Yugoslav Journal of Operations Research
6 (1996), Number 2, 285-288

SIMULATION OF SOME UNIVARIATE DISTRIBUTION APPLIED IN RELIABILITY

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Abstract: This paper presents algorithms for the computer generation of random sampling values of a given random variable X (when its cummulative distribution function over its probability density function is known) which can be used in reliability.

Keywords: Simulation, reliability, failure rate.

1. INTRODUCTION

In order to estimate the reliability of any system it is necessary to know the life-time distribution of its components.

A univariate probability distribution (pdf) can be a life-time distribution (i.e. can be used in reliability) if its cummulative distribution function (cdf) $F(x)$ satisfies the condition $F(0\text{-})=0$.

In reliability theory an important role is played by the failure rate $r(t)=f(t)/R(t)$, where $f(t)$ is pdf and $R(t)=1-F(t)=P(X\geq t)$ is the reliability function.

The problem considered here is to construct an algorithm to generate a random sampling value when its pdf is known.

It is also posible to generate the failure rate r(t) when the cummulative failure rate (cfr) defined by $H(x)=\int_0^x r(t)dt$ is given.

The algorithms have the property when they are called successively n times they produce a sample $X_1, X_2, \ldots, X_n$ over X (i.e. X_i' is be independent and identically distributed as X).

2. SIMULATION OF A UNIVARIATE WEIBULL DISTRIBUTION

The Weibull distribution is one of the most important distributions in reliability.
The standard Weibull (ν) distribution with parameter ν has pdf given by

$$f(x) = \nu x^{\nu-1} \exp(-x^{\nu}), \quad x > 0, \quad \nu > 0$$

A fast and accurate algorithm to generate a standard Weibull (ν),ν>1 variable is based on the ratio of uniforms [1].

0. Input ν, $\nu>1$.
1. Calculate $a = (\nu-1)/\nu, \quad b = (\nu+1)/\nu$
   $u_2 = a^{a/2} \exp(-b/2), \quad v_2 = b^{b/2} \exp(-b/2)$
2. Generate U a uniform (0,1) random variable and calculate $U^* = u_2 U$.
   Generate V a uniform (0,1) random variable and calculate $V^* = v_2 V$.
3. Calculate $R = V^* / U^*$.
   If $\ln U^* + R^{\nu}/2 - (\nu-1)/2 \ln R > 0$ go to 2.
4. X=R (The generated value).

The failure rate is of the form

$$r(t) = \nu t^{\nu-1}$$

and the cummulative failure rate

$$H(t) = t^{\nu}$$

The simplest method to generate T is the inverse method:

1. Input ν
2. Generate U a uniform (0,1) random variable.
3. Calculate $T = \exp(\ln U / \nu)$.

The three-parameter Weibull (α, λ, ν) distribution has the pdf

$$g(y) = \nu / ((y-\alpha)/\lambda)^{\nu-1} \exp((y-\alpha)/\lambda), \quad \alpha \in R; \quad \lambda, \nu > 0; \quad y \geq \alpha$$

and its simulation is done as

$$Y = \alpha + \lambda X$$

where X is a standard Weibull (ν).

In this case, the failure rate is

$$r(t) = \nu / \lambda^{\nu} (t-\alpha)^{\nu-1}$$

A particular case of the Weibull (α, λ, ν) distribution is the Rayleigh (Θ) distribution which is Weibull (0, Θ, 2).

4. SIMULATION OF A UNIVARIATE GAMMA DISTRIBUTION

The standard Gamm (ν) distribution with parameter ν has the pdf given by

$$f(x) = \frac{1}{\Gamma(\nu)} x^{\nu-1} \exp(-x), \quad x > 0, \quad \nu > 0.$$

In order to generate a Gamma random variable with parameter ν (0<ν<1 and ν>1) an algorithm based on the rejection method [2] was considered.

The case 0<ν<1

0. Input ν .
1. Calculate a=(2ν -1)2
   $u_2 = a^{a/2} \exp(a/2)$
   $\nu_1 = r_1(1-r_1)^a \exp\left(-(1-r_1)^2/2\right)$
   $\nu_2 = r_2(1-r_2)^a \exp(-(1-r_2^2)/2)$

   where r_1, r_2 are the roots less than one of the equation $2r^3 - 4r^2 - 2(\nu-1)r + 2 = 0$

2. Generate U a uniform (0,1) random variable and calculate $U^* = u_2 U$.
   Generate V a uniform (0,1) random variable and calculate
   $V^* = v_1 + (v_2 - v_1)U$.
3. Calculate $R = (U^* - V^*)/U^*$
   If $\ln U^* - a \ln R + R^2/2 > 0$ go to 2.
4. $X=R^2$ (The generated value)

The case ν<1

0. Input ν .
1. Calculate b=(ν -1)/2
   $u_2=(2b)^b \exp(-b)$
   $\nu_1 = r_1(1-r_1)^b \exp((r_1-1)/2)$
   $\nu_2 = r_2(1-r_2)^b \exp((r_2-1)/2)$

   where r_1, r_2 are the roots of the equation $r^2 + \nu r - 2 = 0$.

2. Generate U a uniform (0,1) random variable and calculate $U^* = u_2 U$.
   Generate V a uniform (0,1) random variable and calculate
   $V^* = v_1 + (v_2 - v_1)V$.
3. Calculate $R = (U^* - V^*)/U^*$
   If $\ln U^* - b \ln R + R/2 > 0$ go to 2.

4. $X=R$ (the generated value).

The three parameter Gamma (α, λ, ν) distribution has the pdf

$$f(y;\alpha,\lambda,\nu)=\lambda^{\nu}/\Gamma(\nu)(y-\alpha)^{\nu-1}\exp(-\lambda(y-\alpha)); \quad y,\lambda,\nu>0, \quad \alpha\in R, \quad y\geq\alpha.$$

If X is a Gamma $(0,1,\nu)$ standard random variable, then the Gamma (α, λ, ν) random variable Y can be expressed as $Y=\alpha+X/\lambda$, therefore the problem is reduced to generating X.

The cdf $F(x;\alpha,\lambda,\nu)$ may be written as $F(x;\alpha,\lambda,\nu)=\int_0^x f(y;\alpha,\lambda,\nu)dy$

The failure rate is given by $r(t)=\dfrac{f(t;\alpha,\lambda,\nu)}{1-F(t;\alpha,\lambda,\nu)}$ and the cummulative failure rate

$$H(x)=\int_0^x r(t)dt$$

The algorithm for generating X having H distribution is of the form:

1. Input parameters
2. Take $X=H^{-1}(x)$.

In conclusion, the generation of random variables used in the reliability theory is a difficult problem which is always open to new results.

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