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NEW METHODS FOR COMBINING FORECASTS

Sotiris N. PANTAZOPOULOS
Department of Mechanical Engineering, University of Patras, 26500, Rio, Patras, Greece

Costas P. PAPPIS
Department of Industrial Management, University of Piraeus, Karaoli & Dimitriou 80, 18534 Piraeus, Greece

Dedicated to the memory of Professor Jovan Petrić

Abstract: New methods for combining forecasts are introduced. The combined forecast for the next period is made using different forecasts weighted according to variable weights, based on the differences between each forecast and the actual value of the forecasted variable during the previous periods. The methods are implemented on actual data sets and are found to be very stable and accurate.

Keywords: Forecasting, combination of forecasts.

1. INTRODUCTION

Using a combination of forecasts rather than one forecast based on a single forecasting model is increasingly advocated by many researchers and practitioners [1]. An important factor, which encouraged the introduction of methods for combining forecasts and facilitated their acceptability, was the pragmatic need for forecasting based on a wide range of contexts, which guarantees an average accuracy. Indeed, the most serious problem in forecasting model selection is to choose one single model that adequately represents all the complexity characterising the environment, which affects the behaviour of the variable to be forecasted. In a managerial environment, where performance is the major source of validation, various ad hoc approaches, such as those for combining forecasts, become admissible if they generally work well. In addition, running several models in parallel in order to make combined forecasts and making multiple model analyses and syntheses are now computationally feasible due to the development of new software and the evolution of Decision Support Systems. However, while the methodological validity of the practice of combining forecasts is not questionable, an important problem is "which methods of combination work best".

A lot of work has been done on combining forecasts, ranging from earlier theoretical papers [2, 3] to more recent studies [4-9]. The usual method of combining forecasts is Linear Programming. The simplest LP model is the single objective LP model for combining n different forecasts over m observations [10]. The objective in this model is to minimise the total deviation, that means the sum of the forecast errors. The model may be expanded to include multiple accuracy measures. Another approach [9] also based on LP introduces a multiple objective LP model. This model is extended to include the "direction of change" measure. This measure refers to the frequency (number of times) that a combined forecast is successful in correctly identifying the direction of change (positive or negative) in the variable from one period to the next. The objective is to minimise the number of periods in which the forecast direction of change is incorrect.

A lot of other methods have been developed including Bayesian analysis [4]. In practice, the method most commonly used is the simple averaging of the forecasts [11, 12].

2. THE PROPOSED METHODS

The basic idea and the motivation behind the methods for combining forecasts proposed here is to exploit the simple exponential smoothing forecasting approach. According to this method, the forecast for the next period is based on the actual values of the variable to be forecasted during all previous periods, weighted so that the weight assigned to a previous period's actual value of the variable decreases exponentially as that data gets older. Thus, while all past data are taken into account, recent data receive a higher weight than older data. This helps to take account of recent trends, while the past history concerning the values of the variable is not discarded. The forecast for the next period is calculated by adding the actual value of the variable for the present period weighted by a factor w (the smoothing constant) to the forecast for the present period (the so-called "base") multiplied by $1-w$. In a similar way, forecasts may be based on combined past data weighted exponentially according to their distance from the present time.

In particular, let F_{ij} be the forecast of the value of variable X_i made at time period i, $i=1,\dots,m$, using method j ($j=1,\dots,n$). Let w_{ij} be the weight assigned to F_{ij} and F_i be the combined forecast, which is made based on all individual forecasts at period i.

2.1. First method

A simple approach to making a forecast is to put $F_{t+1} = X_t$, where F_{t+1} is the forecast for the next period and X_t is the base (that is, the combined forecast for the present period). Based on this approach the following model is proposed:

$$w_{i-1j} = \frac{1}{| F_{i-1j} - X_{i-1} |}, \quad \forall j \in \{ 1,...,n \},$$

$$F_i = \frac{\sum_{j=1}^{n} w_{i-1j} F_{ij}}{\sum_{j=1}^{n} w_{i-1j}}.$$

where w_{i-1j} is the measure of the jth forecast's accuracy in the previous time period i–1. Thus, each time the combined forecast for the next period is based on forecasts made using different methods, weighted according to the deviation of the corresponding past individual forecasts from the actual value of the variable.

