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GRADIENT METHOD AS A TOOL FOR MATHEMATICAL MODELING IN EARTHQUAKE ENGINEERING

Olivera JOVANOVIĆ
Department of Mechanical Engineering, University of Montenegro, Cetinjski put bb, Podgorica, Yugoslavia

Dimitar JURUKOVSKI
Institute of Earthquake Engineering and Engineering Seismology, University Kiril and Metodij, Skopje, Macedonia

Abstract: The goal of this paper is to investigate the role of viscodamper behavior in the identification of frame models from dynamic response data caused by seismic forcing functions. Including nonlinear damping of viscodampers in the mathematical model for even simple structures significantly affects the distribution of damping, and the accuracy with which response can be predicted. A number of different mathematical models of these structures are evaluated using system identification. Each mathematical model depends on a number of parameters related to the characteristics of the structure. The gradient method is applied to calculate the values of these parameters which best reproduce the measured response of the structure.

This paper presents the mathematical model formulation of a five-story steel frame model using the parameter system identification technique and shaking table experiments. The base isolate system consisting of helical springs and viscodampers was manufactured at GERB, Germany. The experimental work was conducted using the shaking table of the Institute of Earthquake Engineering, University of Skopje, Macedonia.

Keywords: gradient method, parametric identification, nonlinear damping, earthquake.

1. INTRODUCTION

System identification is a tool that can be used to evaluate a model. By systematically adjusting the parameters to provide the best possible correlation between predicted and measured responses, the form of the analytical model can be evaluated. System identification is a generic term for this optimization process, and there are many approaches to applying it to structural engineering. There have been many survey articles written on system identification [1,2,3], so this discussion need not be exhaustive. Evaluating models by adjusting parameters to fit known response data is known as parametric identification.

Base isolation is an anti-seismic strategy by means of which damaging earthquake motion reduces structural responses through a mechanism built into the structural system. Such a system, consisting of helical springs and viscodampers, was developed by the GERB Company in Germany. A five-story steel frame structural model, isolated by GERB vibration base isolation elements, was intensively tested applying a set of different earthquake motions on a biaxial seismic shaking table. During the testing it was observed that the viscous damping of the dampers was continuously decreasing. Namely, with an increase in the number of applications of oscillations to the cylinders in the viscous mass, the temperature of the fluid increased and, consequently, the damping coefficients decreased.

On the basis of this experimental conclusion, research was performed in order to determine the relationship between the change in the damping and the path passed by the cylinder. This means, physically, that the kinetic energy of the cylinder, when it moves in a viscous medium, turns into thermal energy. With an increase in fluid temperature, the fluid tends to lose its viscosity, which in turn causes a decrease in the damping capacity. To determine this relationship, experimental results were obtained by seismic shaking table testing and using the technique of parameter system identification.

2. SYSTEM IDENTIFICATION TECHNIQUE IN EARTHQUAKE ENGINEERING

Mathematical modeling using system identification is the process of defining of mathematical equations for a given physical system whose input functions and responses are known. The problem of mathematical modeling is divided into two categories depending on the degree of previous knowledge of the nature of the considered process. Modeling has the character of "black box" identification for a certain phenomenon and the physical aspect of the problem is unknown. The other category includes all problems whose physical aspects are known. Earthquake engineering problems fall into the second category because either their physical processes or their geometry, properties of the material or structural characteristics are known.

