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UNIFIED TREATMENT OF SUBNETWORKS IN SYMBOLIC ANALYSIS OF LINEAR ELECTRIC CIRCUITS AND SYSTEMS

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Abstract: A comprehensive review of the *S*-parameter theory is given from the viewpoint of symbolic analysis. *S*-parameters are proposed to characterize arbitrary networks in the most general sense. An algorithm is developed to calculate *S*-parameters symbolically. The original program SPAR is presented, which is intended for automated computer-aided symbolic derivation of *S*-parameters of time-invariant linear networks. The original program SALEC is used as an engine in SPAR to find the circuit response symbolically. A real life example is given to illustrate the application of symbolic analysis in the field of automated microwave network analyzer measurements.

Keywords: S-parameters, symbolic analysis, SALEC, SPAR.

1. INTRODUCTION

At lower frequencies classical electronic devices can be modeled by time-invariant linear electric networks (TILEN) containing lumped elements. Usually, two-port devices are of much greater interest. They are described by basic (primary) parameters: immitance (z and y), hybrid (h and k) and transmission (a or ABCD), or by derived (secondary) parameters: input (output) immitance, gain, attenuation etc.

The numerical analysis of TILEN is performed by computer programs similar to SPICE [12] in the time domain by solving state equations [14], or in the complex (frequency) domain by applying modified nodal analysis (MNA) [15].

At higher (microwave) frequencies lumped element modeling is not adequate because elements dimensions are comparable to the signal wavelength. To characterize these devices the most suitable (and from the measurement viewpoint the only possible) are *S*-parameters (*scattering* parameters) [2]. *S*-parameters of time-invariant linear electric networks, containing transmission lines and waveguides, are numerically determined by software tools such as: COMPACT [3], TOUCHSTONE [4], LINPAR [5], LINRES [6] and MATPAR [7].

This paper presents a concise, comprehensive theoretical background, the complete procedure, and the program implementation for computer-aided symbolic evaluation of *S*-parameters (named SPAR) of time-invariant linear electric networks (TILEN) containing uniform transmission lines and waveguides.

Amongst several modern tools for symbolic math [10] *Mathematica*® [17] was chosen, because it appears in the kernel of some operating systems [1] and supports object-oriented programming along with functional and procedural (structured) [11] programming.

A program entitled SALEC has been developed (which functionally incorporates the module SPAR) for symbolic analysis of time-invariant linear electric circuits in the complex domain (*s*-domain), in the environment of the expert system *Mathematica*® [16].

This program has syntax similar to SPICE, but operates on symbolic data. A brief description of this program is given in Section III and some illustrative examples are introduced in section IV.

It is shown that *S*-parameters are the most suitable for unified characterization of linear networks in the symbolic analysis of TILEN.

2. THE CONCEPT OF *S*-PARAMETERS

Current-controlled linear multiport networks are described by the matrix relation $U = zI$ or in the expanded form

$$\underline{U}_j = \sum_{k=1}^{N} \underline{z}_{jk}\underline{I}_k,\ j = 1,2,\ldots,N,\ U = \left[\underline{U}_1,\underline{U}_2,\cdots,\underline{U}_j,\cdots,\underline{U}_N\right]^T,\ I = \left[\underline{I}_1,\underline{I}_2,\cdots,\underline{I}_j,\cdots,\underline{I}_N\right]^T$$

where U and I are vectors of port voltages and currents, assuming the standard reference directions, and

$$z = \begin{bmatrix} \underline{z}_{11} & \cdots & \underline{z}_{1N} \\ \vdots & \ddots & \vdots \\ \underline{z}_{N1} & \cdots & \underline{z}_{NN} \end{bmatrix}$$ is the open-circuited impedance matrix.

The parameter $\underline{z}_{jk}$ can be determined theoretically or experimentally by applying the test current source, $\underline{I}_k$, at port k, leaving other ports open-circuited, finding the voltage at port j, and applying the formula

$$\underline{z}_{jk} = \frac{\underline{U}_j}{\underline{I}_k},\quad \underline{I}_m = 0,\ m = 1,2,\ldots,N,\ m \neq k$$

For voltage-controlled linear multiport networks the corresponding relation is $I = yU$, where

$$y = \begin{bmatrix} \underline{y}_{11} & \cdots & \underline{y}_{1N} \\ \vdots & \ddots & \vdots \\ \underline{y}_{N1} & \cdots & \underline{y}_{NN} \end{bmatrix}$$ is the short-circuited admittance matrix.

