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DECISION THEORY BASED ON NON-ADDITIVE MEASURES

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Abstract: Non-additive measures and integrals based on them as Choquet, Sugeno and fuzzy t-conorm express aggregation operators in the multicriteria decision making problem. A characterization is given of the preferential independence of attributes. The properties of the aggregation operators represented by the Choquet integral are discussed.

Keywords: Decision making problem, Choquet integral, aggregation operator, essential attributes.

1. INTRODUCTION

Theories of fuzzy systems include many concepts that allow the modeling of uncertainty and ambiguity in economics, sociology, industry, the army, etc. If we agree that intelligence is the possibility to manage decisions under uncertainty, then the results obtained in the decision making theory modeled by fuzzy systems also give a good base for artificial intelligence.

Previously used additive probability measures could not model some situations as e.g. the Ellsberg Paradox, although the people's reaction is to prefer to act on the basis of known rather than unknown or vague probabilities ([7], [12]).

For the non-additive set function (measure) m defined on a σ-algebra Σ of subsets of set X (for finite X it is usually taken $\Sigma = \mathcal{P}(X)$ the family of all subsets), the difference $m(A \cup B) - m(B)$ depends on B and can be interpreted as the effect of A joining B [26]. A monotone set function m with $m(\emptyset) = 0$ is usually called a fuzzy measure.

More than contributing to the extension principle of fuzzy sets [32], fuzzy connectives ([6], [12], [17], [36]) and fuzzy measures are important in the problem of modeling the behavior of decision makers. The utility theory ([27], [12]) deals with preference relations describing decision making behavior, and as the basis of decision making theory, is well based axiomatically on fuzzy measures and the Choquet integral.

For a non-negative measurable function f the Choquet integral on $A \in \Sigma$ with respect to fuzzy measure m is defined by

$$C_m = (c)\int_A f dm = \int_0^\infty m(A \cap F_\alpha)\, d\alpha\ ,$$

where $F_\alpha = \{x \mid f(x) \geq \alpha\}$ is the α-cut of f.

The Sugeno integral of f on A with respect to m is defined by

$$(s)\int f dm = \sup_{\alpha \in [0,+\infty]} (\min(\alpha, m(A \cap F_\alpha)))\ .$$

2. DECISION MAKING PROBLEMS AND THE CHOQUET INTEGRAL

Definition 1. *A* **decision making problem** *[12] is a 5-uple* $(A, \Theta, \phi, X, \succeq)$ *where:*

- *A: set of* **alternatives or acts**, *among which the decision maker must choose;*

- *X: set of* **consequences, or results**. *These consequences come from the choice of an alternative;*

- Θ *: set of the* **states of the world**. *According to the state of the world* $\theta \in \Theta$ *(usually unknown), the consequences of the choice of an alternative* $a \in A$ *may differ;*

- $\phi : A \times \Theta \to X$ *specifies for each state of the world* θ *and each alternative a the resulting consequence* $x = \phi(a, \theta)$*;*

- $\succeq$ *: weak order relation on X, i.e. a binary relation verifying:*

(i) $x \succeq y$ *or* $y \succeq x$, $\forall x, y \in X$

(ii) $\succeq$ *is transitive, i.e.* $(x \succeq y, y \succeq z) \Rightarrow x \succeq z$ *;*

$\succeq$ is the preference relation which characterizes the decision maker, and may be strict $(x \succ y)$, and indifferent $(x \sim y)$.

The basic idea behind the utility theory is to transform the weak order $\succeq$ on X into a usual order $\geq$ on real numbers, by means of the so-called **utility function** $u : X \to \mathbf{R}$ whose fundamental property is:

$$x \succ y \Leftrightarrow u(x) > u(y)\ ;$$

when the property is verified, u is said to represent $\succeq$.

There are two subproblems:

- **multicriteria decision** - the state of the world is assumed to be known and X is multidimensional; a consequence is $x = (x_1, \ldots, x_n)$, where $x_i \in X_i$ is an element of the set representing a criterium;

- **decision under uncertainty** - onedimensional set of the consequences is considered when the true state of the world is unknown.

