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# AN ANALYSIS OF SOME TELECOMMUNICATION TRAFFIC MODELS WITH DIFFERENT SERVING INTENSITIES

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**Abstract**: This paper analyses three models with overflow traffic and different serving intensities in primary and secondary trunk groups, which represent specific cases in circuit switching telecommunication networks. Explicit solutions for traffic parameters as well as the numerical iterative procedure to resolve steady state system equations are used in the analysis. For the complex model with two overflow traffic systems with changed serving intensity, we suggest a procedure to overcome the problems of resolving a system with a great number of steady state equations while keeping the advantages of an analytic solution compared to statistics modeling.

**Keywords**: Overflow traffic, circuit switching, alternative routing, serving system.

## 1. INTRODUCTION

In circuit switching telecommunication networks, the advantages of direct routes, alternative routes and trunk groups with one and two direction transmitters are widely used. There is also the possibility of organizing trunk groups with different quality of serving traffics. Such traffic situations are analyzed using some models with overflow traffic. The parameters of the models are obtained numerically from a small number of explicit solutions by resolving steady state system equations. Different approximate equivalent random methods as well as statistical modeling are used, too.

Three fundamental models with overflow traffic and mean service time that differs compared to Poisson traffic from the primary trunk group are considered in this paper. These cases may occur when, because of setting up calls on alternative route complexity, call duration increases or decreases because information is given about an alternative route which is more expensive. It is possible, under the corresponding conditions, to use those models to analyze the case with a variable data transmission rate on the alternative route.

When analyzing the first model, theoretical results from [1] were used, while in the case of the second model, we obtained traffic parameters resolving the system of steady state equations taking into account the developed numerical iterative procedure as well as the experience in paper [2]. For the approximate determination of partial losses, several methods were developed as in [3, 4, 5], but there was always a lack concerning the important regions of parameters values, when the approximate formulae give the mistake. In the very strong analysis, the two-dimensional serving model with the mixture of overflow and Poisson traffic can be resolved numerically for usual trunk group values. There are no special demands concerning the computer performances. Approximate methods should be avoided, too.

In more complex models, as in the case of our three-dimensional model for the mixture of two overflow traffic systems there are a critical number of equations for fewer trunk groups. Thus, the acceptance of approximate methods can be offered as one of the solutions. Starting from the two-dimensional model, the approximate solution for the three-dimensional model is proposed.

## 2. TRAFFIC PARAMETERS FOR THE FIRST MODEL

Fig. 1 shows the first model, with trunk groups with $c$ and $s$ trunks, stream parameters $\lambda_1$ and $\lambda_2$, and corresponding traffic $a_1$ and $a_2$. The losses in the primary trunk group are denoted by $b_c$, and in the secondary by $b_s$. The overflow component of traffic $a_1$ is with mean value $\lambda_1 b_c / \mu$ and serving intensity $\mu$, compared to the primary trunk group where this intensity equals one.

Solving the system of steady state equations using the function technique generating, we obtain a set of formulae for traffic parameters [1]. The formulae used in our analysis will be pointed out.

