D_2^1 = \{(10,0.1),(30,0.4),(50,0.7),(70,1)\}$$

$$D_2^2 = \{(20,0.25),(50,0.5),(80,0.75),(110,0.5),(140,0.25)\}$$

$$D_1^3 = \{(110,0.2),(130,0.4),(170,0.6),(200,0.8),(230,1)\}$$

$$D_2^3 = \{(90,0.2),(150,0.6),(210,1)\}.$$

4. The optimization criteria are: unit price of material, lead time and form of payment.

5. There are three alternatives which are considered, so that: 1- only the first supplier, 2- only the second supplier and 3- combined supply 50% from the first supplier and 50% from the second supplier.

According to expressions (1) to (11), the needed consumption of raw material can be calculated for the production of all products in the treated period. This variable is a fuzzy number and it is presented as:

$$\overline{Q} = \begin{Bmatrix} (60,0.2),(80,0.25),(100,0.4),(120,0.5),(140,0.75), \\ (170,1),(190,0.5),(210,0.25) \end{Bmatrix}$$

The unit price of raw material that is offered by the first and the second supplier is calculated according to the initial fuzzy sets showed in Fig. 1.

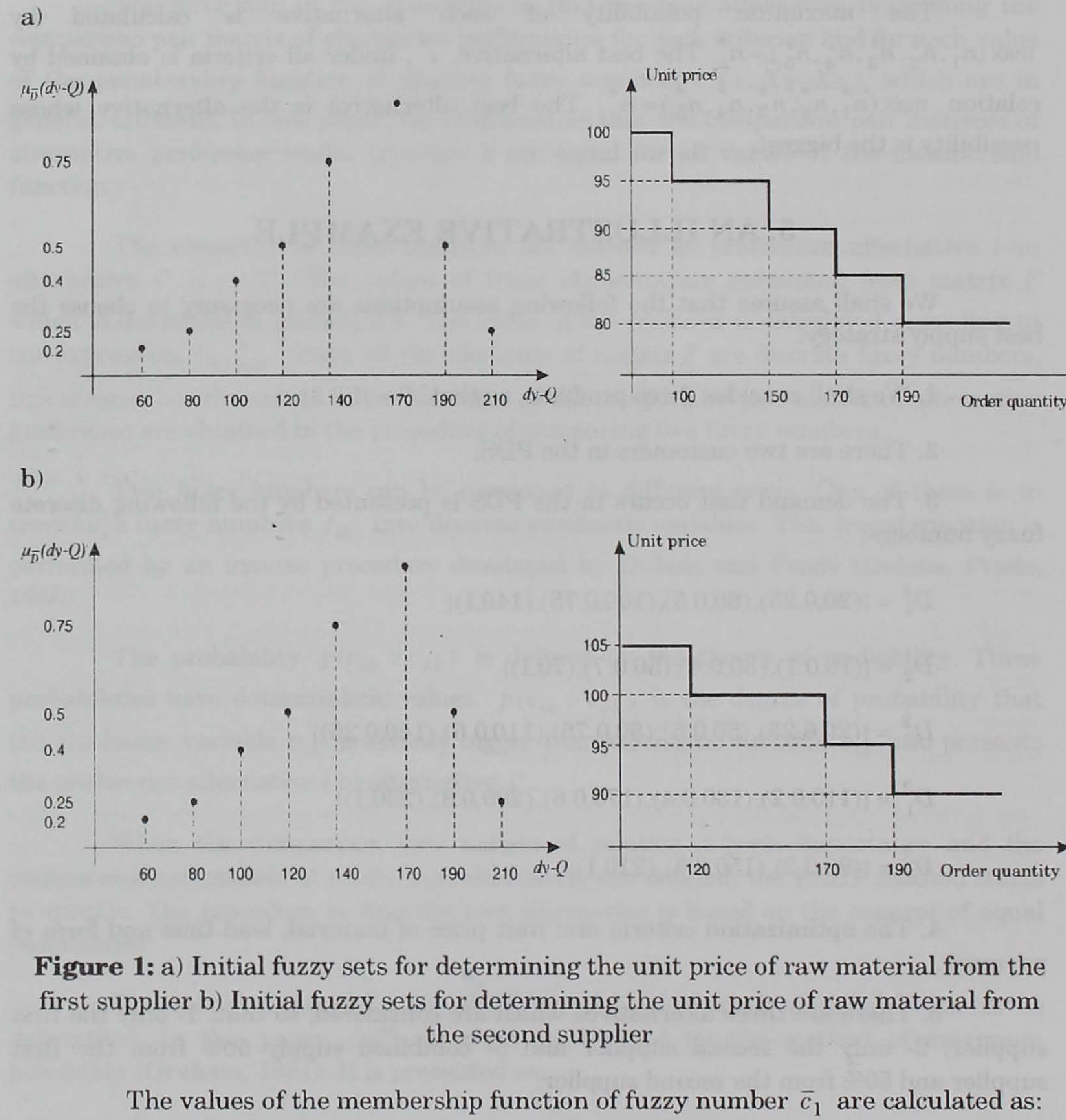


**Figure 1:** a) Initial fuzzy sets for determining the unit price of raw material from the first supplier b) Initial fuzzy sets for determining the unit price of raw material from the second supplier

The values of the membership function of fuzzy number $\bar{c}_1$ are calculated as:

