Yugoslav Journal of Operations Research
9 (1999), Number 1, 35-43

SIMULTANEOUS DETERMINATION OF ECONOMIC PRODUCTION QUANTITY AND ECONOMIC CONTROL CHART DESIGN UNDER TRUNCATED WEIBULL SHOCK MODELS*

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Abstract: In this paper, a Fortran Computer Program for jointly determining economic production quantity, inspection schedule and control chart design is presented. The program is written based on the work of Rahim (1994). A Weibull distributed shock model having increasing failure rate is considered and age-dependent salvage value of the equipment is utilized. The notion of a truncated production cycle is introduced which begins when a new component is installed and ends with a repair after the detection of a failure or after a specified number of inspection intervals, whichever occurs first. The product quality characteristic of the product is assumed to be normally distributed and monitored under the surveillance of a control chart. The optimal values of the economic production quantity and the economic design parameters are determined by minimizing the total expected cost (inventory and quality control cost) per unit time. Both uniform and non-uniform inspection schemes are presented. For a uniform scheme, the length of the inspection intervals is kept constant. However, for a non-uniform scheme, the length of the inspection intervals is regulated to maintain a constant integrated hazard rate over each inspection interval.

Keywords: Economic production quantity, inspection schedule, control chart design, Weibull shock models.

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* The author gratefully acknowledges the financial support of the Natural Sciences and Engineering Research Council of Canada.

1. INTRODUCTION

In many industrial situations, both product quality and production quantity are equally important. The problem of the determination of the economic production quantity (*EPQ*) for manufacturing processes has been well studied in the inventory literature (see, for example, Silver and Peterson [1985]). The effect of deteriorating items on *EPQ* has also received significant attention (for example, Ghare and Schrader [1963], Covert and Philip [1973], Shah [1977], Elsayed and Teresi [1983], and Hollier and Mak [1983], and others). However, very little attention has been paid to studying the effects of a deteriorating production process on *EPQ*. Rosenblatt and Lee (1986), have studied the effect of an imperfect production process on the optimal production cycle. The problem of the joint determination of *EPQ* and inspection schedules was studied by Lee and Rosenblatt (1987). They assumed that the process failure mechanism follows a Markovian shock model. On the other hand, the problem of maintaining statistical quality control is becoming increasingly popular. This can be seen by the considerable attention that the economic design of $\bar{x}$-control charts has received since the pioneering work of Duncan (1956). Details of work in this area may be found in Montgomery (1980, 1991), Vance (1985), McWilliams (1989), Svoboda (1991), Ho and Case (1994) and in some of the references therein. Following the models of Duncan (1956) and Lorenzen and Vance (1986), Rahim (1994) developed an integrated model for inventory control and quality control problems for a class of deteriorating processes where the process failure mechanism follows a truncated Weibull shock model. In this paper, a FORTRAN program for the simultaneous determination of economic production quantity, economic inspection schedule and economic $\bar{x}$-control chart design, is presented based on the economic model of Rahim (1994). The search algorithm used in this paper is similar to the one presented by Rahim (1993) with the exception of the following additional features. An age-dependent salvage value of the equipment is introduced and the possibility of early replacement of the equipment before its failure is considered. A truncated production cycle is defined which begins when a new component is installed and ends with a repair or after a specified number of inspection intervals, whichever occurs first. The search time for detecting a false alarm is assumed to be negligible. Intuitively, the idea that it could be more economically beneficial to scrap or replace a machine prior to its failure is very interesting. The program is quite simple to use. Truncated uniform, truncated non-uniform and non-truncated uniform schemes are presented.

2. DEFINITIONS, ASSUMPTIONS AND NOTATIONS

The output quality characteristic of the product variable is assumed to be normally distributed with mean μ_0 and variance σ_0^2. The production process begins in a state of statistical control and is subject to a single assignable cause which may shift the process mean μ_0 to $\mu_1 = \mu_0 + \delta\sigma_0$, where δ is the shift parameter.

However, the process variance σ_0^2 remains constant. The duration of the in-control period is assumed to follow a Weibull shock model (Banerjee and Rahim [1987, 1988]). The density function for the in-control period is given by

$$f(t) = \lambda \nu t^{\nu - 1} \exp(-\lambda t^{\nu}) \tag{1}$$

where $\lambda > 0$ is the scale parameter, and ν is the shape parameter. The cumulative distribution is given by

$$F(t) = 1 - \exp(-\lambda t^{\nu}) \tag{2}$$

and the hazard rate is defined by

$$r(t) = \lambda \nu t^{\nu - 1} \tag{3}$$

The output quality of the product is monitored by an $\bar{x}$-control chart. The production process is inspected at times $w_1 = h_1$, $w_2 = h_1 + h_2, \ldots$ to determine its state. If any inspection shows that the state of the process is out-of-control, production ceases until the accumulated on-hand inventory is depleted to zero. If the state of the process is found to be in-control, production continues until a predetermined level of inventory is accumulated.