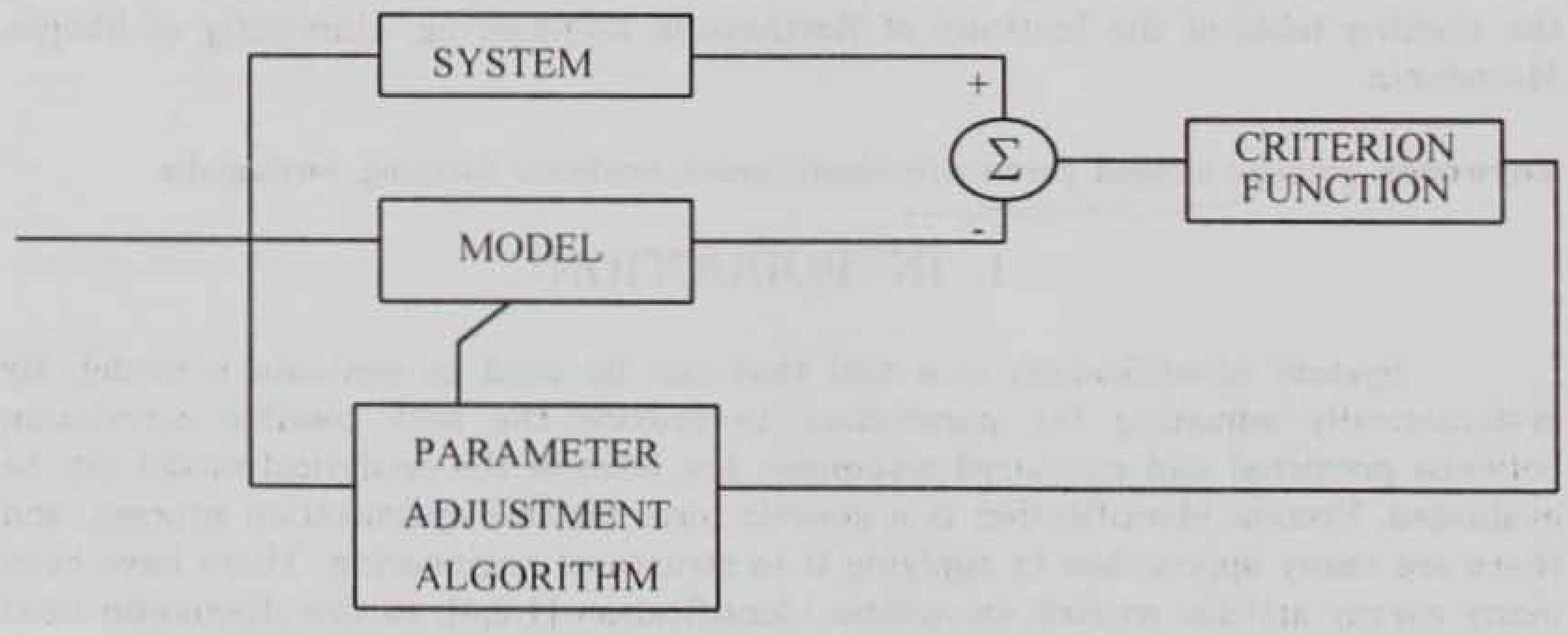

Figure 1. Identification by Bekey

A very good description of this identification technique for application in engineering problems was given by Bekey and the system identification consists of the following three phases:

1. Definition of the form of the model, i.e., selection of the differential equations of the model and extraction of the unknown parameters.

2. Selection of a criterion by means of which the "goodness of fit" of the model responses and physical system responses may be evaluated when both the mathematical model and the physical system have been excited by the same input.

3. Selection of an algorithm or strategy for the adjustment of parameters in such a way that the differences between model and system responses may be minimized [3].

A general configuration of the modeling problem is shown in Fig. 1.

In earthquake engineering the responses of the physical systems can be obtained by experimental investigations of the systems using various test procedures, such as shaking table tests, full scale tests of structures, etc. All these tests provide various experimental results which, depending on the model concept, are used for the determination of the model parameters directly or after filtering.

Our test model is a five-story steel frame, mounted on two heavy base floor girders, supported by four sets of spring-viscodamper elements manufactured by the GERB Company, for simulation of a base isolated model directly or after filtering.

