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SOME ASPECTS OF PIECEWISE LINEAR L_1 APPROXIMATIONS IN BUSINESS OPTIMIZATION

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Abstract: An approximation of the function relationship between decision variables (in the form of statistical data) of the optimization model with the piecewise linear function is used, for which the sum of absolute deviations is minimal. The problem can be transformed to the linear mixed integer optimization model. The piecewise linear function itself can be included in the linear mixed integer optimization model of the multiphase production process.

The piecewise linear function is also used in order to include the quality costs in the linear mixed integer optimization model. Total quality costs can be classified into prevention, appraisal and internal and external failure costs. They can be included in the mixed integer optimization model of the business process by the piecewise linear function.

Keywords: Piecewise linear function, quality costs, sales estimation

1. INTRODUCTION

The business process can be decomposed into production, purchasing and selling activities. The elements of the business process are stated to be purchasing elements, semi-products, products and production capacities. A linear model for the multiphase business process can be written in the form [5]

$$\max\left(\sum_i \sum_k s_{ik} z_{ik} - \sum_i \sum_k c_{ik} y_{ik} - \sum_j v_j x_j\right) \tag{1}$$

subject to nonnegative decision variables z_{ik}, y_{ik}, and x_j and

$$e_i = \sum_{j \in R_j} r_{ij} x_j + \sum_k y_{ik} - \sum_{j \in Q_i} q_{ij} x_j - \sum_k z_{ik} \geq 0, \quad i \in E \tag{2}$$

$d_{ik} \leq z_{ik} \leq D_{ik}$ for some i, k

$b_{ik} \leq y_{ik} \leq B_{ik}$ for some i, k

where

s_{ik} -	the selling price of the i-th element to the k-th customer reduced by the variable selling cost per unit
z_{ik} -	the selling quantity of the i-th element to the k-th customer
c_{ik} -	the purchasing price of the i-th element at the k-th source, increased by the variable purchasing cost per unit
y_{ik} -	the purchased quantity of the i-th element at the k-th source
v_j -	the variable cost per unit of the j-th production activity, excluding the consumption of elements belonging to E
x_j -	the quantity of the j-th production activity
e_i -	the unallocated quantity of the i-th element
r_{ij} -	the quantity of the i-th element produced per unit of the j-th production activity
R_i -	index set of the production activities producing the i-th element
q_{ij} -	the quantity of the i-th element consumed per unit of the j-th production activity
Q_i -	index set of the production activities consuming the i-th element
E -	index set of the relevant elements
d_{ik} -	minimal quantity of the i-th element which has to be sold to the k-th customer
D_{ik} -	maximal quantity of the i-th element which can be sold to the k-th customer
b_{ik} -	minimal quantity of the i-th element which has to be purchased from the k-th source
B_{ik} -	maximal quantity of the i-th element which can be purchased from the k-th source

Using the linear mixed integer optimization model, it is possible to consider the piecewise linear variable costs, the semi-fixed costs, the piecewise linear consumption quantities of the elements of the business process, the piecewise linear income, investments and also some other situations in business optimization [6].

If the function relationship between some decision variables of the linear mixed integer optimization model of the business process is approximated by the piecewise linear function, for which the sum of absolute deviations is minimal, the problem of estimating of the unknown parameters of the linear piecewise function can be transformed into the linear mixed integer optimization model. The piecewise linear function can be included in the linear mixed integer optimization model [5,11].

2. QUALITY COSTS

We frequently hear about the quality revolution. That started in Japan over 40 years ago. This new approach has been titled variously, but every title gives the word quality a new meaning - quality is the customer [4]. This means that every activity we take must be towards meeting the requirements (internal and external) and wishes of customers. The total quality concept is based on defect prevention instead of traditional defect detection and correction [10]. The idea is that the customers get full value for their money. But to reach this idea you need to identify where quality costs arise in order to be able to take whatever action is necessary to eliminate or reduce them. The British Standard (1990) defines four types of manufacturing quality related costs:

Prevention costs are costs of any activity taken to investigate, prevent or reduce defects and failures. For example: costs of quality assurance, quality engineering functions, quality training, the calibration of test and inspection equipment, etc.