The parameter $\underline{y}_{jk}$ can be determined by a dual procedure: applying the test voltage source, $\underline{U}_k$, at port k, leaving other ports short-circuited, and finding the current at port j and applying the formula $\underline{y}_{jk} = \frac{\underline{I}_j}{\underline{U}_k}$, $\underline{U}_m = 0$, $m = 1,2,\ldots,N$, $m \neq k$.

For hybrid-controlled ports, corresponding parameters (h or k) can be determined in a similar way: by short- or open-circuited corresponding ports.

To measure classical primary parameters (immitance, hybrid or transmission) at microwave frequencies, it is practically impossible to implement an open circuit (in waveguides) or a short circuit (in microstrip transmission lines).

A network port, defined as a pair of terminals, does not exist at microwave frequencies. Conductors forming a port exhibit unpredictable parasitic effects which depend on the conductor shape, placement and manner of attachment. A microwave network port is the *cross section* of a transmission line or waveguide; this transmission line (waveguide) belongs to the network and serves to connect the network with the environment. [2].

Voltages and currents, at microwave ports, are not uniquely defined if a port is implemented in a waveguide. No measurement setup exists to find these quantities experimentally, even if they are defined, as in the case of transmission lines with TEM waves.

Consider a microwave network port defined in a uniform, lossless waveguide or transmission line, with a homogeneous dielectric, and a dominant mode. Under the term *uniform* line we shall mean a line with constant geometrical and electromagnetic properties, which are the same in any given cross section. At the same time, this means that permittivity, permeability and conductivity do not depend on the coordinate along which the line (guide) stretches. In the same manner, a uniform waveguide can be defined.

In the following text the term line (guide) will always mean a uniform transmission line (waveguide) with: a) homogeneous dielectric, b) dominant mode, c) negligible losses that do not change the field distribution significantly for the mode being guided.

The process at a port is viewed as a superposition of two progressive waves. The wave traveling towards the network is called *incident*, and the other one, traveling in the opposite direction, is designated as *reflected*. The basic measurable parameter, specifying a wave at a port, is its *average power*.

Instead of voltages and currents, *complex wave signals* are introduced, and defined as complex quantities whose: a) module squared equals the average power carried by the wave, b) argument (phase) equals the phase of the tangent component of the electrical field vector at the port. For some N port network, by definition, the following holds: $a_k^2 = P_{inc,k}$, $b_k^2 = P_{ref,k}$, $P_k = a_k^2 - b_k^2$, $P = \sum_{k=1}^{N} P_k$

where: k is the port number (k=1,2,...,N), a_k is the magnitude of the complex wave signal $\underline{a}_k$ of the incident wave, b_k is the magnitude of the complex wave signal $\underline{b}_k$ of the reflected wave, $P_{inc,k}$ is the incident wave average power, $P_{ref,k}$ is the reflected wave average power, P_k is the input average power at port k, and P represents the total input average power.

For transmission lines, voltages and currents can be uniquely defined. Relations exist between voltages and complex wave signals involving characteristic impedance. $U_{inc}^2 = P_{inc} Z_c$, $U_{ref}^2 = P_{ref} Z_c$, $\underline{a} = \dfrac{\underline{U}_{inc}}{\sqrt{Z_c}}$, $\underline{b} = \dfrac{\underline{U}_{ref}}{\sqrt{Z_c}}$

For waveguides, voltages are not uniquely defined. The above relations do not have a physical background. Formally, it is possible to associate an *equivalent transmission line* to a waveguide. The characteristic impedance of the line should equal the wave impedance for the mode assumed. Due to the assumption that the lines (guides) are lossless, all characteristic and wave impedances are real and positive. If at a (physical) port other modes, or orthogonal elliptically polarized waves exist, except the dominant mode, an independent electrical (logical) port exists for each specific wave. The independence of the ports follows from the principle of the superposition of average powers of particular modes or elliptically polarized waves. This holds for both incident and reflected waves. [2].

In general, incident and reflected waves can exist at all ports. For TILEN without independent sources, according to the principle of superposition, reflected waves are a linear homogeneous combination of incident waves $\underline{b}_j = \sum_{k=1}^{N} \underline{S}_{jk} \underline{a}_k$, $j = 1,2,\ldots,N$ The coefficients, $\underline{S}_{jk}$, are complex quantities and are called S-parameters or scattering parameters. In matrix form:

$$b = Sa,\quad a = \begin{bmatrix} \underline{a}_1 \\ \vdots \\ \underline{a}_N \end{bmatrix},\quad b = \begin{bmatrix} \underline{b}_1 \\ \vdots \\ \underline{b}_N \end{bmatrix},\quad S = \begin{bmatrix} \underline{S}_{11} & \cdots & \underline{S}_{1N} \\ \vdots & \ddots & \vdots \\ \underline{S}_{N1} & \cdots & \underline{S}_{NN} \end{bmatrix}$$

The parameter $\underline{S}_{kk}$ represents the reflection coefficient at port k when other ports are matched and is called the intrinsic reflection coefficient of port k. The parameter $\underline{S}_{jk}$ represents the transmission coefficient from port k (second index) to port j (first index).