The proposed model assumes that a production cycle ends with a true alarm or at w_m, whichever occurs first. In other words, if no true alarm is observed by the time w_{m-1} then the cycle is allowed to continue for an additional time h_m. At time w_m the old component is replaced by a new one. Thus, there is no cost of sampling and charting during the mth sampling interval. The Rahim (1993) model assumes that m is infinity, i.e. a production cycle ends only with a true alarm. In the proposed model, m could be a decision variable along with n, k and h_j, $j = 1, 2, \ldots, m$.

3. PROGRAM DESCRIPTION

The objective of the program is to derive the optimal decision variables. These are the simple size n, the control limit coefficient k, the number of inspection intervals m, and the length of the jth sampling interval h_j $(j = 1, 2, \ldots, m)$. The program achieves this by minimizing the expected cost per hour $ECT(m)$.

A computer program was developed based on the search algorithm of Hooke and Jeeves (1961) and similar to the one developed by Rahim (1989, 1993). For a non-uniform scheme, the lengths of the sampling intervals are chosen to maintain a constant integrated hazard rate over each sampling interval. For the Weibull shock model, the expression for h_j is given by:

$$h_j = [j^{1/\nu} - (j-1)^{1/\nu}]h_1 \tag{4}$$

Three different schemes are considered in this program: Scheme A is used for truncated non-uniform, Scheme B for truncated uniform, and Scheme C for non-truncated uniform.

The following notations will be used for developing the economic model of an $\bar{x}$-control chart:

m = the number of inspection intervals

n = the sample size

h_i = the length of the jth sampling (inspection) interval

k = the control limit coefficient for the $\bar{x}$-control chart

Z_1 = the expected time to repair the process if a failure is detected (or the expected preventive maintenance time at the end of the mth sample if no true failure is detected)

a = the fixed sampling cost

b = the cost per unit sampled

Y = the cost per false alarm

W = the cost to locate and repair the assignable cause

D_0 = the quality cost per hour while producing in-control

D_1 = the quality cost per hour while producing out-of-control

α = P_r (exceeding control limits | process in-control)

β = P_r (not exceeding control limits | process out-of-control)

$w_j = \sum_{i=1}^{j} h_i,\ j = 1,2,\ldots,m;\ w_0 = 0$

$F(w_j)$ = the cumulative distribution function

$\Delta F(w_j) = F(w_j) - F(w_{j-1});\ j = 1,2,\ldots,m$

$\bar{F}(w_j) = 1 - F(w_j)$

$S(x)$ = the salvage value for working equipment of age x

$E(T)$ = the expected total length of the production cycle including repair time

$E(C)$ = the total expected quality control cost per production cycle including the repair cost

The notation for the *EPQ* model will be used as follows:

D = the demand rate

P = the production rate

C_h = the holding cost per item per unit time

C_o = the set-up cost for each production cycle

T_p = the length of the production cycle including the mth inspection, during which inventory is built up

T_I = the total length of the inventory cycle

ETC = the expected total cost per unit time for the integrated production, inventory and quality control cycle

Expressions for the $E(T)$, $E(C)$ and ETC are shown in the Appendix. For details the readers are referred to Rahim (1994).

4. PROGRAM OPERATION

The program requires the user to specify the expected time to repair the process Z_1, the fixed sampling cost a, the variable cost per unit sampled b, the cost per false alarm Y, the quality cost per hour while producing in-control D_0, the quality cost per hour while producing out-of-control D_1, the salvage of the machine at age zero $S(0)$, the demand rate D, the production rate P, the inventory holding cost per unit time C_h, and the set-up for each production cycle C_0. Further, the program prompts the user for the type of search scheme desired (non-uniform or uniform) and the initial values of k and h_1.

The output consists of the optimal control limit coefficient k, the sampling interval h_1, the probability of a point falling outside the control limits when the process is in-control α, the probability of a point falling outside the control limits after a shift of δ (the power), and the value of the expected cost per hour. These quantities are computed for the range of sample sizes n. The program determines the overall optimal design parameters (n, k, h_1) corresponding to the minimum cost value for a given m. The program then searches for the optimal value of m, determined by the following inequalities $ECT(m-1) \geq ECT(m) \geq ECT(m+1)$. The optimal production run length w_m is then determined. The program is limited to a Weibull type shock model having a non-decreasing failure rate.

4.1. Examples

Example 1. Truncated Non-Uniform Sampling

Assume that the values of the time parameters, cost parameters, and shift parameter are as follows: $Z_1 = 1.00$ hours, $D_0 = \$50.00$, $D_1 = \$950.00$, $W = \$1100.00$ and $Y = \$500.00$, $a = \$4.00$, $b = \$1.20$, $\delta = 0.50$, $D = 1400$ units, $P = 1500$ units, $C_h = \$0.10$ and $C_0 = \$20$. Suppose that the process-failure mechanism is governed by a Weibull distributed with parameters $\lambda = 0.05$ and $\nu = 2.0$. A non-uniform sampling scheme is desired. The resulting optimal plan is $n = 50$, $h_1 = 1.91$ hours, $k = 2.50$, $\alpha = 0.0124$ and $1 - \beta = 0.8494$. The expected cost per hour is \$429.12, and the optimal number of sampling intervals m is 25. The optimal solution yields a production run length of $w_m = 4.34$ time units and provides an $EPQ = 6513$.

