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STOCHASTIC BEHAVIOUR OF A ROBOT-SAFETY DEVICE SYSTEM

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Abstract: Innovations in the field of microelectronics and micromechanics have enhanced the involvement of "smart" robots in various technical applications. Unfortunately, no robot is completely reliable. Therefore, up-to-date robots are often connected with a (repairable) safety device. Such a device prevents possible damage, caused by a robot failure, in the robot's neighbouring environment. However, the random behaviour of the entire system (robot, safety device, repair facility) could jeopardize some prescribed safety requirements. Therefore, an appropriate statistical analysis is quite indispensable to support the system designer in problems of risk acceptance and safety assessments. We introduce a robot-safety device system attended by two statistically different repairmen. Our system is characterized by the natural assumption of cold standby and by an admissible "risky" state. In order to describe the random behaviour of the system, we introduce a stochastic process endowed with probability kernels satisfying Kolmogorov-type equations. The solution procedure is based on advanced methods of renewal theory. Next, we derive the invariant measure of the T-system and the long-run availability of the robot. Finally, we consider the particular but important case of fast repair.

Keywords: Robot, safety device, invariant measure, availability, risk-criterion.

1. INTRODUCTION

Innovations in the field of microelectronics and micromechanics have enhanced the involvement of robots in all kind of manufacturing systems [2].

Nowadays, "smart" robots are used in various technical applications, such as monitoring a complex standby system operating in remote areas.

Unfortunately, no robot is completely reliable. In general, the failure-free time of a robot is a random variable characterized by a survival function [9]. Therefore, up-to-date robots are often connected with a (repairable) safety device [4], [5], [6]. Such a device prevents possible damage, caused by a robot failure, in the robot's neighbouring environment. However, the stochastic behaviour of the entire system (robot, safety unit, repair facility) could jeopardize some prescribed safety requirements. For instance, if we allow the robot to operate during the repair time of the failed safety device. Such a "risky" state is called admissible if the associated event: "The robot is operative but the safety device is in repair", constitutes a rare event. Therefore, an appropriate statistical analysis is quite indispensable to support the system designer in problems of risk acceptance and safety assessments.

In order to avoid undesirable delays in repairing failed units, we introduce a robot-safety device system attended by two statistically different repairmen. Each repairman has his own particular task. Repairman S is skilled in repairing the safety unit, whereas repairman R is an expert in repairing robots. The system satisfies the usual conditions (independent identically distributed random variables and perfect repair [8]). Both repairmen are jointly busy, if and only if, both units (robot + safety device) are down. In the other case, at least one repairman is idle. In any case, the safety device always waits, in cold standby [1], until the repair of the robot has been completed.

In order to describe the random behaviour of the entire system (henceforth called a T-system) we introduce a stochastic process endowed with probability kernels satisfying Kolmogorov-type equations. We transform the basic equation into an integro-differential equation of the (Stieltjes) convolution type. The solution procedure is based on advanced methods of renewal theory.

Next, we derive the invariant measure of the T-system and the long-run availability of the robot-safety device.

Finally, we consider the particular but important case of fast repair.

2. FORMULATION

Consider a T-system satisfying the usual conditions.

The robot has a constant failure rate $\lambda > 0$ but a general repair time distribution $R(\cdot)$. $R(0) = 0$ with finite mean ρ. Let $\bar{R}(\cdot) := 1 - R(\cdot)$. The operative safety system has a constant failure rate $\lambda_S > 0$ but a zero failure rate in standby (the so-called cold standby state) and a general repair time distribution $R_S(\cdot)$. $R_S(0) = 0$, with finite mean ρ_S.

Without loss of generality (see forthcoming remark), we may assume that both distributions have density functions (in the Radon-Nikodym sense) defined on $[0, \infty)$.

In order to describe the random behaviour of the T-system, we introduce a stochastic process $\{N_t, t \geq 0\}$, with arbitrary discrete state space $\{A, B, C, D\} \subset [0, \infty)$ characterized by the following events:

$\{N_t = A\}$: "Both units are operating in parallel at time t".

Note that A is a renewal state. Figure 1 shows a functional block-diagram of the T-system operating in state A.