This parameter can be determined analytically (numerically) or experimentally if, at port k, a test source is applied, other ports are matched, the complex incident wave signal is determined at port k, reflected at port j, and the following formula is applied:

$$\underline{S}_{jk} = \frac{\underline{b}_j}{\underline{a}_k}, \quad \underline{a}_m = 0,\ m = 1,2,\ldots,N\ ,\ m \neq k.$$

Characteristic impedances of transmission lines and equivalent lines for which the ports are defined are called *nominal or reference* port impedances. Their standard value is 50 Ω. In some cases (commercial TV systems) the 75 Ω value is used.

Relations between complex wave signals, nominal (reference) impedances, voltages and currents are given by: $\underline{a}_k = \dfrac{\underline{U}_k + Z_{ck}\underline{I}_k}{2\sqrt{Z_{ck}}}$, $\underline{b}_k = \dfrac{\underline{U}_k - Z_{ck}\underline{I}_k}{2\sqrt{Z_{ck}}}$, $k = 1,2,\ldots,N$

According to the idea in [2], in order to determine S-parameters, the simplest way, is to connect a real voltage test source at port k, $\underline{U}_g = 2\,V$, impedance $\underline{Z}_g = Z_{ck}$, match other ports, and find port voltages (in volts). In that case the following holds [2]:

$$\underline{S}_{kk} = \underline{U}_k - 1,\quad \underline{S}_{jk} = \underline{U}_j\sqrt{\frac{Z_{ck}}{Z_{cj}}},\quad j = 1,2,\ldots,N\ ,\ j \neq k$$

In practice, characteristic port impedances are almost always reference impedances for the evaluation of S-parameters. Strictly speaking, reference port impedances can differ from characteristic impedances of (equivalent) lines on which ports are defined. In the most general case for TILEN containing lines and guides a real positive number (resistance in nature) must be specified for each port and it is called reference (or nominal) impedance. S-parameters are determined with respect to these numbers. If S-parameters are known for one set of reference impedances, simple transformations can be applied to obtain S-parameters for another set of nominal impedances [2].

If a TILEN contains independent sources, according to the superposition principle, the following equation will hold $b = S\,a + b_g$, where S represents a scattering matrix of the same network with independent sources disconnected, b_g is a column matrix of known complex wave signals generated within the network and emanating from it when all the ports are matched.

The parameter b_g is the direct consequence of the independent sources inside the network. It can be calculated or measured if all the ports are matched, port voltages are found, and the following formula is applied: $\underline{b}_{gk} = \dfrac{\underline{U}_k}{\sqrt{Z_{ck}}}$, $\underline{a}_k = 0$, $k = 1,2,\ldots,N$

Each TILEN with regular elements, including lines and guides, always has S-parameters. This holds even in the case when a TILEN contains ideal operational amplifiers (nullors), if feedback exists between their ports.

In order to meet the requirements for the symbolic analysis of a TILEN of known S-parameters, it is necessary to form an equation to characterize the TILEN in terms of voltages and currents. It can be shown that the following equation holds:

$$Y_c^{1/2}U - Z_c^{1/2}I = S\left(Y_c^{1/2}U + Z_c^{1/2}I\right) + 2b_g$$

$$Y_c^{1/2} = \begin{bmatrix} \sqrt{Y_{c1}} & 0 & \cdots & 0 \\ 0 & \sqrt{Y_{c2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{Y_{cN}} \end{bmatrix}, U = \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \vdots \\ \underline{U}_N \end{bmatrix}, I = \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \vdots \\ \underline{I}_N \end{bmatrix}$$

$$Z_c^{1/2} = \begin{bmatrix} \sqrt{Z_{c1}} & 0 & \cdots & 0 \\ 0 & \sqrt{Z_{c2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{Z_{cN}} \end{bmatrix}, \quad b_g = \begin{bmatrix} \underline{b}_{g1} \\ \underline{b}_{g2} \\ \vdots \\ \underline{b}_{gN} \end{bmatrix}$$

where: Z_{ck}, $Y_{ck} = \dfrac{1}{Z_{ck}}$ are nominal impedance, admittance, of port k.