The uncertainty measure on Θ might be known, and as the set of consequences X is onedimensional, the elements of A are acts $f : \Theta \to X$ and the preference relation is defined on the set of acts.

Probability measures can be identified from real experiments by the frequency interpretation, but not when subjective probability measures are concerned. Subjective belief that an event will occur or that has occurred can be modeled as a subjective probability measure on the states of the world, so that the set of acts can be compared.

Different authors [1], [8], [12], [27], [28] have examined the conditions for the existence of the utility functions $u : X \to \mathbf{R}$ and the unique probability measure P^* on Θ representing the preference relation $\succ$ on A through the expected utility E of the action with respect to P^*

$$f \succ g \Leftrightarrow E[u(f(\theta_i)), P^*] > E[u(g(\theta_i)), P^*],$$

or, in the case of simple probability measures on $\mathbf{R}$ as the elements of X ($P = (p_1 \cdot y_1; \ldots; p_n \cdot y_n)$ where y_i may be negative), $u : \mathbf{R} \to \mathbf{R}$ such that

$$f \succ g \Leftrightarrow \int Eu(f(\theta_i))dP^* \geq \int Eu(g(\theta_i))dP^* .$$

The previous result puts specific requirements on the preference relation and if the requirements are not so strict (independence is replaced with co-monotonic independence - for this and other definitions [12], [26] may be consulted), decisions may be compared by the Choquet integrals with respect to the unique fuzzy measure m^* on Θ

$$f \succeq g \Leftrightarrow (c)\int Eu(f(\theta_i))dm^* \geq (c)\int Eu(g(\theta_i))dm^* .$$

(Each of the utility functions is unique up to the positive linear transformation).

The clear interpretation of the Choquet integral as the generalization of the expectation in decision making is an important result. It has also been proven that the Choquet integral comparison for decisions exists when independence is replaced with max-min independence existing for the relations of which either the best lottery (money lotteries are the elements of X) is assigned or the worst lottery is assigned for each pair of decisions considered for independence.

If there is no information about uncertainty measures on Θ, there are different classical criteria to make the decision that can all be represented by the Choquet integral with respect to the suitable fuzzy measure.

3. PREFERENTIAL INDEPENDENCE

In multidimensional non-probabilistic problems two alternatives differ according to their different consequences. Sets of the components X_i of X are called attributes or factors, or criteria in multicriteria decision making. To represent the preference relation, a utility function $u: X \to \mathbf{R}$ has the property

$$x \succ y \Leftrightarrow u(x) > u(y).$$

Definition 2. *Let* $J \subset I = \{1,2,\ldots,n\}$. *The space of attributes* $X_J = \times_{i \in J} X_i$ *is said to be* **preferentially independent** X_{J^c} *iff, for every* (x_J, y_J) *of elements of* X_J^2

$$(x_J, x_{J^c}) \succeq (y_J, x_{J^c}) \text{ for some } x_{J^c} \Rightarrow (x_J, x_{J^c}) \succeq (y_J, x_{J^c}) \text{ for all } x_{J^c} \in X_{J^c}.$$

The whole set of attributes is said to be mutually preferentially independent if X_J *is preferentially independent of* X_{J^c} *for every* $J \subset I$.

The existence of an additive utility function implies mutual preferential independence, but the converse is not true. Equivalent conditions are given by theorems too complicated to be used in practice. Murofushi and Sugeno have established important results on the relation between the additivity of the utility function in the form of the Choquet integral and mutual preferential independence.

Definition 3. *An attribute* i *is said to be* **essential** *iff there exist* $x_i, y_i \in X_i$ *and* $x_{i^c} \in X_{i^c}$, *such that*

$$(x_i, x_{i^c}) \succ (y_i, x_{i^c}).$$

An attribute which is not essential is said to be inessential.