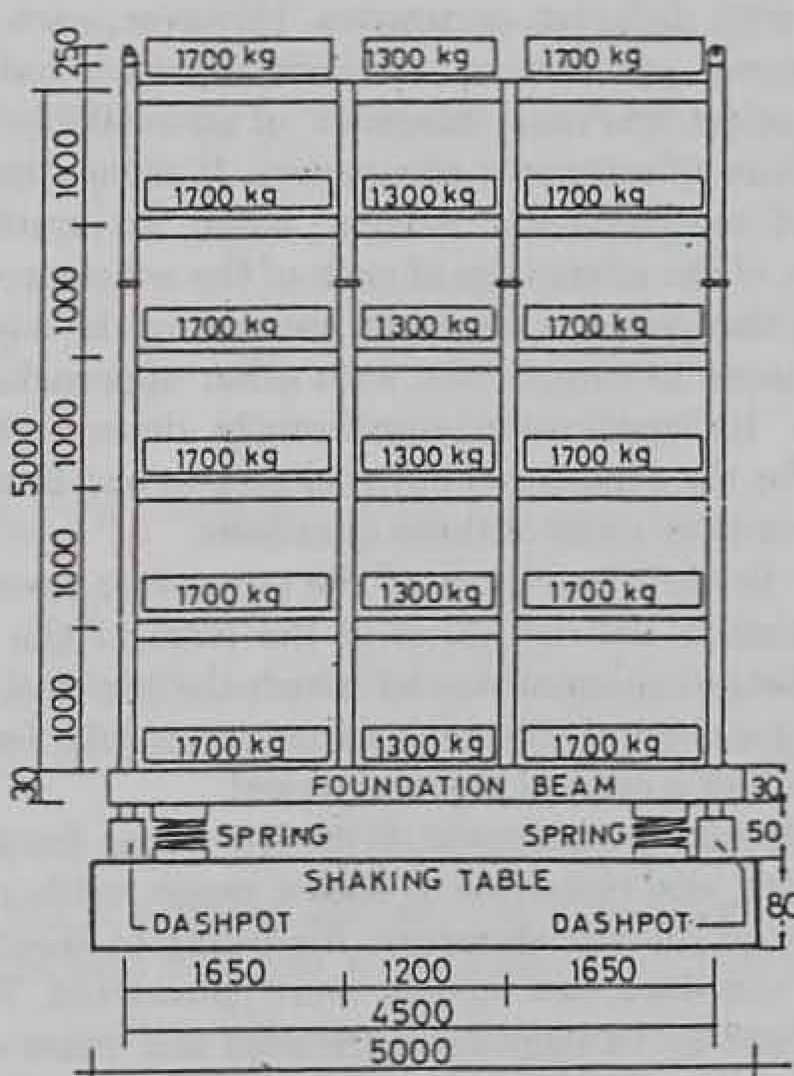

Figure 2. Structural model on the shaking table with vibration base isolation elements

The position of the set, consisting of a spring and viscodamper elements, is shown in Fig. 2. The viscodamper axis and the column axis are overlapping. The springs and viscodampers are designed for the real dead load of the model, which is approximately 80 kN.

This experimental program was planned in a way to ensure the collection of maximum useful experimental data. So, the displacement and acceleration time histories were recorded for various sets of earthquakes of different excitation levels on each floor.

The aim of installing viscodampers is to absorb the energy brought into the system and to decrease the amplitude of the system vibrations. The frame has a linear behavior, while the nonlinear behavior of the system stems from the viscodampers.

The experimental model was instrumented by 30 channels which measured the accelerations, displacements and stresses. The displacements were recorded by linear potentiometers with respect to a reference beam located on the foundation block. The horizontal displacements were measured on the base girder and each floor. The horizontal displacements were used in the system identification. The earthquake Petrovac 1979, was simulated on the shaking table.

3. MATHEMATICAL MODELING

From a theoretical point of view, a large number of mathematical models would be possible for each structural system, whose solutions present the considered dynamic response with different accuracies. However, each model involves unknown functions and unknown parameters and each approach has its own advantages. The problem is how to select "the most adequate" of all available models and how to reduce the unknown functions of unknown parameters. It seems that only by the technique of the construction of mathematical models, based on system identification, can an objective evaluation of the advantage of each of the considered mathematical models be provided. Although this paper is aimed at presenting the advantages of the parameter identification technique in comparison with other approaches, it cannot be applied to all practical cases. Its application significantly depends on the kind of available experimental data for the considered physical system and their reliability. In this paper, efforts are made to answer some of these questions.

Following Bekey's procedure of the parameter system identification, the first step in its application is the definition of the form of the mathematical model, i.e., selection of the differential equations by which the physical system is mathematically described. A large number of different dynamic models, having different complexity levels, can be applied for a certain physical model.