In the special case, when there are no independent sources, the z matrix can be calculated from the known S matrix: $z = Z_c^{1/2}(1-S)^{-1}(1+S)Z_c^{1/2}$, (1 is the identity matrix).

3. PROGRAM IMPLEMENTATION OF S-PARAMETER EVALUATION IN A SYMBOLIC FORM

According to the theory previously presented, an algorithm was established to find S-parameters and the program module SPAR is written to determine the S matrix of a TILEN in a symbolic form. In the particular program implementation real voltage test sources are replaced by real *current* (Norton) test sources, in order to avoid adding extra nodes. The network whose S-parameters are to be found is specified in a plain text (ASCII) file containing: the number of nodes, the list of components and their interconnection and parameters, the number of components, the number of ports, port locations and their nominal impedances. The program module SPAR automatically creates all the test circuits required and determines the corresponding S-matrix column by column. Its nucleus (engine) for symbolic analysis of the linear electric circuit is the program SALEC [16].

The source code of the module SPAR, solving networks *without* independent sources, in the programming environment of the expert system *Mathematica*®, follows:

```
Print["SPAR 1.0 Dejan Tosic (C)1995"];
<< SALEC23.M
BeginPackage["SALEC`"]
SPAR::usage = "SPAR[netfile]"
SPAR[netfile_String]:=Block[{},
  Clear[numberofnodes, component, numberofcomponents];
  Clear[numberofports, port];
  Clear[irows, icols, irowsm, icolsm, iport];
  Clear[smatrix]; Get[netfile];
  For[iport = 1, iport <= numberofports,
    iport++; numberofcomponents++,
    component[numberofcomponents+1] =
    Flatten[{Z,port[iport]}]
    ];
  numberofcomponents++;
  smatrix = Table[ 0, {irowsm,1,numberofports},
              {icolsm,1,numberofports}];
  For[icols = 1, icols <= numberofports, icols++,
    component[numberofcomponents] =
    {I, port[icols][[2]], port[icols][[1]],
     2/port[icols][[3]]};
    Print["Intermediate response"];
    SALEC[""]; PrependTo[response,0];
    For[irows =1, irows <= numberofports, irows++,
      If[irows==icols, smatrix[[icols,icols]] =
      Simplify[response[[port[icols][[1]]+1]]-
           response[[port[icols][[2]]+1]]-1
          ],
      smatrix[[irows,icols]] = Simplify[
   Sqrt[port[icols][[3]]/port[irows][[3]]]*
    (response[[port[irows][[1]]+1]]-
     response[[port[irows][[2]]+1]])
                                    ]
      ]
     ];
   ]
  Print["S-parameters"]; Print[MatrixForm[smatrix]];
smatrix;
]
EndPackage[ ]
```

4. EXAMPLES: FINDING *S*-PARAMETERS OF A SYMMETRICAL POWER SPLITTER

A three-port resistive network, Fig. 1, was made out of three identical resistors of resistance R. The nominal impedances of the ports are Z. The network is matched at all three ports if $R=Z/3$. In that case diagonal elements of the corresponding S-matrix are zero and the off diagonal equal 1/2. The insertion loss equals 6 dB.