Example 2. Truncated Uniform Sampling

Using the same parameter values as in Example 1, the resulting scheme for a uniform sampling yields the following values. The resulting optimal plan is $n = 56$, $h = 0.90$ hours, and $k = 2.49$. The characteristics of this plan are $\alpha = 0.0127$, power $= 0.8944$, the expected cost per hours is \$455.86, the optimal number of sampling intervals m is 6 with $EPQ = 6016$ and the optimal production run length w_m is 4.81 hours. Thus, a non-uniform sampling scheme results in about 5.87% lower cost than a uniform sampling scheme.

5. DISCUSSION ON THE LIMITATIONS OF THE MODEL

The assumption that has been used in this study to stop the production run with the occurrence of an assignable cause and not to restart until accumulated inventory is used up may not be practical in many industrial situations. In a realistic model, the *EPQ* would involve a situation where production capacity exceeds demand. In the course of a year, many inventory cycles may occur where production is started and continued until a desired level of inventory is reached. Then production of the product of interest is ceased. Considering the impact of the multi product environment and how little time it takes to restart the run, it may not be logical to stop the process and restart again until all accumulated inventory is depleted. Furthermore, starting and stopping may affect the scheduling of other runs, which leads to costs not captured in the model.

In the cost calculations, it is clear that a cost penalty is incurred whenever the system shifts to an out-of-control state. However, it appears that, regardless of whether a production cycles ends with an out-of-control signal or normal termination after m cycles, the implicit-assumption is that all items produced are conforming and can be used to meet demand. A more realistic model would consider that, if a production cycle ends with an out-of-control signal, the items produced in the most recent sampling interval are of questionable quality. That might mean they need to be scrapped, 100% inspected, etc. Costs related to this problem may have been captured in the model. However, it left out the timing issues which will affect cycle lengths as well as costs. If all of the accumulated inventory is not usable, then the time to deplete the inventory to meet demand is affected. The assumption that all items made can be used to meet demand, regardless of how the production run is terminated, may not be a general one.

6. CONCLUSION

The paper provides a model incorporating the effect of economic production quantity on the $\bar{x}$-control chart design of an imperfect production process. The model allows joint optimization of sample size, the sampling intervals, control limit coefficients and the economic production quantity to minimize the total expected cost.

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APPENDIX

$$E(T) = Z_1 + \sum_{j=1}^{m} h_j \overline{F}(w_{j-1}) + \beta \sum_{j=1}^{m-1} \Delta F(w_j) \sum_{i=j+1}^{m} h_i \beta^{i-j-1}$$

That is, the expected cycle length $E(T)$ can be described as the repair time, the expected time for inspection intervals when the process is in-control, and the expected time for detecting the presence of an assignable cause.

And,

$$E(C) = (D_0 - D_1) \int_0^{w_m} xf(x)dx + (D_1 - D_0) \sum_{j=1}^{m} w_j \Delta F(w_j) +$$

$$+ D_0 \sum_{j=1}^{m} h_j \overline{F}(w_{j-1}) + \alpha Y \sum_{j=1}^{m-1} \overline{F}(w_j) + D_1 \beta [\sum_{j=1}^{m-1} \Delta F(w_j) \sum_{i=j+1}^{m} h_i \beta^{i-j-1}] + W +$$

$$+ (a + bn)[1 + \sum_{j=1}^{m-2} \overline{F}(w_j) + \beta \sum_{j=1}^{m-2} \Delta F(w_j) \{(1-\beta) \sum_{i=1}^{m-1-j} i\beta^{i-1} + (m-1-j)\beta^{m-1-j} \}] -$$

$$- \overline{F}(w_m) S(w_m);$$

That is, the expected cost per cycle $E(C)$ can be described as the expected cost of operating while the process was initially in-control and then went out-of-control, subsequently triggering a true alarm; the expected cost of operating while in-control with no alarm; the expected cost of false alarms; the expected cost of operating while out-of-control with no alarm; the repair cost and the expected cost of sampling; minus the salvage value for working equipment of age x.

$$ETC = \frac{C_0 + \frac{1}{2}\frac{P}{D}(P-D) \cdot E(T_p^2) \cdot C_h + E(C)}{E(T_I)} =$$

$$= \frac{\frac{D}{P} C_0 + \frac{1}{2}(P-D) \cdot C_h \cdot E(T_p^2) + E(C) \frac{D}{P}}{E[T_p]}$$

That is, the expected total cost for the integrated production, inventory and quality control cycle per unit time can be described as the set-up cost, the expected inventory cost, and the expected quality control cost.

Here

$T_p = T - Z_1$

$E(T_p) = E(T) - Z_1$

derivation of $E(T_p^2)$ can be found in Rahim (1994).