If the utility function is the Choquet integral, the characteristic of the inessential attributes is $u_i(x_i) = const.$, $\forall x_i \in X_i$, or $\{i\}$ is the null set.

Theorem 1. *Let* $J \subset I$. *Then:*

J is preferentially independent of J^C iff either J is a positive semi-atom (see [12]), or $\{J, J^C\}$ is an inter-additive partition of I (see [12]).

(See [12] for the proof.) For the next characterization of preferential independence we need the following result.

Lemma 1. *Let i be an essential attribute. Then $\{i\}$ is preferentially independent of $\{i\}^C$ iff $\{i\}$ is positive.*

Theorem 2. *(i) If a set of attributes has exactly two essential attributes i and j, then the following conditions are equivalent to each other:*

1. *the attributes are mutually preferentially independent;*
2. *$\{i\}$ and $\{j\}$ are both positive;*
3. *$m(\{i,j\}) > \max(m(\{i\}), m(\{j\}))$.*

(ii) If the set I of attributes has at least three essential attributes, the following two conditions are equivalent:

1. *the attributes are mutually preferentially independent;*
2. *m is additive.*

Proof (i):

$1 \Rightarrow 2$ follows as $\{i\}$ and $\{i\}^C$ are preferentially independent, as well as $\{j\}$ and $\{j\}^C$ (Lemma 1).

$2 \Rightarrow 3$ follows from the definition of the positive set.

$3 \Rightarrow 2$ if $\{i\}$ or $\{j\}$ is not positive, then $m(\{i,j\}) = m(\{j\})$ or $m(\{i,j\}) = m(\{i\})$ considering the measure space over $\{i,j\}$.

$2 \Rightarrow 1$ from Lemma 1 it follows that $\{i\}$ is preferentially independent of $\{i\}^C$ and that $\{j\}$ is preferentially independent of $\{j\}^C$;

$$J \supset \{i,j\},\ K \subset J^C,\ m(K) < m(K \cup \{i\}) < m(K \cup \{i,j\}) \leq m(K \cup J)$$

as $\{i\}$ and $\{j\}$ are positive; it follows that ($\{i\}$ is its subset) J is not a semi-atom and (Theorem 1) the following must hold

$$m(J \cup K) = m(J) + m(K),$$

which is true as the Choquet integral is additive for the comonotonic functions [26, Corollary 7.8] (constant function $\chi_K u(I)$ and $\chi_J u(I)$ are comonotonic, $K \subset J^C$ is a subset of the set of inessential attributes (that are not null sets), or the Choquet integral form of the utility function has the same value for the functions that differ from the null set);

if $J \ni i,\ J^C \ni j$,

$$C_m(u(x_J), u(x_{J^C})) < C_m(u(y_J), u(x_{J^C})) \text{ for } x_i \prec y_i$$

($\{i\}$ is preferentially independent of $\{i\}^C$) for $\forall x_{J^C} \in X_{J^C}$ - interadditive partition $(J \setminus \{i\}, \{i, j\}, J^C \setminus \{j\})$ can be considered or the integrated functions are comonotonic as the elements of $I \setminus \{i, j\}$ are inessential and $\{i\}$ is a positive semi-atom; thus any J is preferentially independent of J^C.

4. AGGREGATION OPERATOR

Using an **aggregation operator** $\mathcal{H}$, a suitable utility function [9], [12] is constructed to represent the preference relation, starting from the onedimensional utility functions:

$$u(x) = \mathcal{H}(u_1(x_1), \dots, u_n(x_n)),$$

$$x \succ y \Leftrightarrow \mathcal{H}(u_1(x_1), \dots, u_n(x_n)) > \mathcal{H}(u_1(y_1), \dots, u_n(y_n)).$$

Values in multicriteria decision making are not interesting in themselves, but for the ordering implied by these values.