$$\mu_{\bar{c}_1}(80) = \min(\mu_{\bar{Q}_m}(190), \mu_{\bar{Q}_m}(210)) = \min(0.5, 0.25) = 0.25$$

$$\mu_{\bar{c}_1}(85) = \min(\mu_{\bar{Q}_m}(170)) = \min(1) = 1$$

$$\mu_{\bar{c}_1}(90) = \min(\mu_{\bar{Q}_m}(120), \mu_{\bar{Q}_m}(140)) = \min(0.5, 0.75) = 0.5$$

$$\mu_{\bar{c}_1}(95) = \min(\mu_{\bar{Q}_m}(100), \mu_{\bar{Q}_m}(120)) = \min(0.4, 0.5) = 0.4$$

$$\mu_{\bar{c}_1}(100) = \min(\mu_{\bar{Q}_m}(60), \mu_{\bar{Q}_m}(80)) = \min(0.2, 0.25) = 0.2$$

The unit price of raw material offered by the first supplier, $\bar{c}_1$ is presented by $\bar{c}_1 = \{(80,0.25), (85,1), (90,0.5), (95,0.4), (100,0.2)\}$

The values of the membership function of fuzzy number $\overline{c}_2$ are calculated as:

$$\mu_{\overline{c}_2}(120) = \min(\mu_{\overline{Q}_m}(190), \mu_{\overline{Q}_m}(210)) = \min(0.5, 0.25) = 0.25$$

$$\mu_{\overline{c}_2}(130) = \min(\mu_{\overline{Q}_m}(170)) = \min(1) = 1$$

$$\mu_{\overline{c}_2}(140) = \min(\mu_{\overline{Q}_m}(120), \mu_{\overline{Q}_m}(140)) = \min(0.5, 0.75) = 0.5$$

$$\mu_{\overline{c}_2}(150) = \min(\mu_{\overline{Q}_m}(60), \mu_{\overline{Q}_m}(80), \mu_{\overline{Q}_m}(100)) = \min(0.2, 0.25, 0.4) = 0.2$$

The unit price of raw material which is offered by the first supplier, $\overline{c}_2$ is presented by $\overline{c}_2 = \{(120, 0.25), (130, 1), (140, 0.5), (150, 0.2)\}$

The procedures for determining the values of all elements of matrix $F$ are explained below.