2.2. Second method

The series $(w_{1j}, w_{2j}, \ldots, w_{i-1j})$ may be smoothed the same way as in the single exponential smoothing forecasting method. Thus, if the goal is to include more information about the past of the jth forecast's accuracy, the combining method may be modified as follows:

$$w_{i-1j} = \frac{1}{| F_{i-1j} - X_{i-1} |}, \quad \forall j \in \{ 1,...,n \},$$

$$\hat{w}_{ij} = \alpha w_{i-1j} + (1-\alpha)\hat{w}_{i-1j}, \quad \forall j \in \{ 1,\ldots,n \}.$$

$$F_i = \frac{\sum_{j=1}^{n} \hat{w}_{ij} F_{ij}}{\sum_{j=1}^{n} \hat{w}_{ij}}.$$

2.3. Third method

An alternative approach may be to use weights equal to the inverse squared error instead of the inverse absolute error. In that case the simple combination method may be defined as follows:

$$w_{i-1j} = \frac{1}{(F_{i-1j} - X_{i-1})^2}, \quad \forall j \in \{ 1,...,n \},$$

$$F_i = \frac{\sum_{j=1}^{n} w_{i-1j} F_{ij}}{\sum_{j=1}^{n} w_{i-1j}}.$$

2.4 Fourth method

The weights' series may be smoothed the same way as above and the resulting combination method may be described by the following equations:

$$w_{ij} = \alpha \frac{1}{(F_{i-1j} - X_{i-1})^2} + (1-\alpha) w_{i-1j}, \quad \forall j \in \{ 1,...,n \},$$

$$F_i = \frac{\sum_{j=1}^{n} w_{i-1j} F_{ij}}{\sum_{j=1}^{n} w_{i-1j}}.$$

3. NUMERICAL TESTS

In order to test the performance of the proposed combination methods, 48 series of data have been used. The data sets were taken from the Agricultural Bank of Greece [13] and refer to vegetable production during the years 1980-1993 in the Periphery of Patras. The percentage standard deviations of the above sets lie within a large interval ($0.1\% - 78\%$). Consequently the set of these 48 series of data is considered to be sufficient for testing the methods' performance. In addition, in some of the sets, a decreasing shift of the base takes place, so the methods may be tested for quick adaptation.

For each data set, four series of forecasts have been produced using the following well known forecasting methods:

- Single exponential smoothing ($\alpha = 0.3$)
- Whybark's method [14], $\{ \alpha_B, \alpha_M, \alpha_H \} = \{ 0.2, 0.4, 0.8 \}$
- Dennis' method [15], $\{ N, \alpha_B, \Delta \} = \{ 2, 0.2, 0.6 \}$
- Trigg Leach method [16], $\phi = 0.3$.

The mean accuracy of the above methods is presented in Table 1.

The series of forecasts generated by the above forecasting methods were combined by simple averaging, LP optimisation (which is reduced to the least squares method), Bayesian analysis and the proposed methods. The comparative results obtained [17] may be characterised as very encouraging and are summarised in Tables 2-5 (mean values of errors have been considered).

Although more results may be needed, the proposed methods seem to be very stable and accurate since, for the data sets used, they outperform any other combining method.

Table 1: Comparative accuracy of forecasting methods (mean values of errors)

	single smoothing	Whybark's method	Dennis' method	Trigg-Leach
MAPE	51.25	42.41	33.56	28.03
MPE	-45.14	-35.90	-26.03	-20.37
RMSPE	34.47	28.46	21.32	20.26

MAPE: Mean Absolute Percentage Error
MPE: Mean Percentage Error
MRSPE: Mean Root Squared Percentage Error.

Table 2: Comparative accuracy of existing combining methods

	simple averaging	LP	Bayesian analysis
MAPE	37.97	38.62	39.19
MPE	-31.86	-31.61	-33.25
RMSPE	24.94	24.64	25.56

Table 3: Comparative accuracy
1st and 3rd proposed methods

	1st method $w = 1/\mid e_t \mid$	3rd method $w = 1/e_t^2$
MAPE	29.03	26.61
MPE	-23.96	-21.62
RMSPE	20.08	18.45

Table 4: Method accuracy for several values of the smoothing parameter.
2nd proposed method

α=	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
MAPE	40.64	38.31	36.51	35.48	34.95	34.59	34.05	33.2	31.24
MPE	-34.14	-31.8	-30	-28.98	-28.47	-28.14	-27.65	-26.9	-25.25
RMSPE	27.72	26.78	26.02	25.61	25.43	25.31	25.12	24.89	23.58

Table 5: Method accuracy for several values of the smoothing parameter.
4th proposed method

α=	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
MAPE	40.59	38.22	36.34	35.03	33.97	32.84	31.43	30.16	29.32
MPE	-34.09	-31.74	-29.89	-28.63	-27.67	-26.71	-25.56	-24.54	-24.04
RMSPE	27.68	26.72	25.89	25.3	24.8	24.09	22.82	21.22	20.22

4. CONCLUSION

Four new methods for combining forecasts were presented in this paper. According to these methods, each forecast contributes to the forecast of the value of the forecasted variable for the next period according to a variable weight. This weight is determined by the differences between the forecasts and the actual values of the forecast variable during preceding periods.

The correspondence between the proposed methods and extrapolative forecasting methods should be noted. For example, if there is a trend in some forecasts' accuracy characterising a particular method, then a combining method may be constructed corresponding to the Holt's trend forecasting method and the forecast weights will be smoothed accordingly.

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