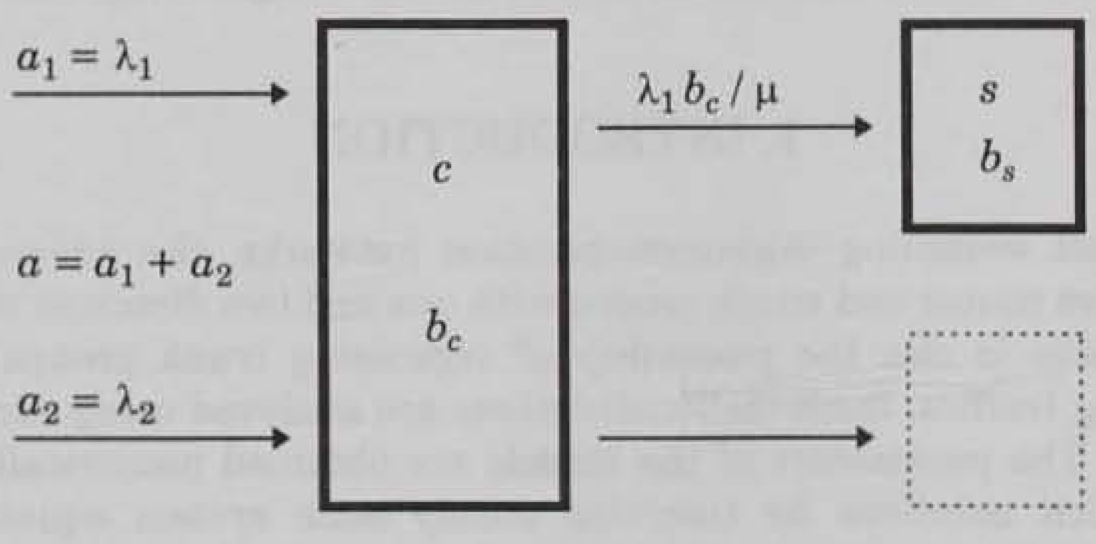


**Figure 1**. The model for overflow traffic components with changed serving intensity

The losses in the primary trunk group can be obtained using the First Erlang formula, i. e.

$$b_c = B(c, a = a_1 + a_2) = \frac{R_0}{R_1}. \tag{1}$$

The auxiliary parameters $R_{\mu n}$ are of the form

$$R_0 = \frac{a^c}{c!} \tag{2}$$

and

$$R_{\mu n} = R_0 + \sum_{i=1}^{c} \binom{\mu n + i - 1}{i} \frac{a^{c-i}}{(c-i)!}, \quad n \geq 1 \tag{3}$$

while for noninteger $\mu$, they are

$$\binom{\mu n + i - 1}{i} = \frac{(\mu n + i - 1)\ldots\mu n}{i!}. \tag{4}$$

For stream call losses with parameter $\lambda_1$, we have

$$b_{cs} = \frac{\dfrac{(a_1/\mu)^s}{s!} \prod_{i=0}^{s} \dfrac{R_{\mu i}}{R_{\mu i+1}}}{\sum_{n=0}^{s} \dfrac{(a_1/\mu)^{s-n}}{(s-n)!} \prod_{i=n+1}^{s} \dfrac{R_{\mu i}}{R_{\mu i+1}}} \tag{5}$$

under the condition

$$\prod_{i=s+1}^{s} (\bullet) = 1. \tag{6}$$

The call loss in the secondary trunk group is

$$b_s = \frac{b_{cs}}{b_c} = b_{cs} \frac{R_1}{R_0}, \tag{7}$$

while the time loss, as the probability of the occupancy of all trunks in the secondary trunk group has the form

$$b_t = b_{cs} \frac{R_{\mu s+1}}{R_{\mu s}}. \tag{8}$$

Beside the mean value of the overflow part of the traffic,

$$m_1 = \frac{\lambda_1}{\mu} b_c \tag{9}$$

its variance can be determined, too:

$$v_1 = m_1 \left( \frac{\lambda_1}{\mu} \frac{R_\mu}{R_{\mu+1}} + 1 - m_1 \right) \tag{10}$$

as well as the peakedness factor $z_1 = v_1 / m_1$.

The dependence of both traffic systems on trunk group values which change for various serving intensities in the secondary trunk group, is illustrated in Fig. 2.