The considered structure was in elastic range for all simulated earthquakes. The supporting spring also remained in elastic range, while non-linearity was observed only in the behavior of the viscodampers. According to these experimental results and using the available test data, two models were considered. The first model (Fig. 2) has the structure idealized by 72 degrees of freedom and mass concentrated at the joints. Each mass has three degrees of freedom, two translations and one rotation. The model is, basically, supported by a system of springs and nonlinear viscodampers.

The second model has the structure idealized as a rigid body which is elastically supported by elastic springs and nonlinear viscodampers. Analysis showed that this model with only three degrees of freedom, two translations and one rotation provides very good results.

With that in mind, in this case the mechanical properties of the viscodampers, having nonlinear relationship, are not known. The strategy of the formulation of the mathematical model for the given physical problem was directed towards identification of the change in the viscous damping of the dampers depending on temperature. Literature offers many relationships experimentally defined as different empirical and theoretical models, but they always apply to known fluids.

The viscous material of the GERB dampers, in this case as a physical medium, is considered to have unknown properties. Therefore, the technique applied for formulation of the mathematical model is based on parameter system identification.

The differential equations of the dynamic behavior of the physical system in a relative coordinate system in matrix form are expressed as follows:

$$[M]\{\ddot{u}(\underline{\beta})\}+[R(\dot{u})]\{\dot{u}(\underline{\beta})\}+[K]\{u(\beta)\}=-[M]\{\ddot{n}(t)\}, \tag{1}$$

$$[M]=\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_c \end{bmatrix} \quad \text{the mass matrix}, \tag{2}$$

$$[R]=\begin{bmatrix} r_z & 0 & r_{z\varphi} \\ 0 & r_y & r_{y\varphi} \\ r_{\varphi z} & r_{\varphi y} & r_{\varphi} \end{bmatrix} \quad \text{the damping matrix}, \tag{3}$$

$$[K]=\begin{bmatrix} k_z & 0 & k_{z\varphi} \\ 0 & k_y & k_{y\varphi} \\ k_{\varphi z} & k_{\varphi y} & k_{\varphi} \end{bmatrix} \quad \text{the stiffness matrix}. \tag{4}$$

The mass matrix and stiffness matrix can be determined on the experimental base if the mass, dimensions of the system and the stiffness of the springs are known.

The vector of motion $\{u(\underline{\beta})\}=\{z(\underline{\beta}),y(\underline{\beta}),\varphi(\underline{\beta})\}$ indicates the total motion for the two translational degrees of freedom and one rotational degree of freedom for the mathematical model. Vectors $\ddot{u}(\underline{\beta})$ $\dot{u}(\underline{\beta})$ and are the vectors of acceleration and velocity, expressed as functions of an unknown vector. As it is obvious from the above equation, its solution is parametrically dependent on vector $\underline{\beta}$. The quality and quantity of the experimental data affect the selection of "the most adequate" of all available models for the considered problems.

To date, the damping matrix has been calculated as a linear combination of the mass matrix and stiffness, or as a function of the velocity v of a body in a viscodamper, but this work presents another point of view, i. e., damping is given through three unknown parameters in the function of the path s of the cylindrical body

$$r_z = b + (a - b)\exp(-c^{-1})\int |v| dt \tag{5}$$

$\{\beta\} = \{a, b, c\}$ - vector of unknown parameters.

A number of experiments have shown that the damping, caused by constant displacement of the body in the cylinder, decreases within physically permitted limits, leading to the conclusion that it is an exponentially decreasing function. That is why the damping is expressed through this function, as shown in (5). The unknown parameters $\{a, b, c\}$ will be determined by parameter identification of the system. Fig. 3 shows the meaning of parameters in the physics.