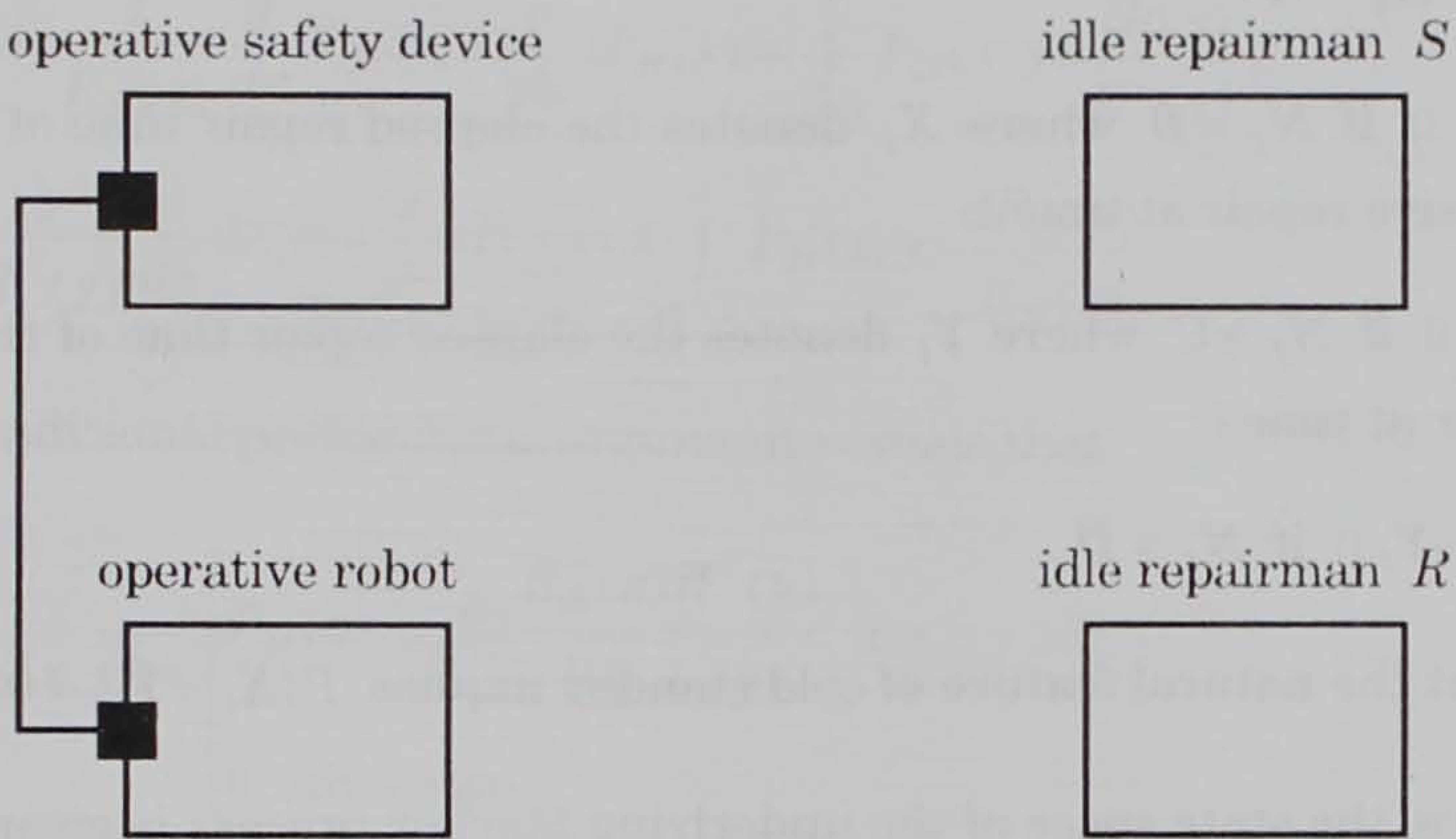

Figure 1: Functional block-diagram of the T-system operating in state A

$\{N_t = B\}$: "The robot is operative but the safety device is in repair at time

Figure 2 shows a functional block-diagram of the T-system operating in so-called "risky" state B.