The **equivalence** of the aggregation operators is defined to lead to the same ranking of consequences. Weak equivalence is defined by the implication

$$\mathcal{H}_1(x) > \mathcal{H}_1(x') \Rightarrow \mathcal{H}_2(x) \geq \mathcal{H}_2(x'),$$

where the arguments are from the product space $X_1 \times X_2 \times \ldots \times X_n$ and aggregated values are in $\mathbf{R}$, and coincides with the concept of co-monotonic functions ensuring that no contradictory decisions will be made.

Level surface or **indifference surface** [12] of the operator $\mathcal{H}: [0,1]^n \to [0,1]$

$$\mathcal{H}^{-1}(z) \stackrel{\Delta}{=} \{a \in [0,1]^n \mid \mathcal{H}(a) = z\}$$

is the locus of the alternatives left undecided by the operator. The elementary algebra result is the equivalence of the two aggregation operators having the same indifference surfaces and the existence of the unique bijection $u(\mathcal{H}_1(x)) = \mathcal{H}_2(x), \forall x \in X$. So, the strict equivalence of the two operators is equivalent to the bijection u being increasing. Weak equivalence is equivalent to u being "non - decreasing": $\forall y_1, y_2 \in R_1$, $y_1 > y_2 \Rightarrow \forall z_1 \in u(y_1)$, $\forall z_2 \in u(y_2)$, $z_1 \geq z_2$ when $\mathcal{H}_1$ and $\mathcal{H}_2$ are mappings from X to $R_1, R_2 \subseteq \mathbf{R}$ respectively, and $u: R_1 \to R_2$ is multi-valued such that

$\mathcal{H}_2(x) \in u(\mathcal{H}_1(x))$, $\forall x \in X$. The level surfaces of the two operators cannot cross each other.

If the aggregation operators are strictly monotone, weak equivalence implies strict equivalence.

There are $n!$ canonical regions of $[0,1]^n$ depending on $\sigma \in \sigma$ of the permutations of the index set $I = \{1,\dots,n\}$, in which the Choquet integral is linear: they are $(n-1)$ -dimensional hyperplanes R_σ. The indifference surfaces, [12]:

$$C_m^{-1}(z) = \{a \in [0,1]^n \mid a_{(1)} + (a_{(2)} - a_{(1)})m(A_{(2)}) + \cdots + (a_{(n)} - a_{(n-1)})m(A_{(n)}) = z\}$$

or

$$C_m^{-1}(z) = \{a \in [0,1]^n \mid a_{(1)}(1 - m(A_{(2)}) + a_{(2)}(m(A_{(2)}) - m(A_{(3)}) + \cdots + a_{(n)}m(A_{(n)}) = z\}$$

of the Choquet integral are the same as for the nondecreasing continuous operator with level surfaces $\sum_{i=1}^n k_i^\sigma a_{\sigma(i)} = z$, with $\sum_{i=1}^n k_i^\sigma = 1$ in the canonical region R_σ when the unique fuzzy measure

$$m_{\mathcal{H}}(\{x_{\sigma(n-l+1)},\dots,x_{\sigma(n)}\}) = \sum_{i=n-l+1}^{n} k_i^{\sigma'}$$

is given by the permutation σ' which coincides with σ in at least l elements.

Because of continuity and monotonicity, the indifference surfaces of the Choquet integral are connected when passing from one R_σ to another. The equivalence class in the strict sense is given by

$$\widetilde{C}_m = \{\mathcal{H} \mid \mathcal{H}(a_1,\dots,a_n) = u[\sum_{i=1}^{n} k_i^\sigma a_{\sigma(i)}]\}$$

if

u is a strictly increasing real function defined on $[0,1]$,

$\sum_{i=1}^n k_i^\sigma = 1, \forall \sigma \in \sigma$,

σ is a permutation such that $a_{\sigma(1)} \le \ldots \le a_{\sigma(n)}$,

$k_i^\sigma = m(A_{\sigma(i)}) - m(A_{\sigma(i+1)})$, where $A_{\sigma(i)} = \{x_{\sigma(i)},\dots,x_{\sigma(n)}\}$.