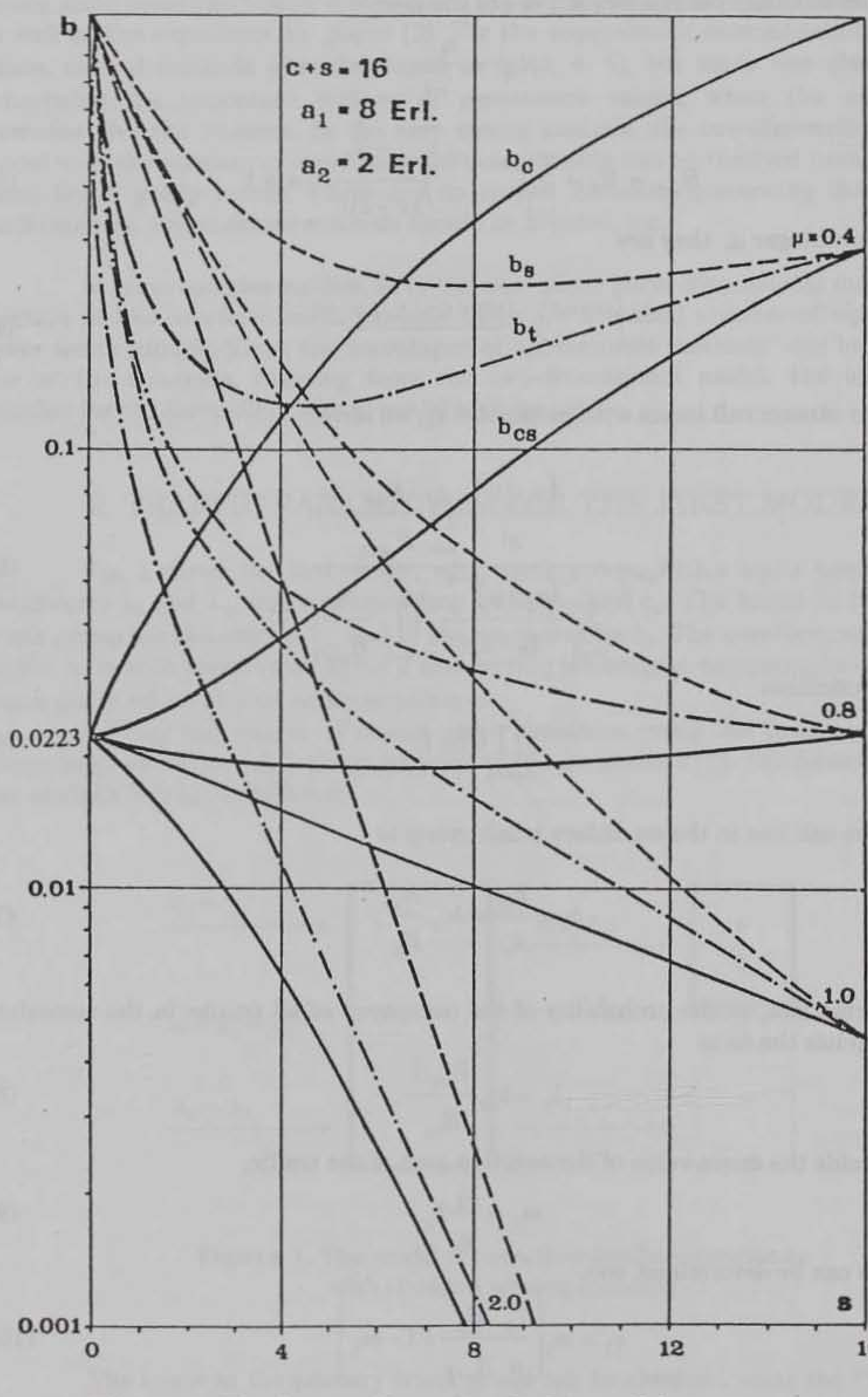


**Figure 2**. The change of losses for the first model using different serving intensities

It can be shown that the previous formulae hold for overflow traffic from the second Poisson traffic, when we have some other serving intensity in the secondary trunk group. It is clear that the solution is generalized to more overflow components from the common primary trunk group.

## 3. SECOND MODEL TRAFFIC PARAMETERS DETERMINATION

In the second model, we have a serving problem concerning the mixture of overflow and Poisson traffic in the secondary trunk group, while the serving intensity is different compared to the primary trunk group (Fig. 3.).