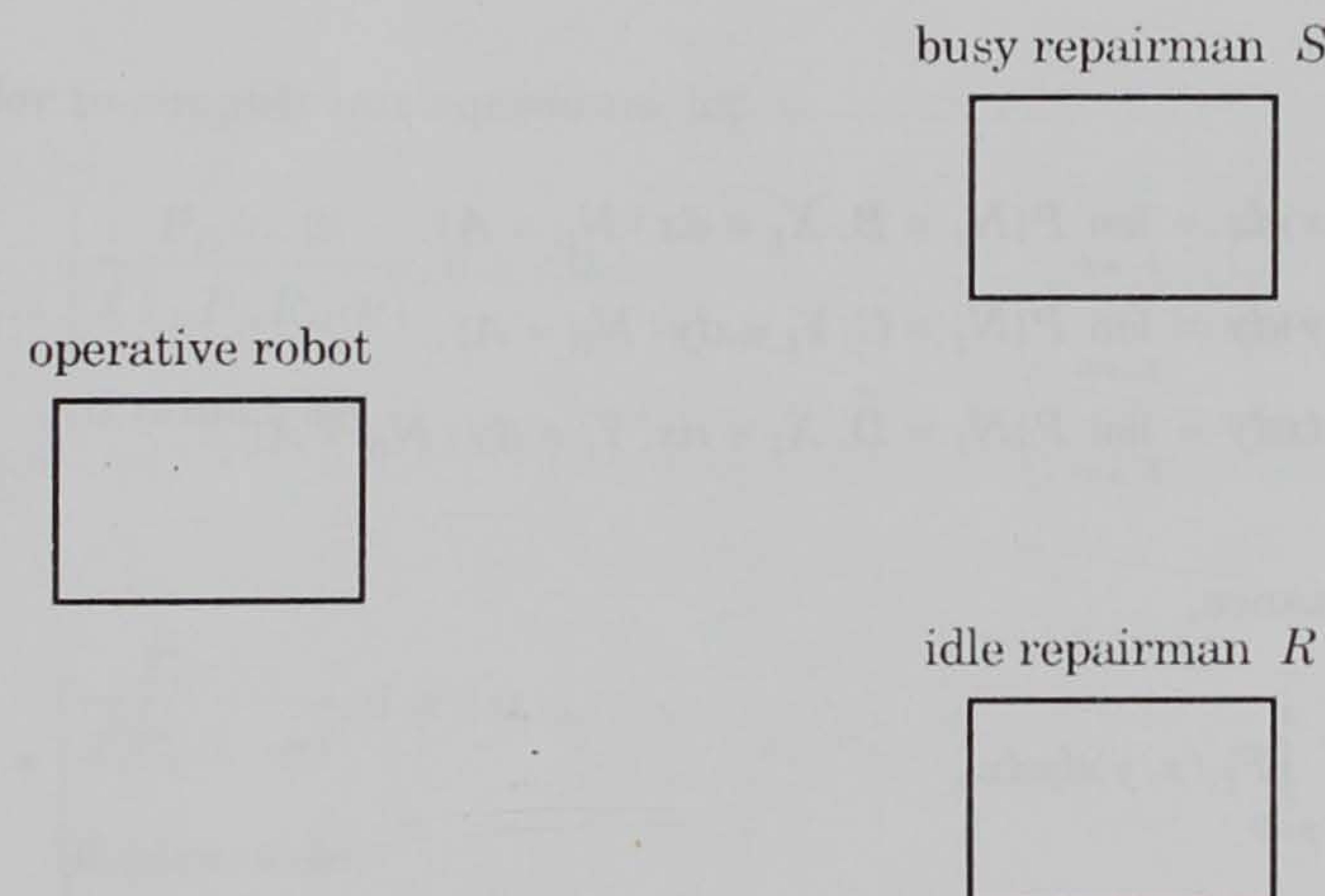

Figure 2: Functional block-diagram of the T-system operating in state B

$\{N_t = C\}$: "The safety device is in cold standby and the robot is in repair at time t".

$\{N_t = D\}$: "Both repairmen are jointly busy at time t".

A Markov characterization of the process $\{N_t, t \geq 0\}$ is piecewise and conditionally defined by:

$\{N_t\}$ if $N_t = A$.

$\{(N_t, X_t)\}$ if $N_t = B$ where X_t denotes the elapsed repair time of the safety device in progressive repair at time t.

$\{(N_t, Y_t)\}$ if $N_t = C$ where Y_t denotes the elapsed repair time of the robot in progressive repair at time t.

$\{(N_t, X_t, Y_t)\}$ if $N_t = D$.

Note that the natural feature of cold standby implies $P\{X_t < Y_t\} = 0$.

Therefore, the state space of the underlying Markov process is given by

$\{A\} \cup \{(B, x): x \geq 0\} \cup \{(C, y): y \geq 0\} \cup \{(D, x, y): x \geq y \geq 0\}$.