Appropriate analysis of the indifference surfaces of the Sugeno integral is based on the expression of the integral as the median: in the region $a_1 \le \ldots \le a_n$

$$S_m(a_1,\ldots,a_n) = med(a_1,\ldots,a_n, m_{2,\ldots,n},\ldots,m_n)$$

where $m_{i,i+1,\ldots,n} = m(\{x_i, x_{i+1},\ldots,x_n\})$ and

$$med(a_1,\ldots,a_{2q+1}) = a_{(q+1)}, \quad a_{(1)} \leq \ldots \leq a_{(2q+1)}.$$

The restricted fuzzy t-conorm integral [20], [12] based on a system of the form $(\Delta, S, \Delta, \Diamond)$ is interesting for a multicriteria analysis as it is regarded as the mean value of the integrands. If the generating functions of Δ and S are h and g, $g(1) = 1$, the integral

$$(\mathcal{F})\int f dm = h^{-1}[(c)\int h \circ f \, d(g \circ m)]$$

can be analyzed analogously to what is known about equivalence classes of the Choquet integral.

5. MULTIATTRIBUTE UTILITY THEORY

Multicriteria evaluation is present in a very wide field of applications, such as resource allocation, designing new goods, environmental planning, quality control, evaluation of creditworthiness, etc, and some concrete examples in [12], [27], [33] are given to illustrate this variety.

There are two approaches to the multiattribute utility theory:

– **the ordinal approach** compare alternatives two by two, but the formed Hasse diagram might be with objects not all comparable to each other as the transitivity property of the preference relation is lost in the aggregation step; the approach might derive some fundamental results and references from [12] can be followed towards them;

– **the cardinal approach**, given in the sequel, gives an absolute evaluation to every alternative for a given criterion and real values are aggregated to get a global evaluation; all objects are comparable, whatever their characteristics.

The problem considers a list of **similar objects** $S = \{s_1,\ldots,s_n\}$ described by the set **attributes** $Z = \{z_1,\ldots,z_p\}$. **Information**, precise or not, about the values of the attributes for each object exists. Each object is then evaluated according to a set of **criteria** $X = \{x_1,\ldots,x_p\}$, which are defined by the decision maker himself. The general form of expressing a criterion is assumed to be

$$x_j : z_j \text{ is } A_j$$

where A_j is a fuzzy set on the universe of z_j. To solve the problem, **marginal** (partial) and **global** evaluations are to be performed.

Each object s_i is evaluated with respect to one particular criterion x_j defining a mapping $h_j : S \rightarrow [0,1]$. It tells the degree of an object from S to satisfy the criterion x_j. $h_j(s_j)$ is the degree of compatibility between the fuzzy set A_j and the value of x_j for object s_i.

The mapping may be defined by $h_j(s_i) = A_j(z_j^i)$ when the precise value z_j^i of attribute z_j of s_i is known. When z_j^i as a fuzzy number better represents the value of the attribute, the definition of Zadeh based on the extension principle

$$\tilde{h}_j(s_i)(y) = \sup_{z_j | y = A(z_j)} z_j^i(z_j)$$

results with the fuzzy set considering the compatibility between the two fuzzy sets.

For an object with all marginal evaluations $h_1(s_i), \ldots, h_p(s_i)$, the problem is to determine a single evaluation with respect to all the criteria. The global value is subjective as decision makers differ in their decisions when the criteria are the same:

– **criteria weights** express ideas about what is important and what could be neglected;

– **behavior** can be tolerant or disjunctive-oriented when "it is sufficient if some criteria are met", or intolerant or conjunctive-oriented if it is declared that "all criteria must be equally well met".

Quasi-Sugeno, t-norms or fuzzy t-conorm integrals can be the operators used over the marginal evaluation values for they are normalized and the utility function should be determined up to the positive linear transformation.