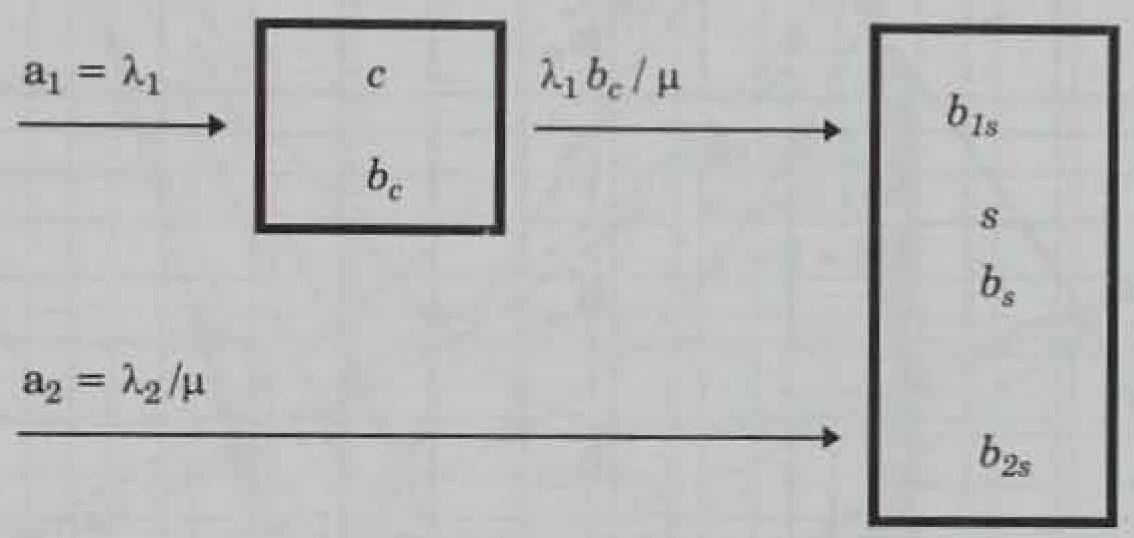


**Figure 3.** Model for the mixture of overflow and Poisson traffic

In this model there is no solution in explicit form for the stated probabilities and losses. Thus, it is necessary, to solve numerically the system of $(c+1)(s+1)$ steady state equations. The iterative procedure has been shown to be the most convenient, because of the real trunk groups volume as well as other reasons.

Some components typically have different losses in the secondary trunk group, $b_{1s}$ and $b_{2s}$, as shown in Fig. 4., where the losses for this model are presented by broken lines and (″). The losses for the first model are represented using solid lines and (′). In both cases, the condition is that $\mu=1$. Here, it is convenient to observe the mutual relation between the mean loss in the secondary trunk group, $b_s''$, and the loss obtained by the equivalent random method, $b_{se}''$. In practice, the first one is approximated very often by the second one [6]. Notice that in order to find the equivalent trunk group and traffic, a special numerical procedure is developed. Instead of the Integral Erlang formula, a qualitative recurrence formula for the noninteger trunk group is used [2].

The importance of Fig. 4. lies in the fact that the two models can be compared from the loss point of view. Thus, the total trunk group, which is always the same, can be characterized. It can be easily seen that for the same loss in one traffic system in the corresponding trunk group organization, the loss for the second traffic system is smaller in the case of the first model. The shaded area, is of importance for the two losses. This phenomenon is of interest for the case of such a trunk group organization that some traffic are served with a different serving quality.