Finally, we consider the system in stationary state (the so-called ergodic state) with invariant measure $\{P_A, P_B, P_C, P_D\}$, $P_A + P_B + P_C + P_D = 1$, where P_K; $K = A, B, C, D$ is defined by

$$P_K := \lim_{t \to \infty} P\{N_t = K \mid N_0 = A\}.$$

Furthermore, let

$$\begin{aligned} P_B(x)dx &:= \lim_{t \to \infty} P\{N_t = B, X_t \in dx \mid N_0 = A\}, \\ P_C(y)dy &:= \lim_{t \to \infty} P\{N_t = C, Y_t \in dy \mid N_0 = A\}, \\ P_D(x, y)dxdy &:= \lim_{t \to \infty} P\{N_t = D, X_t \in dx, Y_t \in dy \mid N_0 = A\}. \end{aligned}$$

Note that, for instance,

$$P_D = \int_{x=0}^{\infty} \int_{y=0}^{x} P_D(x, y)dydx.$$

3. INTEGRO-DIFFERENTIAL EQUATION

In order to construct a set of differential equations, we apply our usual technical manipulations related to a version of the supplementary variable technique [10].

For $x > 0$, $y > 0$, we obtain the Kolmogorov-type equations

$$(\lambda + \frac{1}{R_S^{-}(x)}\frac{d}{dx}R_S(x) + \frac{d}{dx})P_B(x) = \int_{y=0}^{x} P_D(x,y)\frac{dR(y)}{R^{-}(y)},$$

$$(\frac{1}{R^{-}(y)}\frac{d}{dy}R(y) + \frac{d}{dy})P_C(y) = \int_{x=y}^{\infty} P_D(x,y)\frac{dR_S(x)}{R_S^{-}(x)}.$$

A simple conditional probabilistic argument reveals that

$$P_D(x,y) = \begin{cases} P_D(x-y,0)\dfrac{R_S(x)R^{-}(y)}{R_S(x-y)}, \text{ if } x \ge y \ge 0, \\ 0, \text{ otherwise.} \end{cases}$$

The boundary conditions are,

$$P_B(0) = \lambda_S P_A,$$
$$P_C(0) = \lambda P_A,$$

$$P_D(x,0) = \begin{cases} \lambda P_B(x), \text{ if } x \ge 0, \\ 0, \text{ otherwise.} \end{cases}$$

In order to simplify our equations, let

$$\phi_D(u) := \begin{cases} \dfrac{P_D(u,0)}{\lambda\,\lambda_S P_A R_S(u)}, \text{ if } u \ge 0, \\ 0, \text{ otherwise,} \end{cases}$$

and

$$\phi_C(u) := \begin{cases} \dfrac{P_C(u)}{\lambda\, P_A R^{-}(u)}, \text{ if } u \ge 0, \\ 0, \text{ otherwise,} \end{cases}.$$

Invoking the boundary conditions and some technical manipulations reveals that

$$P_B(x) = \lambda_S P_A \phi_D(x) R_S^-(x),$$
$$P_C(y) = \lambda P_A \phi_C(y) R^-(y),$$
$$P_D(x, y) = \lambda\lambda_S P_A \phi_D(x-y) R_S^-(x) R^-(y).$$

Finally, inserting the above transforms into the corresponding Kolmogorov equations yields the following preliminary result:

For $y > 0$,

$$\frac{d}{dy}\phi_C(y) = \lambda_S \int_{x=y}^{\infty} \phi_D(x-y) dR_S(x),$$

$$\phi_C(0) = 1.$$

The function $\phi_D(x)$ satisfies the integro-differential equation

$$(\lambda + \frac{d}{dx})\phi_D(x) = \lambda \int_{y=0}^{x} \phi_D(x-y) dR(y),$$

with boundary condition $\phi_D(0) = 1$.

4. SOLUTION PROCEDURE

Without ambiguity, we employ the notation

$$(F * u)(t) = \int_0^t u(t-\tau) dF(\tau),$$

for the convolution of a function $u(t)$, bounded on compact intervals, with an arbitrary probability distribution $F(t), t \geq 0$. Then the n-fold convolution of F is denoted by F^{n*}, where F^{0*} denotes the Heaviside unit-step function with unit-jump at $t = 0$.

The following theorem, stated for direct reference, is a basic tool of advanced renewal theory [3].

Theorem.

Suppose that $a(t)$ is a bounded function.
Then the integral equation

$$A(t) = a(t) + (F * A)(t),$$

has a unique solution, bounded on compact intervals, given by

$$A(t) = (\sum_{n=0}^{\infty} F^{n*} * a)(t).$$

Moreover, if $a(t)$ *is directly Riemann integrable on* $[0,\infty)$ *and if* $F(t)$ *is nonlattice, then*

$$\lim_{t\to\infty} A(t) = \mu^{-1} \int_0^{\infty} a(\tau)d\tau,$$

where

$$\mu := \int_0^{\infty} \tau dF(\tau).$$

In order to determine the solution of our integro-differential equation

$$(\lambda + \frac{d}{dx})\phi_D(x) = \lambda(R * \phi_D)(x),$$

we first transform the equation into an appropriate integral equation.