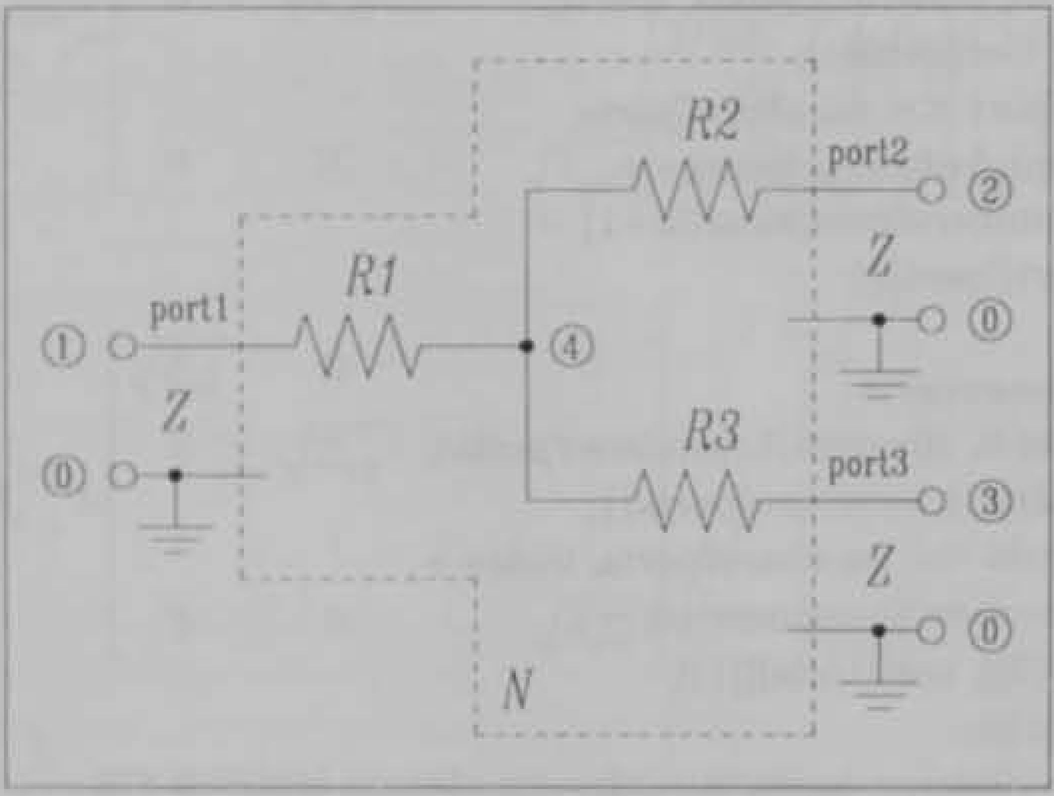

Figure 1. A symmetrical resistive power splitter.

The network shown above is specified in the file SPLITTER.SP.

```
(* A SYMMETRIC RESISTIVE POWER SPLITTER *)
numberofnodes = 4
component[1] = {"R", "R1", 1,4, R}
component[2] = {"R", "R2", 2,4, R}
component[3] = {"R", "R3", 3,4, R}
numberofcomponents = 3
port[1] = {1,0, Z}
port[2] = {2,0, Z}
port[3] = {3,0, Z}
numberofports = 3
```

Interactive execution of the module SPAR, with the above file as input results in the session log that follows (S-matrix printed by rows).

$$-1+\frac{2\,(\,3\,R+Z\,)}{3(\,R+Z\,)} \qquad \frac{2\,Z}{3(\,R+Z\,)} \qquad \frac{2\,Z}{3(\,R+Z\,)}$$

$$\frac{2Z}{3(R+Z)} \qquad -1+\frac{2(3R+Z)}{3(R+Z)} \qquad \frac{2Z}{3(R+Z)}$$

$$\frac{2Z}{3(R+Z)} \qquad \frac{2Z}{3(R+Z)} \qquad -1+\frac{2(3R+Z)}{3(R+Z)}$$

The analysis takes several seconds on a PC 486 DX CPU 50 MHz. The S-parameters determined are stored in the matrix (variable) *smatrix* and are available for further processing. As the first example of this post processing let us find the value of R which makes all the ports matched. The command line In[2] specifies an equation in R of the form $\underline{S}_{11}(R)=0$. The known result R=Z/3 appears as Out[2] line. Next, this value is substituted into the S-matrix found. Again, the expected (known) result appears; zeros on the main diagonal, 1/2 off diagonal. The insertion loss is determined in the last line In[5] and proves the value 6 dB.

```
In[2]:= Zmatch = Solve[smatrix[[1,1]]==0,{R}]

Out[2]= {{R -> Z/3}}

In[3]:= Smatch = smatrix /. Flatten[%]
```

Out[3]= $\{\{0,\frac{1}{2},\frac{1}{2}\},\ \{\frac{1}{2},0,\frac{1}{2}\},\ \{\frac{1}{2},\frac{1}{2},0\}\}$

In[4]:= MatrixForm[%]

Out[4]//MatrixForm = $\begin{matrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{matrix}$

```
In[5]:= insertionloss =
-20*Log[10,Smatch[[1,2]]] //N

Out[5]= 6.0206
```

ANALYSIS OF REFLECTION MEASUREMENTS USING THE NETWORK ANALYZER

The symbolic analysis tools SPAR and SALEC can be easily applied in the analysis of network analyzer reflection measurement test setups. In this example, we consider the Hewlett Packard 8754A Network Analyzer and its basic accessory, the HP 8502A Transmission/Reflection Test Set. We focus on the reflection measurements of one-port devices. The device under test is assumed to be passive and its reflection coefficient is to be measured [9].

The network analyzer (N/A) provides an RF signal to the test device (N/A output). The test set routes the signal to the device under test (DUT) and separates the incident and reflected signals at the test device. The incident signal is routed to N/A receiver R input, and the reflected signal is fed to the N/A receiver A input. These two signals are further processed in the N/A receiver to obtain the DUT reflection coefficient, its magnitude and phase.