Appraisal costs are costs of assessing the quality achieved. For example: cost of inspectors and their equipment, conducting field performance tests, etc.

Internal failure costs arising within the manufacturing organization or failure to achieve quality specified (before dispatch to the customers). For example: scrap, rework and repair, re-inspection and re-testing, etc.

External failure costs arising outside the organization or failure to achieve quality specified (after dispatch to the customers). For example: customer complaints, service activities, insurance, loss of goodwill and image, etc.

The proportion of type of quality related cost is different in different companies. Generally speaking about 95% of quality related costs are appraisal and failure costs [3]. The top 20% of the causes of failure cover 80% of the costs. After some of the quality techniques and methods have been used, we should expect different proportions between quality related costs as shown in Fig. 1.

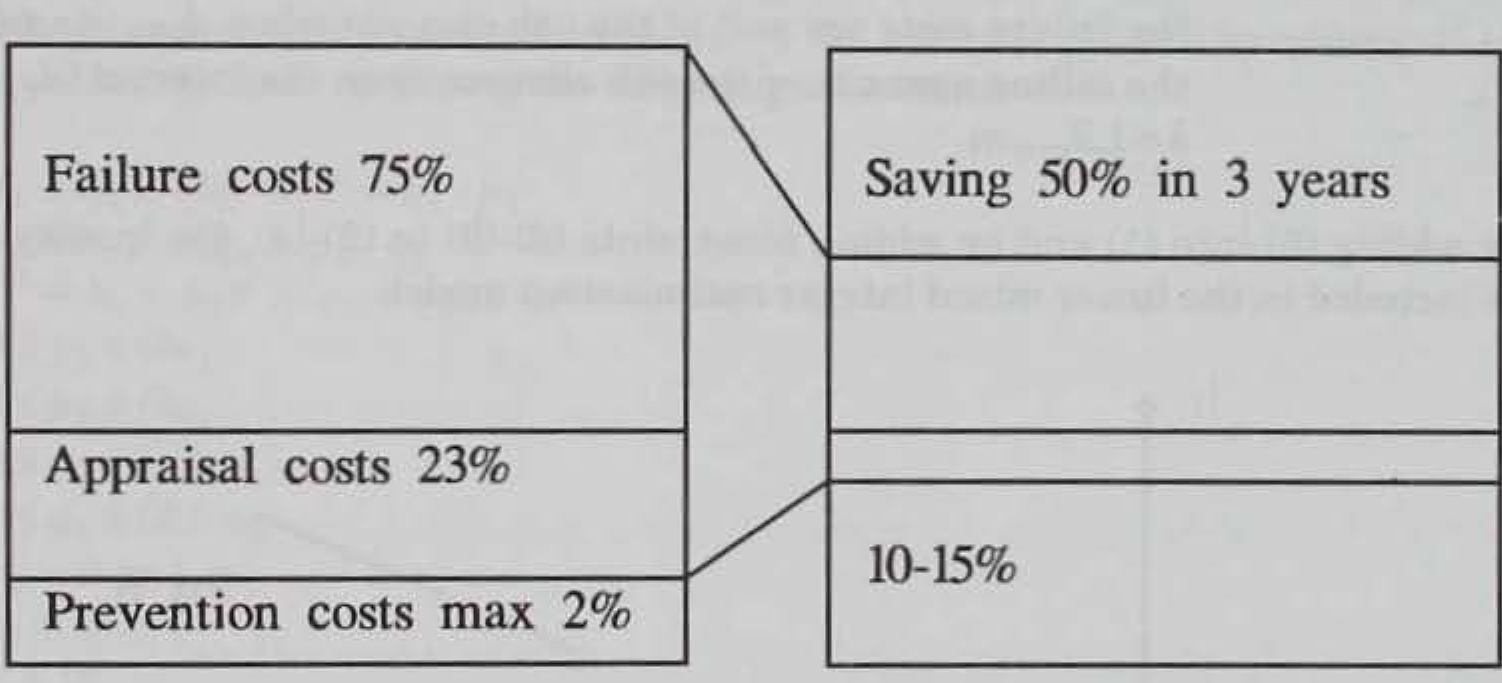

Figure 1: Quality costs before and after using quality techniques and methods.