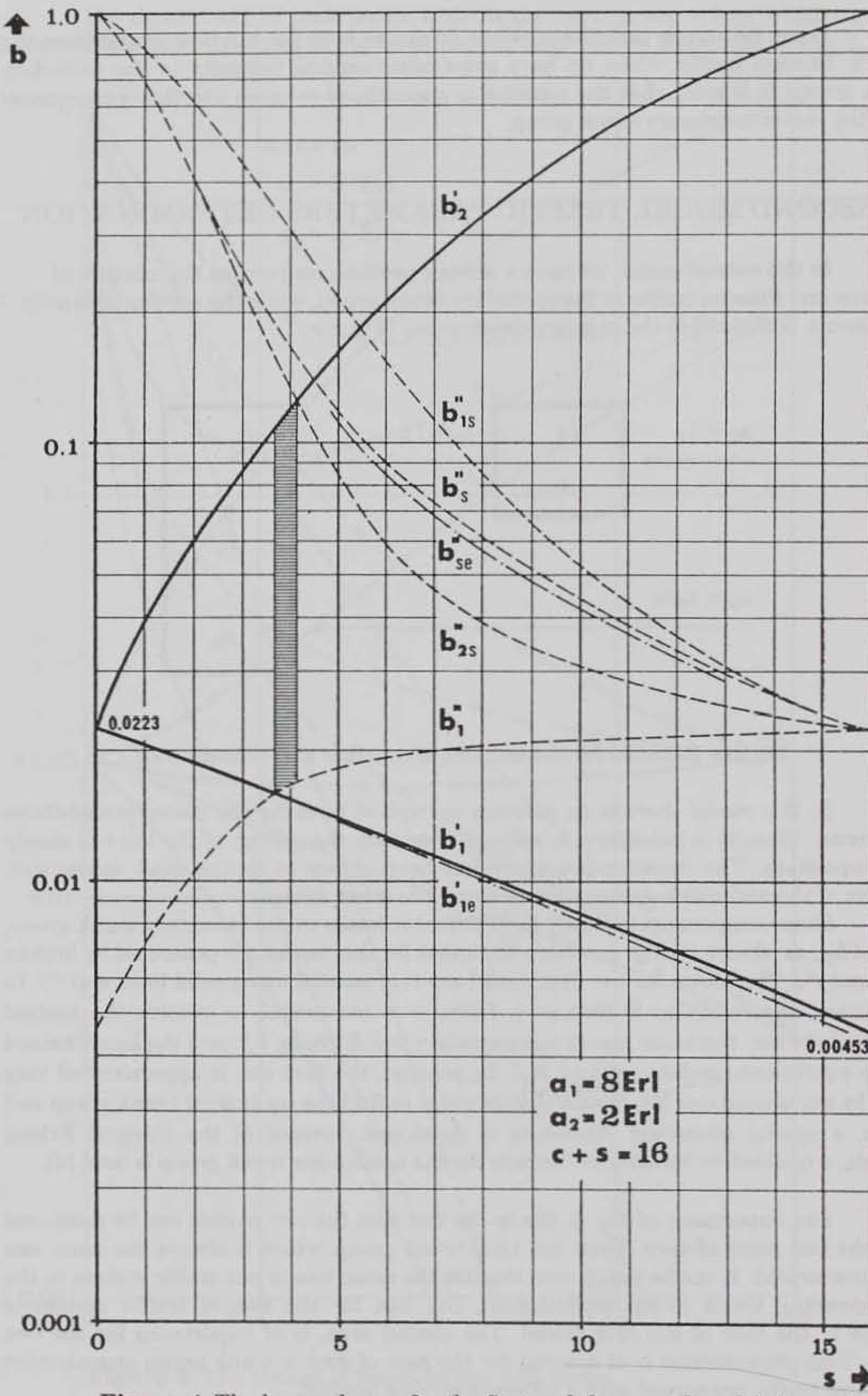


**Figure 4**. The losses change for the first and the second model, for $\mu=1$

The dependence of second model losses versus trunk groups values as well as the serving intensity in the secondary trunk group is for a constant total trunk group, illustrated in Fig. 5.