First, an equivalent linear circuit, for the test setup involved is established. It is decomposed into simple blocks to emphasize different functions of the setup components. According to Fig. 2 it can be seen that the RF source (N/A output) is represented by a real Norton current generator, sub-circuit N1. The source resistance equals the reference (nominal) impedance Z which is 50 Ω. Both N/A inputs, the R-input for incident (N4) and A-input for reflected wave (N6), are also assumed to be equal to the reference impedance Z.

The complete reflection test setup is shown in Fig. 2.

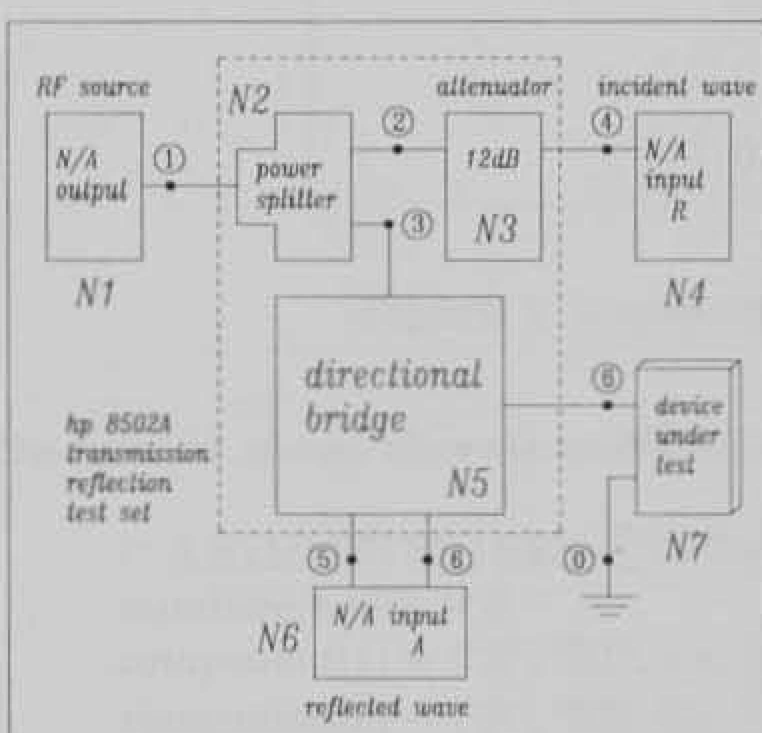

Figure 2. Reflection measurements setup.

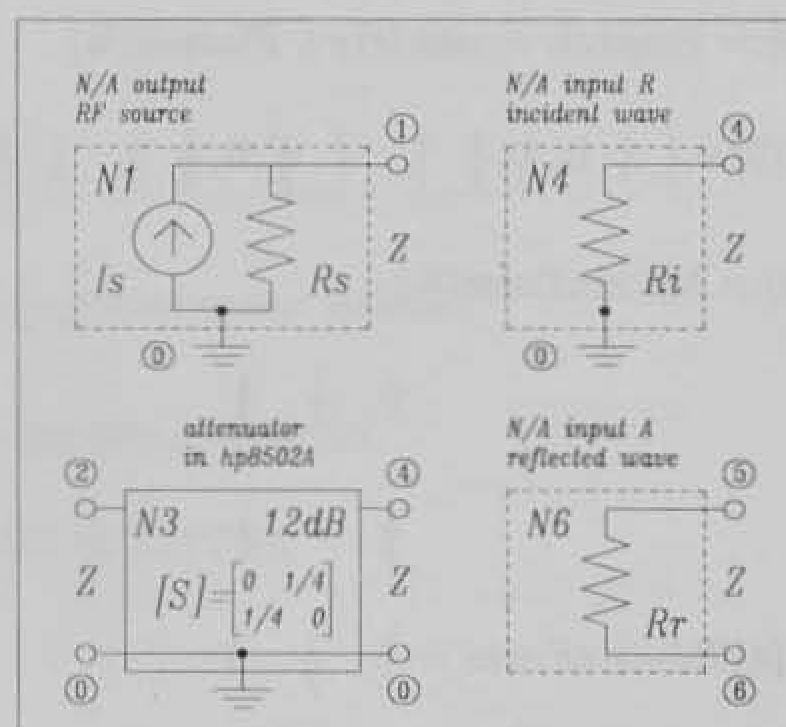

Figure 3. Equivalent sub circuits of the test setup

The 12 dB attenuator, N3, inside the HP 8502A is viewed as an ideal, symmetrical, double matched two-port attenuator of known S-matrix.

$$S_{atten} = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{4} & 0 \end{bmatrix}, \text{ attenuation [dB]} = -20\log_{10}(S_{21}) = 6$$

Besides the attenuator, the HP 8502A contains a two-way resistive power splitter (6 dB loss each path), Fig. 4, and a directional bridge (12 dB coupling), Fig. 5.