The failure costs are variable costs, while appraisal and prevention costs have a semi-fixed character. Both can be represented together by the piecewise linear function of the selling quantity of the i-th product and included in the linear mixed integer optimization model (the model (1)-(4) considers that the produced quantity and the quantity sold of the i-th product are the same). Semi-fixed costs occur if the level of fixed costs increases by leaps at a certain selling (or produced or purchased) quantity. Within the given bounds of the selling quantity they remain the same.

Using the methodology described in the literature [5], the manufacturing quality costs of the i-th element can be represented by the piecewise linear function

$$t_i = \sum_{k=1}^{m} b_{ik}u_{ik} + \sum_{k=1}^{m} r_{ik}g_{ik} \tag{5}$$

subject to

$$z_i = \sum_{k=1}^{m} g_{ik} \tag{6}$$

$$(d_{ik} - d_{i(k-1)}u_k) - g_{ik} \geq 0 \qquad \text{for } k = 1,2,...,m \tag{7}$$

$$g_{ik} - (d_{ik} - d_{i(k-1)})u_{k+1} \geq 0 \qquad \text{for } k = 1,2,...,m-1 \tag{8}$$

$$u_k = 0 \text{ or } 1 \qquad \text{for } k = 1,2,..., m \tag{9}$$

where

z_i -	the selling quantity of the i-th element
t_i -	the manufacturing quality costs of the i-th element
d_{ik}-	the selling quantity of the i-th element at the end of the k-th interval
b_{i1} -	the base level of the appraisal and prevention costs, occurring regardless of sales volume
b_{ik} -	the amount of the appraisal and prevention costs occurring if z_i increases beyond d_{k-1}, $k = 2, 3,..., m$
r_{ik} -	the failure costs per unit of the i-th element when $d_{i(k-1)}<z_i\leq d_{ik}$
g_{ik} -	the selling quantity of the i-th element from the interval $(d_{i(k-1)}, d_{ik})$; $k=1,2,..., m$

By adding (5) into (1) and by adding constraints (6)-(9) to (2)-(4), the quality costs can be included in the linear mixed integer optimization model.