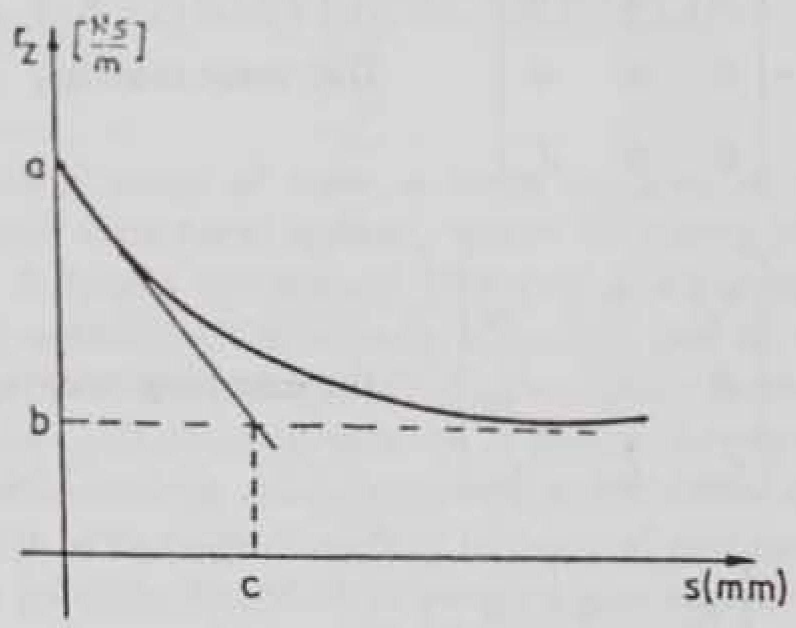

Figure 3. Curve of damping in the viscodamper

Unknown parameters a and b show the initial and ultimate value of the damping ratio of the matter in the damping cylinders..

Equation (1) presents the dynamic behavior of the mathematical model of the discussed physical system. It can be solved numerically by step-by-step integration.

The solution of this equation presenting the horizontal displacement of the system is compared with the experimental measure of the horizontal displacement of the system and is used for determining the error function.

The next problem is the selection of a criterion function or an error function. It is explained as follows.

Suppose we have a system subjected to a time dependent input $\{\ddot{n}(t)\}$, which produces a set of measurable outputs $y_j(t), j = 1,...,n$. If we have a model which we believe represents the system, this means we have some means by which, given an input, $\{\ddot{n}(t)\}$and some information about the system in terms of a vector of unknown constants, $\underline{\beta}$, we can predict the output of the system, $x_j(\underline{\beta}, t)$. Here we include $\underline{\beta}$ as an argument to emphasize the dependence of the predicted output of the system on the information supplied by the model.

One measure of how the predicted response matches the measured response is the squared-error loss function over time interval $0 < t < T$:

$$J(\underline{\beta}) = \sum_{j=1}^{n} \int_{0}^{T} g_j \left[x_j(\underline{\beta}, t) - y_j(t) \right]^2 dt, \tag{6}$$

In this equation g_j, $j=1,...,n$ represent the weight coefficients for evaluation of various physical variable influences. For this case g_j has the value of one, since the error function is formed only by displacement. Again, $\underline{\beta}$ is included as an argument to emphasize the dependence of J on $\underline{\beta}$. If $J(\underline{\beta}) = 0$, then the predicted response would exactly match the measured response. We would like to know what value of $\underline{\beta}$, if any, minimizes J.

Unfortunately, very few models permit a closed form solution for $\underline{\beta}$ which minimizes J globally. It is, however, often possible to generate an iterative scheme that will produce a $\underline{\beta}$ that is a local minimum.

The third step in the identification technique is very important and represents the selection of the algorithm for parameter adjustment to minimize the criterion function (6). There are a large number of methods in mathematical optimization theory that can be used in the identification process. Most of them are based on the iterative technique.

Gradient methods are the most suitable for determination of the function minimum in a multi-dimensional space.