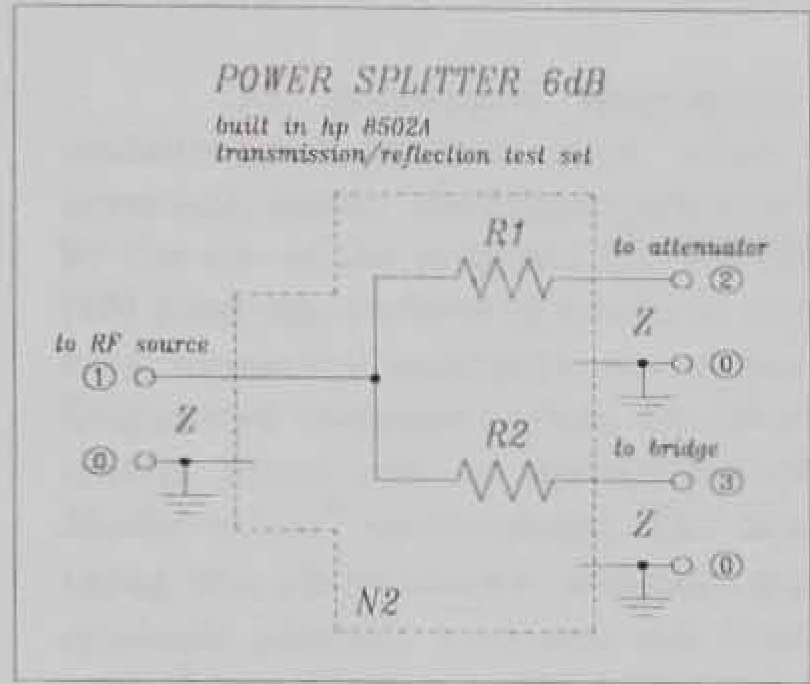

Figure 4. The two-way resistive power splitter subcircuit.

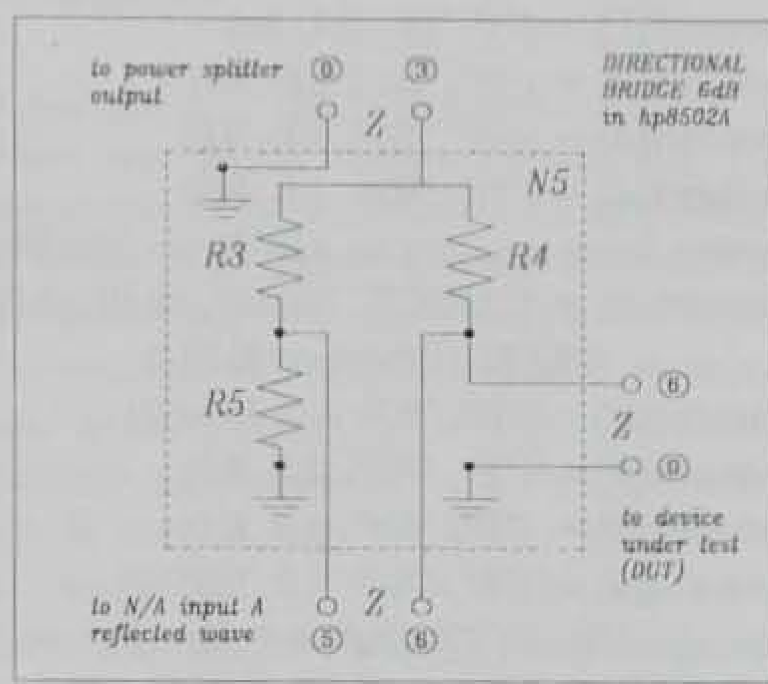

Figure 5. The directional bridge subcircuit.

The power splitter, N2, is modeled by identical resistors whose resistances equal the reference impedance Z: $R_1 = R_2 = Z = 50\,\Omega$, as shown in Fig. 4.

The directional bridge, N5, is represented by three identical resistors whose resistances equal the reference impedance Z: $R_3 = R_4 = R_5 = Z = 50\,\Omega$.