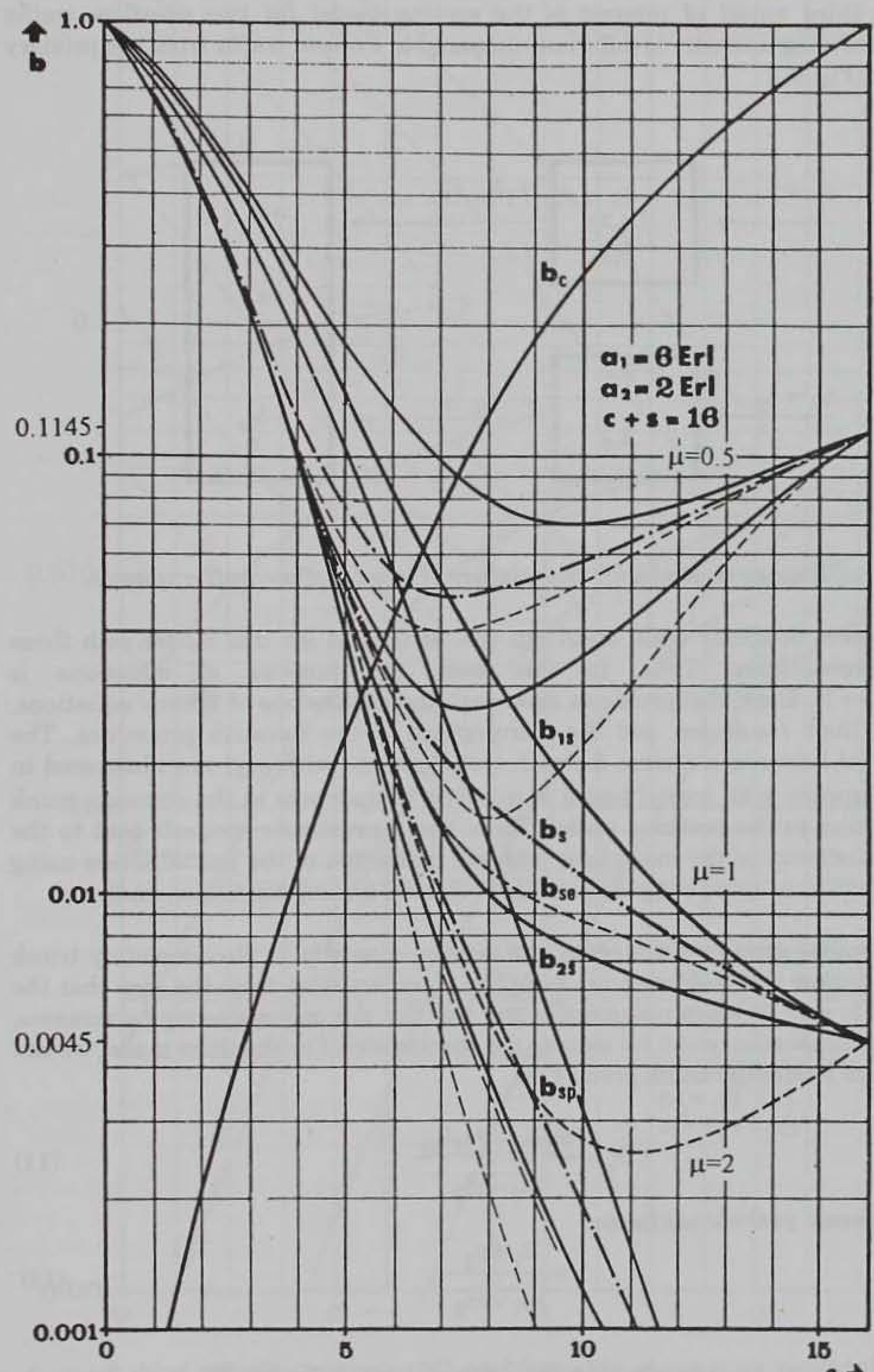


**Figure 5**. Second model loss changes, for different values of μ

# 4. LOSS DETERMINATION POSSIBILITIES FOR TWO OVERFLOW TRAFFIC SYSTEMS

The third model of interest is the serving model for two overflow traffic systems. The serving intensity is different compared to Poisson traffic from the primary trunk groups (Fig. 6.).