Invoking the integrating factor $e^{\lambda x}$ and the boundary condition $\phi_D(0) = 1$, reveals that $\phi_D(x), x \geq 0$ satisfies the equation

$$\phi_D(x) = e^{-\lambda x} + (F * \phi_D)(x),$$

where

$$F(x) = \int_0^x (1 - e^{-\lambda(x-u)})dR(u).$$

Note that

$$\int_0^{\infty} x dF = \rho + \lambda^{-1}.$$

Consequently, a straightforward application of our theorem reveals that

$$\phi_D(x) = \sum_{n=0}^{\infty} F^{n*}(x) * e^{-\lambda x}.$$

Observe that

$$\lim_{x\to\infty} \phi_D(x) = (1 + \rho\lambda)^{-1}.$$

Therefore, ϕ_D is bounded in $[0,\infty)$. Moreover, ϕ_D is decreasing (simply note that $\phi'_D(x) < 0$).

Finally, we remark that ϕ_D is absolutely continuous (with respect to the Lebesgue measure) on $(0,\infty)$, irrespective of the canonical structure of R.

A similar remark holds for ϕ_C.

As a matter of fact,

$$\phi_C(y) = 1 + \lambda_S \int_{u=0}^{y} \int_{x=u}^{\infty} \phi_D(x-u) dR_S(x) du .$$

Consequently, the Radon-Nikodym theorem [7] ensures that ϕ_C is also absolutely continuous on $(0,\infty)$, irrespective of the canonical structure of R_S. Therefore, our initial assumptions concerning the existence of repair time densities on $[0,\infty)$ are totally superfluous to ensure the existence of an invariant measure.

5. THE INVARIANT MEASURE

In order to derive the invariant measure of the T-system, we first recall the relations

$$P_B(x) = \lambda_S P_A \phi_D(x) R_S^-(x),$$
$$P_C(y) = \lambda P_A \phi_C(y) R^-(y),$$
$$P_D(x,y) = \lambda \lambda_S P_A \phi_D(x-y) R_S^-(x) R^-(y).$$

and we define

$$\tau_B := \lambda_S \int_0^\infty \phi_D(x) R_S^-(x) dx ,$$

$$\tau_C := \lambda \int_0^\infty \phi_C(y) R^-(y) dy ,$$

$$\tau_D := \lambda \lambda_S \int_{x=0}^{\infty} \int_{y=0}^{x} \phi_D(x-y) R^-(y) R_S^-(x) dy dx ,$$

Using the relations
$P_A + P_B + P_C + P_D = 1$, $\tau_B + \tau_D = \lambda_S \rho_S$, $\tau_D + \tau_C = \lambda \rho (1+\tau_B)$-, reveals that

$$P_A = \frac{1}{(1+\tau_B)(1+\lambda\rho)},$$

$$P_B = \frac{\tau_B}{(1+\tau_B)(1+\lambda\rho)},$$

$$P_C = \frac{\tau_B(1+\lambda\rho)+\lambda\rho-\lambda_S\rho_S}{(1+\tau_B)(1+\lambda\rho)},$$

$$P_D = \frac{\lambda_S\rho_S-\tau_B}{(1+\tau_B)(1+\lambda\rho)}.$$

The robot-safety device system is only operative in state A. Therefore, the long-run availability, denoted by **A**, is given by P_A.

Finally, we propose the following risk-criterion: State B is admissible if P_B satisfies the relation $P_B < \delta << 1$, for some $\delta > 0$, called the security level.

Next, we consider the case of fast repair, i.e. let $\lambda_S\rho_S << 1$ and $\lambda\rho << 1$. Note that the notion of "fast" repair does not necessarily imply a small average repair time. The mean repair time is only supposed to be considerably smaller than the average life time of the corresponding unit. A natural assumption that covers almost all engineering applications!

We recall that $\forall u \geq 0$ and for any R with mean ρ, $(1+\rho\lambda)^{-1} < \phi_D(u) \leq 1$.

Whence,

$$\lambda_S\rho_S(1+\rho\lambda)^{-1} < \tau_B < \lambda_S\rho_S.$$

Or

$$\mathbf{A} \geq \frac{1}{(1+\lambda\rho)(1+\lambda_S\rho_S)}.$$

Consequently, the case of fast repair induces a very tight lower bound for **A**.

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