All the ports in the measurement setup have the same reference impedance Z (50 W). The device under test (DUT) is assumed to be passive one-port, N7, so its impedance can be expressed in terms of its reflection coefficient, $\underline{S}$, and vice versa, $\underline{Z}_{test} = Z(1+\underline{S})/(1-\underline{S})$. Network analyzer operation/measurement theory and practice state that, within the limits of component errors, the following formula holds: $\underline{S} = \dfrac{\underline{U}_{reflected\_at\_DUT}}{\underline{U}_{incident\_at\_DUT}} = \dfrac{\underline{U}_{at\_port\_A}}{\underline{U}_{at\_port\_R}} = \dfrac{\underline{V}_6 - \underline{V}_5}{\underline{V}_4}$

The measurement setup is symbolically evaluated by the SALEC program. The input file 8502A.S2, describing components and their interconnection, looks like this:

```
(* 8502A TRANSMISSION/REFLECTION TEST SET *)

Is = Us/Z; Rs = Z;                    (* RF source *)
R1 = Z; R2 = Z;                       (* power splitter *)
Satten = {0,1/4,1/4,0};               (* attenuator *)
RincR = Z;                            (* R input *)
R3 = Z; R4 = Z; R5 = Z;               (* bridge *)
RrefA = Z;                            (* A input *)
Ztest = Z*(1+S)/(1-S);                (* DUT *)
```

```
numberofnodes = 6
component[1] = {"I", "Is", 0,1, Is}
component[2] = {"R", "Rs", 1,0, Rs}
component[3] = {"R", "R1", 1,2, R1}
component[4] = {"R", "R2", 1,3, R2}
(* convert S-parameters to ABCD via S2ABCD function *)
component[5] = {"ANET", "atten", {2,0},{4,0},
          S2ABCD[Satten,{Z,Z}]}
component[6] = {"R", "Ri", 4,0, RincR}
component[7] = {"R", "R3", 3,5, R3}
component[8] = {"R", "R4", 3,6, R4}
component[9] = {"R", "R5", 5,0, R5}
component[10] = {"R", "Rr", 5,6, RrefA}
component[11] = {"R", "DUT", 6,0, Ztest}
numberofcomponents = 11
```

Execution of the SALEC program, with this file as input, yields:

```
In[1]:= Get["SALEC23.M"]; SALEC["8502A.S2"]
SALEC 2.3 Dejan Tosic (C)1995

       -8 Ug                          (2 + S) Ug
V1 = -----------              V5 = -(-----------------)
      -16 + S                           -16 + S

       -4 Ug                         -2 (1 + S) Ug
V2 = -----------              V6 = --------------------
      -16 + S                           -16 + S

       (4 + S) Ug
V3 = -(------------------)    Execution time = 1.71 Seconds
         -16 + S
                              (* node voltages are in the array result *)
          Ug
V4 = -(------------)          In[2]:= UrefA = result[[6]]
        -16 + S                              - result[[5]]

           S Ug                             Ug
Out[2] = -(-------------)     Out[3] = -(------------)
           -16 + S                        -16 + S

                              In[4]:= Stest = UrefA/UincR
In[3]:= UincR = result[[4]]   Out[4] = S
```

The above result shows that the symbolic analysis gives the same result as the previously presented theory.

5. CONCLUSION

At microwave frequencies S-parameters are preferred, from the measurement point of view, when devices are to be characterized. For time-invariant, linear, electrical networks S-parameters can be determined symbolically by the use of the program module SPAR, which is an inherent part of the SALEC [16] program. Instead of symbolic expressions for network parameters, as a special case, numerical constants can appear. This enables the calculation and plotting of frequencys response curves, amplitude and phase diagrams, etc. The resultant S-matrix found can be post-processed through an interactive or batch mode of *Mathematica*® environment. For instance, it can be approximated (symbolically) using the *Approximate* program module available in SALEC [13]. Due to the symbolic analysis approach the circuit/network analysis is performed only once, and then other specific (required) transformations and presentations of the response are performed. Typically, the same test setup can be (generally) evaluated symbolically, and then particular, measured S-parameters could be substituted for the corresponding devices (attenuators, splitters, bridges, connectors, lines, terminations, etc.), at particular frequencies.

The program module SPAR can be used in all cases where (time-invariant, linear, electric) device analysis, design, characterization and testing are of interest [8].

This paper presents a concise review and clear theoretical background for S-parameter symbolic analysis. It clarifies the equations required to handle networks characterized by scattering parameters. The SPAR algorithm and the complete source code listing are also given.

The SPAR/SALEC operation is demonstrated on two real-life examples and the whole interactive sessions are shown.

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