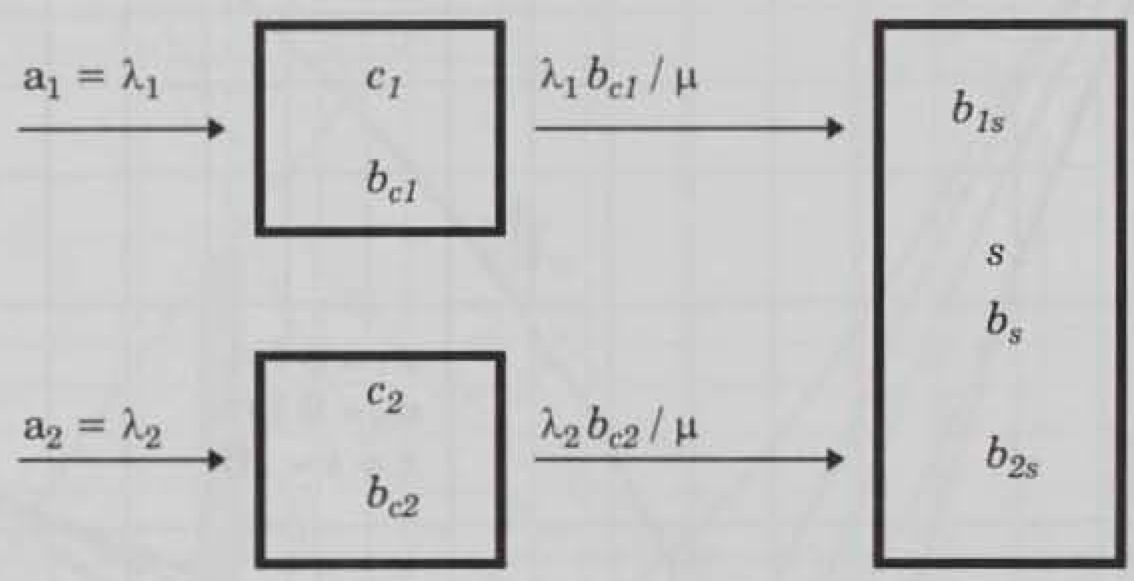


**Figure 6.** Model for the mixture of two overflow traffic systems

A system of steady state equations can be formed for this model with three parameter probabilities. Thus, in this case, the number of equations is $(c_1+1)(c_2+1)(s+1)$. Here, real problems arise with the dimensions of system equations, the speed of their resolution and the convergence of the iterative procedure. The changes in total losses in a system for each traffic system with $\mu=1$ are illustrated in Fig. 7. The situation with partial losses of overflow components in the common trunk group $s$ is similar to the previous model. Thus, the approximate methods lead to the approximate discovery of the mean loss, with the correction of the partial losses using simpler proportions or more complex formulae obtained a using regression analysis.

Our model presumes the change in serving intensity in the secondary trunk group, which makes the problem more complex. Here, we start from the idea that the second model is easy to solve numerically and that for the corresponding parameters, can represent the starting point for solving the third model. For the third model we can define the mean loss in the trunk group $s$ as

$$b_s = \frac{m_1 b_{1s} + m_2 b_{2s}}{m_1 + m_2} \tag{11}$$

as well as the mean peakedness factor

$$z = \frac{v_1 + v_2}{m_1 + m_2} \tag{12}$$

while the variance $v_1 = m_1 z_1$ can be obtained from (10), for $a_2=0$. Similar holds for $v_2$.