4. OPTIMIZATION ALGORITHM

Given a set of parameters $\underline{\beta}$, we would like a systematic method to discover a new set $\underline{\beta}_{i+1}$ such that $J(\underline{\beta}_{i+1}) < J(\underline{\beta}_i)$. Repeated often enough, this will lead to a minimum for J. If the function J is approximately quadratic in the neighborhood of $\underline{\beta}_i$, there will be little error in the approximation

$$J(\underline{\beta}_{i+1}) = J(\underline{\beta}_i) + \nabla J^T(\underline{\beta}_i)(-\underline{\beta}_{i+1} - \underline{\beta}_i) + 1/2(\underline{\beta}_{i+1} - \underline{\beta}_i)^T \nabla^2 J(\underline{\beta}_i)(\underline{\beta}_{i+1} - \underline{\beta}_i)$$

where

$$\nabla J_p = \partial J / \partial \beta_p$$

and

$$\nabla^2 J_{ps} = \partial^2 J / \partial\beta_p \partial\beta_s$$

To minimize J, its gradient with respect to $\underline{\beta}_{i+1}$ is set to the zero vector. If the Hessian matrix is invertible, it follows that

$$\underline{\beta}_{i+1} = \underline{\beta}_i - [\nabla^2 J(\underline{\beta}_i)]^{-1} \nabla J(\underline{\beta}_i),$$

Since J will not, in general, be exactly quadratic, we will want to be able to adjust the size of the correction to $\underline{\beta}_i$. Thus we modify the equation by adding a step size variable, α:

$$\underline{\beta}_{i+1} = \underline{\beta}_i - \alpha [\nabla^2 J(\underline{\beta}_i)]^{-1} \nabla J(\underline{\beta}_i).$$

The components of ∇J and $\nabla^2 J$ are found by taking the appropriate derivatives of the error function:

$$\nabla J_p = \frac{\partial J}{\partial \beta_p} = 2\sum_{j=1}^{n}\int_0^T g_j \left\{ [x_j(\underline{\beta} - y_j(t)] \frac{\partial x_j(\underline{\beta},t)}{\partial \beta_p} \right\} dt$$

$$\frac{\partial^2 J}{\partial\beta_p \partial\beta_s} = 2\sum_{j=1}^{n}\left\{ \int_0^T g_j \frac{\partial x_j(\underline{\beta},t)}{\partial\beta_p}\frac{\partial x_j(\underline{\beta},t)}{\partial\beta_s} dt + \int_0^T g_j (x_j(\underline{\beta},t) - y_j(t)) \frac{\partial^2 x_j(\underline{\beta},t)}{\partial\beta_p \partial\beta_s} dt \right\}.$$

Experience has shown that the second integral, particularly when $\underline{\beta}_i$ is close to a minimum, is negligible when compared with the first. The Gauss-Newton iteration scheme, therefore, is to choose α and calculate

$$\underline{\beta}_{i+1} = \underline{\beta}_i - \alpha [\underline{\mathrm{AH}}(\underline{\beta}_i)]^{-1} \nabla J(\underline{\beta}_i),$$

where the approximate Hessian matrix, $\underline{\mathrm{AH}}$, is defined as

$$\underline{\mathrm{AH}}_{ps} = 2\sum_{j=1}^{n}\left\{ \int_0^T g_j \frac{\partial x_j(\underline{\beta},t)}{\partial\beta_p}\frac{\partial x_j(\underline{\beta},t)}{\partial\beta_s} dt \right\}.$$

The technique for choosing α is known as a line search algorithm since the multidimensional minimization problem has been reduced to a single dimension.

5. LINE SEARCH

By establishing a search direction, the error function is reduced to a function of the variable α

$$J(\alpha) = J[\underline{\beta}_i - \alpha \underline{AH}^{-1}(\underline{\beta}_i)\nabla J(\underline{\beta}_i)]$$

whose derivative is

$$\frac{d}{d\alpha}J(\alpha) = -\nabla J^T(\underline{\beta}_{i+1})\,\underline{AH}^{-1}(\underline{\beta}_i)\nabla J(\underline{\beta}_i).$$

If we are pointed in the right direction, $J'(0) < 0$. If the error surface were quadratic, then the exact minimum would be at $\alpha = 1$. If $J'(1) > 0$ then there must be a minimum for $0 < \alpha < 1$. In order to find a point closer to the minimum, a cubic polynomial is constructed so that its values and derivatives match J at the end points, and the minimum of the cubic is used as a new trial point. If, on the other hand, $J'(1) < 0$ and $J(1) < J(0)$, then a quadratic extrapolation is made. In this way, successive approximations to the functional minimum are made until some stopping criterion is met.