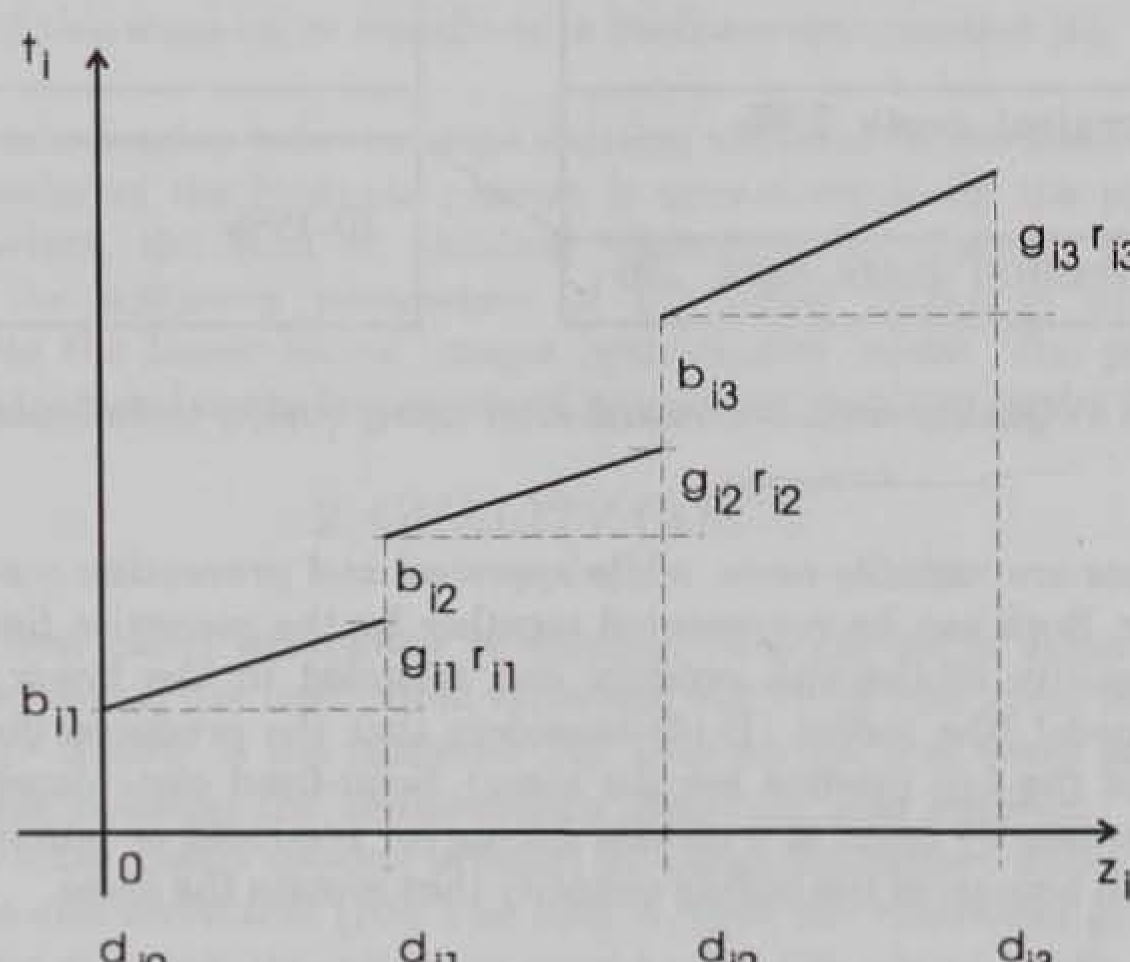

Figure 2: Quality costs of the i-th element for $m = 3$.

The semi-fixed and variable quality costs as the function of the selling quantity of the i-th element are represented in Fig. 2.

3. SALES ESTIMATION BY THE PIECEWISE LINEAR FUNCTION

Let us take an example of business optimization, where the function relationship between the selling quantity of the product I and the advertising costs is given in the form of empirical data (in a sample of n elements). The approximation of the function relationship is to be obtained by the piecewise linear function, for which the sum of the absolute deviations is minimal, in the form

$$D^I = \max(0; \max(a_0 + a_1 t^I ; b_0 + b_1 t^I) \qquad (10)$$

where

D^I - the maximal selling quantity of element I
t^I - the advertising costs for element I

Using the methodology described in the literature [6], the linear mixed integer model for the estimation of the parameters of the function (10) can be formed and solved by available software [9]. The function (10) with the estimated parameters can be included in the linear mixed integer optimization model of the business process by adding $-t^I$ to the objective function and by adding the following constraints

$$\begin{aligned}
&a_0 + a_1 t^I - b_0 - b_1 t^I = q_1 - p_1\\
&b_0 + b_1 t^I + q_1 = p_2 - q_2\\
&D^I = b_0 + b_1 t^I + q_1 + q_2\\
&0 \le p_1 \le G u_1\\
&0 \le p_2 \le G u_2\\
&0 \le q_1 \le G(1-u_1)\\
&0 \le q_2 \le G(1-u_2)\\
&u_1 = 0 \text{ or } 1\\
&u_2 = 0 \text{ or } 1\\
&z^I \le D^I
\end{aligned}$$

where G is a suitable constant and z^I is the selling quantity of element I, to the other constraints of the linear mixed integer optimization model.

4. CONCLUSION

In this paper the semi-fixed and variable manufacturing quality costs are examined and presented by the piecewise linear function. Given that semi-fixed costs exist in most industrial marketing situations [8], it is possible to include other costs as well which have a semi-fixed character, to the linear business optimzation model, in the same way.

If the selling quantity of the product is estimated by the piecewise linear function of the predictor random variables for which the sum of absolute deviations is minimal, the number of the variables of the corresponding model for estimating the unknown parameters of the piecewise linear function, with regard to the number of the elements of the data set, must be taken into account.

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