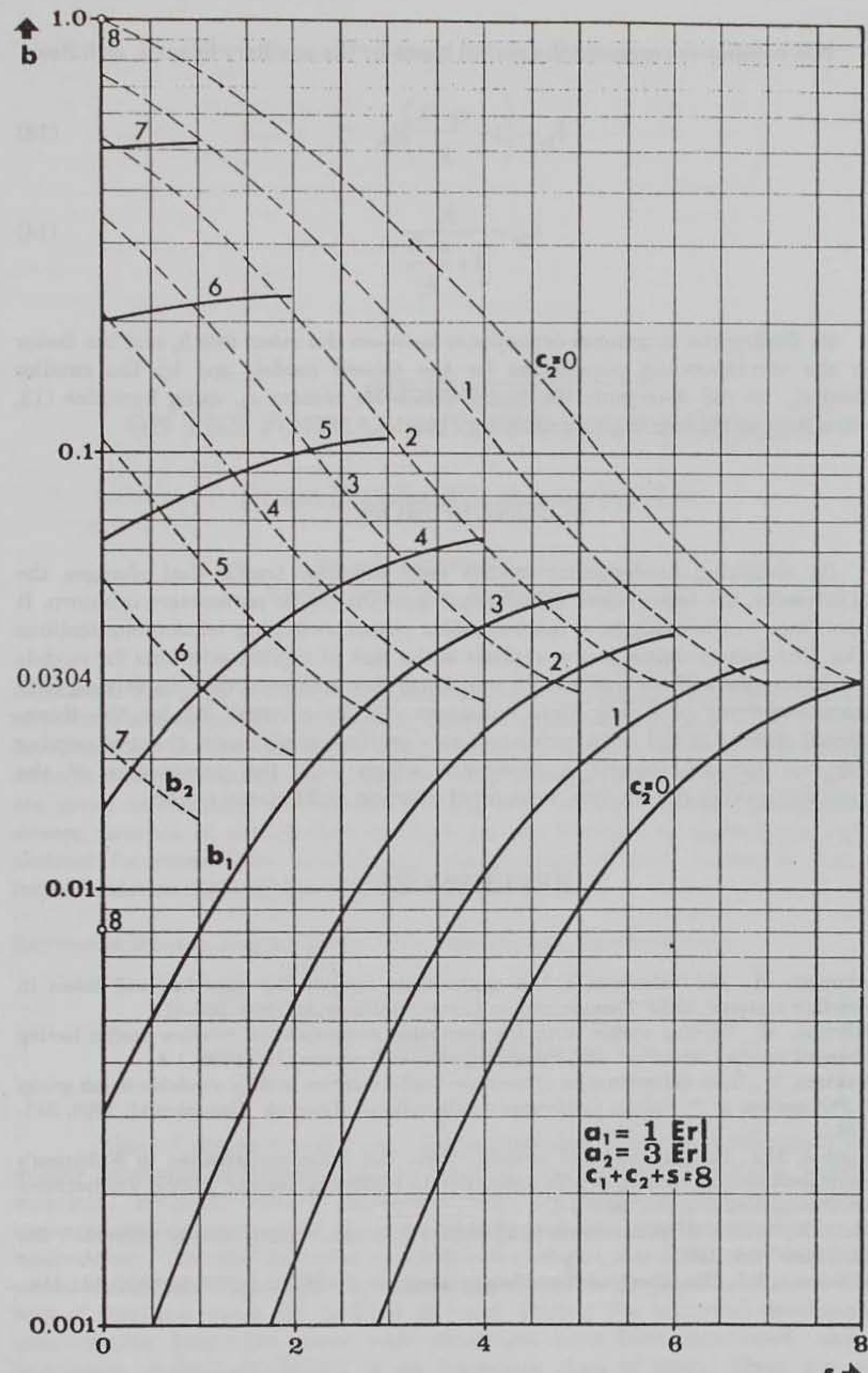


**Figure 7.** Losses changes in the third model for $\mu=1$

It is possible to represent the partial losses by the auxiliary factor $k$, as follows

$$b_{1s} = \left(1 + \frac{z_1 - 1}{k}\right) b_{2s} \tag{13}$$

$$b_{2s} = \frac{b_s}{1 + \frac{z-1}{k}} \tag{14}$$

By finding the functional dependence between the mean loss $b_s$ and the factor $k$ from the corresponding parameters for the second model, and by the smaller parameter $c_2$, we can determine the partial losses for greater $c_2$, using formulae (13, 14). Initial tests in this sense give satisfactory results.

## 5. CONCLUSION

By analyzing fundamental models with overflow traffic that changes the serving intensity, the influence of this change upon the traffic parameters is shown. It has importance in the analysis of more complex circuit switching telecommunications networks. The basic problem in the analysis is the lack of explicit solutions for models with a mixture of overflow traffic. The numerical iterative procedure in solving two-dimensional systems of steady state equations can be adopted. As for the three-dimensional serving model for a mixture of two overflow traffic with changed serving intensity, we can recommend a procedure which uses the parameters of the corresponding serving system with mixture of overflow and Poisson traffic.

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