6. SOME PROPERTIES OF AGGREGATION OPERATORS

A suitable p-place operator $\mathcal{H}$ should be consistent with the preference relation of the decision maker to give a global evaluation and should possess the following properties:

1. $\mathcal{H}(0,\ldots,0) = 0$, $\mathcal{H}(1,\ldots,1) = 1$;

– idempotence (I) $\mathcal{H}(a,\ldots,a) = a, \forall a$;

– continuity;

– monotonicity (M) (usually non decreasing) with respect to each argument;

2. properties requested in evaluation and measurement problems:

– decomposability (D)

$$\mathcal{H}^{(p)}(a_1,\ldots,a_k,a_{k+1},\ldots,a_p)=\mathcal{H}^{(p)}(a,\ldots,a,a_{k+1},\ldots,a_p)$$

where $a=\mathcal{H}^{(k)}(a_1,\ldots,a_k)$ for all $(a_1,\ldots,a_p)$;

- order linkage property (OL)

$$\mathcal{H}^{(p+1)}(\mathcal{H}^{(p)}(a_{(1)},\ldots,a_{(p)}),\mathcal{H}^{(p)}(a_{(2)},\ldots,a_{(p+1)}),\ldots$$
$$\mathcal{H}^{(p)}(a_{(p+1)},\ldots,a_{(2p)}))=\mathcal{H}^{(p)}(\mathcal{H}^{(p+1)}(a_{(1)},\ldots,a_{(p+1)}),$$
$$\mathcal{H}^{(p+1)}(a_{(2)},\ldots,a_{(p+2)}),\ldots,\mathcal{H}^{(p+1)}(a_{(p)},\ldots,a_{(2p)})),$$

where (i) denotes the particular permutation such that $a_{(1)}\le\ldots\le a_{(2m)}$;

- ordered linkage property with permutation (OLP) (Grabisch [10])

$$\mathcal{H}^{(p+1)}([\mathcal{H}^{(p)}(a_{(1)},\ldots,a_{(p)}),\mathcal{H}^{(p)}(a_{(2)},\ldots,a_{(p+1)}),\ldots$$
$$\mathcal{H}^{(p)}(a_{(p+1)},\ldots,a_{(2p)})]_\sigma)=\mathcal{H}^{(p)}(\mathcal{H}^{(p+1)}([a_{(1)},\ldots,a_{(p+1)}]_\sigma),$$
$$\mathcal{H}^{(p+1)}([a_{(2)},\ldots,a_{(p+2)}]_\sigma),\ldots,\mathcal{H}^{(p+1)}([a_{(p)},\ldots,a_{(2p)}]_\sigma)),\quad \forall\sigma\in\sigma$$

where $[a_1,\ldots,a_{p+1}]_\sigma$ means $a_{\sigma(1)},\ldots,a_{\sigma(p+1)})$ (a permutation of the indices) and σ is the set of all permutations on a given set;

- stability under the same positive linear transformation (SPL)

$$\mathcal{H}(ra_1+t,\ldots,ra_m+t)=r\mathcal{H}(a_1,\ldots,a_m)+t,\quad \forall r>0,\ \forall t\in\mathbf{R};$$

(changing the scale does not change the result);

- stability under positive linear transformation with the same unit, co-monotonic zeros (SPLUC)

$$\mathcal{H}(ra_{\sigma(1)}+t_{\sigma(1)},\ldots,ra_{\sigma(p)}+t_{\sigma(p)})=r\mathcal{H}(a_{\sigma(1)},\ldots,a_{\sigma(p)})+T(t_{\sigma(1)},\ldots,t_{\sigma(p)}),$$
$$\forall a_1\le\ldots\le a_p,\ \forall r>0,\ \forall t_1\le\ldots\le t_p\in\mathbf{R},\ \forall\sigma\in\sigma;$$

3. possibility of expressing weights of importance on criteria if this is necessary;

4. possibility of expressing the behavior of the decision maker (extreme examples are max and min);

5. possibility of expressing a compensatory effect, or an interaction between criteria (**redundancy** when the criteria express more or less the same thing; and **support or reinforcement** when the criteria with little importance taken separately become very important when considered jointly);

6. possibility of an easy semantical interpretation: we should be able to relate parameters defining $\mathcal{H}$ to the behavior implied by $\mathcal{H}$.