The stopping criterion for the line search will affect the relative amount of time spent on finding search directions and doing line searches. In general, spending too much time on either is not economic. In practice, a good deal of trial and error is necessary to find a reasonable distribution of effort. In this case four or five iterations in the line search is probably a good compromise.

6. DISCUSSION OF THE RESULTS

The algorithm for parameter adjustment is based on the modified Gauss-Newton method which usually provides convergence. Using this developed algorithm the model presented in Fig. 3 was first analyzed.

The mathematical model was proposed with three unknown parameters:

$$\{\underline{\beta}\} = \{a, b, c\}.$$

Following the parameter system identification technology for the given mathematical model and developed algorithm for parameter adjustment, the initial values of the parameter had to be estimated. The values of this vector can be defined as random values, but in order to have a smaller number of iterations the following estimation procedure was used. Using the empirical relations, the initial values were estimated as:

$$\{\underline{\beta}_0\} = \{550000, 340000, 6\}$$

After the fourth iteration, vector $\{\underline{\beta}\}$ took the following values:

$$\{\underline{\beta}_4\} = \{929900, 640000, 5.80\}.$$

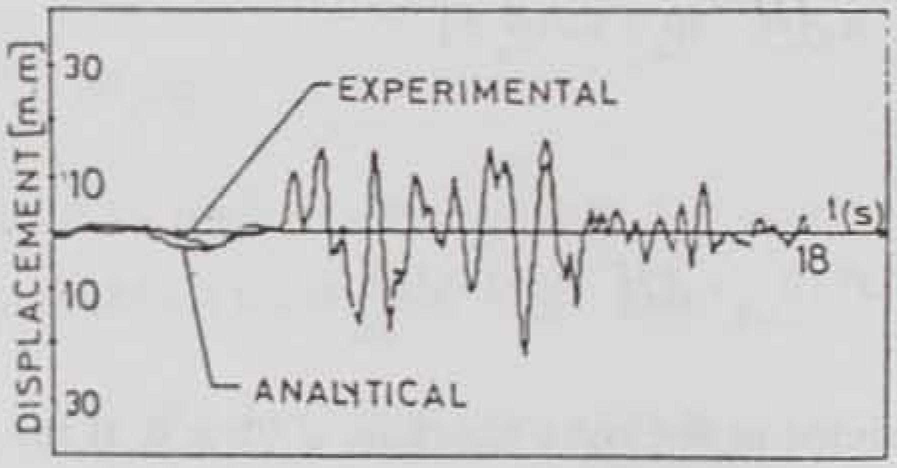

Figure 4. Analytical and experimental displacement time histories at the center of the model for Petrovac 1979 earthquake, *N-S* component