**1. Unit price of raw material - $f_{i1}$**

$$f_{11} = \overline{c}_1 = \{(80, 0.25), (85, 1), (90, 0.5), (95, 0.4)(100, 0.2)\}$$

$$f_{21} = \overline{c}_2 = \{(90, 0.25), (95, 1), (100, 0.5), (105, 0.2)\}$$

$$f_{31} = 0.5(\overline{c}_1 + \overline{c}_2) = \{(85, 0.25), (90, 1), (95, 0.5), (100, 0.2)\}$$

**2. Lead time - $f_{i2}$**

We shall assume that the lead time for the first supplier is in the interval (16-26) days and the lead time for the second supplier is in the interval (12-20) days. In this paper, the values of the elements are calculated by expressions:

$$f_{12} = \overline{LT}_2 \cap \overline{LT}_3 = \{(16, 0.1), (18, 0.8), (20, 1), (22, 0.8), (24, 0.6), (26, 0.2)\}$$

$$f_{22} = \overline{LT}_1 \cap \overline{LT}_2 = \{(12, 0.2), (14, 0.1), (16, 0.1), (18, 0.8), (20, 1)\}$$

$$f_{32} = 0.5(f_{12} \cap f_{22}) = \{(16, 0.2), (18, 0.8), (20, 1), (22, 0.6)\}$$

**3. Form of payment - $f_{i3}$**

In this paper, we shall assume that the first supplier enables credit payment and the second supplier enables partial advanced payment and the rest on credit. According to the scale of measures this means that

$$f_{13} = 5$$

$$f_{23} = 3$$

$$f_{33} = 0.5(f_{13} + f_{23}) = 4$$

### 5.1. Choosing the best alternative by FAHP

In this paper, the relative importance of each pair of criteria is defined by: 1:2 = "important",1:3= "very important" and 2:3 = "less important". It should be mentioned that 1:2 means the relative importance of criterion 1 to criterion 2, up to 2:3 which means the relative importance of criterion 2 to criterion 3.

The comparison pair matrix of relative importance of the criteria is presented by:

$$\begin{bmatrix} - & \text{important} & \text{veryimportant} \\ & - & \text{lessimportant} \\ & & - \end{bmatrix}$$

The values of the comparison pair matrix of alternative preference under the first criterion-unit price of raw material are:

$$\begin{bmatrix} - & 0.4 & 0.7 \\ & - & 0.3 \\ & & - \end{bmatrix}$$

The values of the comparison pair matrix of alternative preference under the second criterion-lead time are

$$\begin{bmatrix} - & 0.5 & 0.5 \\ & - & 0.7 \\ & & - \end{bmatrix}$$

The values of the comparison pair matrix of alternative preference under the criterion-form of payment are:

$$\begin{bmatrix} - & 1.3 & 1.4 \\ & - & 0.8 \\ & & - \end{bmatrix}$$

Using the FAHP method the following results are obtained

| Values of membership functions | Values of alternatives | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 0 | 0.219 | 0.313 | 0.468 |
| | 0.216 | 0.293 | 0.492 |
| 0.25 | 0.219 | 0.304 | 0.478 |
| | 0.216 | 0.292 | 0.492 |
| 0.5 | 0.219 | 0.298 | 0.484 |
| | 0.216 | 0.292 | 0.492 |
| 0.75 | 0.219 | 0.293 | 0.487 |
| | 0.218 | 0.291 | 0.491 |
| 1 | 0.222 | 0.298 | 0.468 |

1: max (0.219,0.218,0.222)=0.222

2: max (0.313,0.304,0.298,0.293)=0.313

3: max (0.492,0.491,0.468)=0.492

In this paper, the best alternative is given by expression max (0.222,0.313,0.492)=0.492, so that the best alternative is alternative 3.

## 6. CONCLUSION

In this paper, a new mathematical model is presented for selection of the best supply strategy in a PDS. This model is a multicriteria optimization problem. It is shown that:

1. The problem of the integration of suppliers into the production distribution system is immanently a multicriteria optimization problem.

2. The uncertainties which appear in this problem are adequately described by discrete fuzzy numbers.

3. The best supply strategy with respect to the criteria considered is found using fuzzification of the AHP method.

4. The AHP method is adequate for solving the problem for two main reasons. The first is that it allows the hierarchical structuring of criteria, subcriteria and alternatives. The second is that entering input data by comparison pairs of criteria is a very appropriate way to define input data alternatives.

5. The best supply strategy offers the possibility of integrating suppliers into the production system in the process of production integration.

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