The operators used were different models to express both conjunctive and disjunctive behavior (Zimmermann and Zysno [36], compensatory operators [17]), or were the generalization of the averaging operators as the weighted operators, or were symmetric sums defined to be auto-dual. They were either easily interpretable, but restricted and particular, or with a wider range, but not interpretable; they may exhibit bias in experiments, which is explainable by the interaction or dependency between criteria; even if they express the compensatory effect of human decision makers (Zimmermann, Hayashi), they may lack the theoretical foundation.

Knowing previously used aggregation operators, their properties and drawbacks, discrete fuzzy integrals are a good candidate for aggregation.

The fuzzy t-conorm integral is a generalization of the Sugeno and Choquet integral defined by the **t-conorm system for integration** $(\Delta, S, \Delta, \Diamond)$:

$$(\mathcal{F})\int f \Diamond dm = \underline{S}_{i=1}^{m}((a_{(i)} - \Delta a_{(i-1)}) \Diamond m(A_{(i)})),$$

where $a_{(1)} \leq \ldots \leq a_{(n)}$, $A_{(i)} = \{a_{(i)}, \ldots, a_{(n)}\}$, and the spaces of values of the integrand $([0,1], \Delta)$, measure $([0,1], S)$ and integral $([0,1], \underline{S})$ should be chosen to adequately represent the needed model: if, as in the decision making theory, an integral is regarded as the mean value of the integrands, the t-conorm system has the form $(\Delta, S, \Delta, \Diamond)$.

The fuzzy measure represents weights on criteria, either on individual criteria or any group of criteria (represented by a subset of $X = \{x_1, \ldots, x_p\}$), that enables the fuzzy integrals to express interaction between criteria.

The properties for the aggregation are the characteristics of the integrals, or require their particular form or the restricted fuzzy measure values (commutativity or associativity for example). The characterization of discrete fuzzy integrals and their relations with existing aggregation operators is given in [12]. As a mapping, the fuzzy integral is defined by a set of 2^n parameters and a t-conorm system. We lack any complete method for constructing a fuzzy measure from semantical considerations only, but [12] summarizes the results about the interpretation of fuzzy measures as an important step towards the solution.

The contribution of every element of an index set $I = \{1, \ldots, n\}$ is characterized and the known result (Theorem 2.(ii)) stated as the introduction to the problem of connecting some kind of interaction among attributes with some kind of non-additivity of the fuzzy measure.

As intuitive facts, it is accepted that super-additivity between attributes i and j entails a strengthening interaction, **synergy**, or support between i and j and that sub-additivity between i and j entails a weakening interaction, **redundancy**, or destructive effect between i and j. The recent definition of an interaction index (Murofushi, Soneda [12]) can help the formalization of these intuitive facts.

The algorithm identifying the fuzzy measure, according to Yoneda et al ([12]) combines the semantical analysis and the optimization approach. The criterium to be minimized is the quadratic form of the difference between the resulting measure and the additive equidistributed measure $m_j = \frac{1}{n}$. The constraints are monotonicity constraints, constraints from the training data, and constraints coming from semantical considerations: the importance of the criteria expresses the decision maker's preference or his knowledge and the dependency and support between criteria (interaction) are modeled with suitable parameter values equivalent to the linguistic representation.

7. CONCLUSIONS

The paper summarizes some results of the decision making theory with the aim of introducing fuzzy integrals (Choquet, Sugeno, t-conorm) as the base for the analyzed phenomenon or process. To identify the model, such integrals require the parameter identification of the suitable fuzzy measure. The consistent theory of the semantical interpretation of the fuzzy measure is still open and so are practical procedures for the identification algorithms.

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