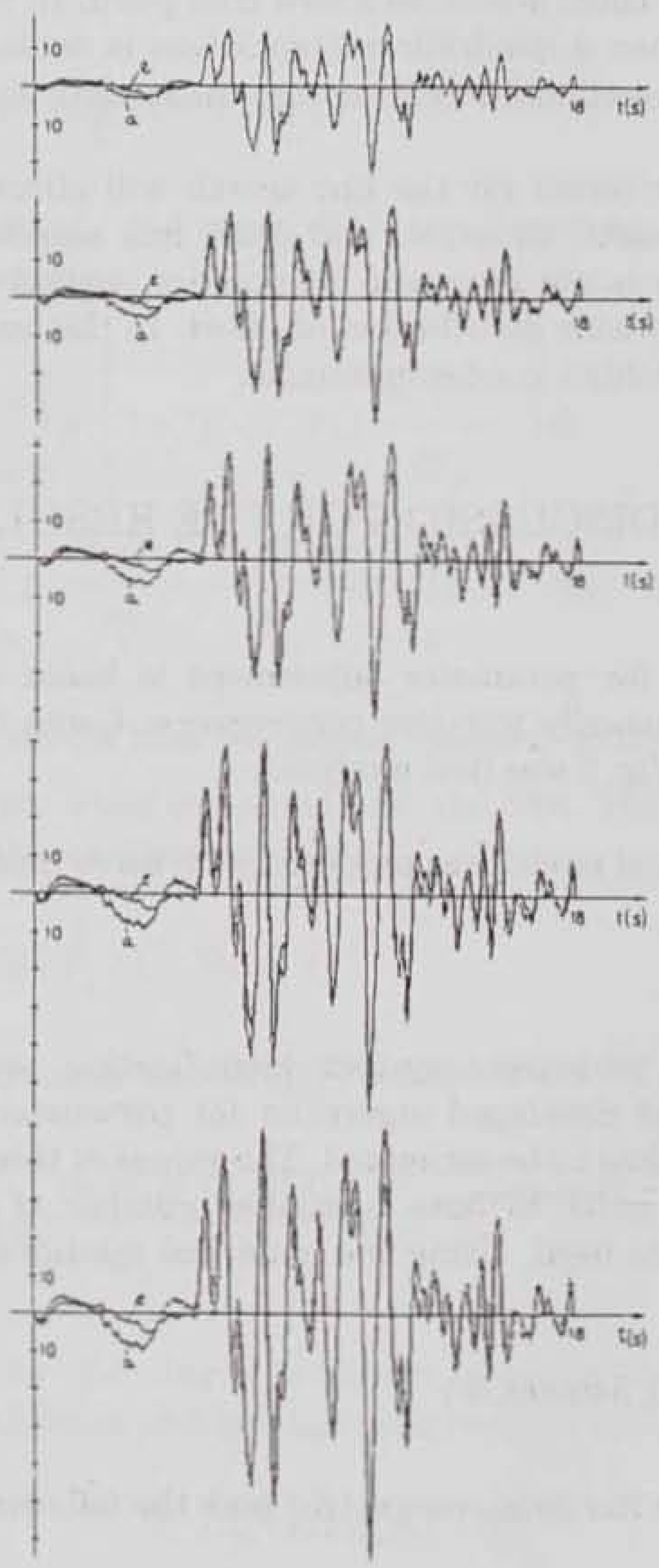

Figure 5. Analytical and experimental displacement time histories on the all five floors of the model for Petrovac 1979 earthquake, *N-S* component

The responses of the mathematical model were calculated for the vector of parameter $\underline{\beta}$ and the results were compared with the experimental ones. The results of this identification technique are given in Fig. 4. The experimental results as displacement of the system and the mathematical results as displacement of the same point of the frame are shown in the same Figure.

From that Figure, visually, a good correlation between the experimental and analytical time histories can be seen. The same analyses were performed for the model presented in Fig. 2, and the correlation of the analytical and experimental displacement time histories on the all five floors, for the earthquake Petrovac 1979, is shown in Fig. 5.

It is not necessary to comment on the differences between the responses since they are numerically qualified.

For the second model, which is more complex than the previous one, the error function calculated for one story is less than the error function calculated for the simplest model. But the effort made for the second model is much greater compared with the effort made for the analyses of the first model.

It is difficult to give an accurate answer to the question as to which model should be applied in the analysis or design of a real structure, although the system identification technique makes it possible to quantify some answers. But, practically speaking, it seems that the physics of the problem should be of primary importance for deciding which model should be applied.

7. CONCLUSION

The application of the parameter system identification technique in the field of earthquake engineering, in the case where valuable experimental results are available for the considered physical systems, is a very powerful tool for an objective definition of mathematical models.

It has been shown that the use of the Gauss-Newton iterative identification procedure greatly reduces the computational effort required. At the same time, the technique appears to be quite stable and converges rapidly.

The use of vibration isolators is possible in many buildings, such as nuclear power plants as well as ordinary structures, as a protection against earthquake engineering. Vibration isolators reduce the earthquake loads on a structure by more than five to ten times with respect to those of a fixed base structure. Therefore, the increase in cost due to isolating devices and double-layer foundations is much less than the extra cost of anti seismic design requirements of the conventional method.

Changes of damping in viscodampers are found in this paper and given the opportunity to quantify the effects of any parameters on the mathematical model response. For that reason, damping is a calculated mathematical response of displacement for both considered models. Continually good agreement between experimental and numerical results has been shown.

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