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PRODUCTION PLANNING IN MINES USING FUZZY LINEAR PROGRAMMING

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Abstract: *Mine production systems* (MPS) are characterized, more or less, by the following properties: uncertainty, subjectivity, impreciseness, polysemy, instability, indefiniteness and lack of data. From the aspect of modern studies these MPS mathematically belong to the class of fuzzy systems. With this in mind, the usage of fuzzy sets is recommended as a methodology that treats the impreciseness, indefiniteness and complexity of mine production systems in a satisfactory manner. Fuzzy modelling, discussed in this paper, is applied as a mathematical implement. The objective of the paper is to show production optimization in practice on the example of bauxite open pits where *fuzzy linear programming* (FLP) was used following the criterion per income. The variables in the criterion function are linguistic variables while both variables and variable restrictions are presented by *triangular fuzzy numbers* (TFN). The term "satisfactory income" presented through triangle fuzzy numbers was introduced instead of "maximizing the income". Sensitivity of the obtained solutions has been analysed.

Keywords: Mine production systems, uncertainty, fuzzy sets, fuzzy linear programming.

1. INTRODUCTION

Mining, is an enormous businesssystem. The dominant role has been given to the mine production system (MPS) which is particularly important for its influence on the overall running of the business. In both the mine production system and other systems/subsystems the presence of uncertainty, subjectivity, impreciseness and often polysemy has proved to be evident. Connections between particular attributes within the mine production systems or the mine production system together with the environment or other systems/subsystems in the business system differ according to their content and intensity. Both uncertainty in the mine production system and the complexity of the phenomena result from the specific and complex nature of the working medium, working conditions, interactive connections, investment tasks mining has an economic branch is characteristic for regarding to other branches etc.

Linear programming (LP) is one of the most frequently used methods of operational research in mining. Using linear programming the extreme values of objective functions are determined taking into account the fulfilment of limitating conditions. These values, in fact, represent optimum solutions. However, very often the precise data on certain input parameter values appearing within a determined mathematical model do not exist. On addition the uncertainty is obvious (change in working conditions, quality of mineral raw materials, economic conditions, income, etc.). The means that some problems emerging duringlinear (or dynamic) programming should be solved for the cases of environmental unceratinty. In order to solve this class of problems the theory on possibilities, namely the theory on fuzzy sets (Zadeh 1965, 1978), should be used as it formulates and solves the problem of optimization under conditions when objective functions and constraints faile to be precisely formulated. This concept was introduced by Belman and Zadeh (1970) through the criterion treating both the aim and limitation as fuzzy sets and defines a set that meets the demands of both the aim and limitations of fuzzy linear programming (FLP). It also allows fuzzyfication (transfer) when necessary as well as operations with defuzzyfied values when the need for fuzzyfied ones does not exist any longer. The relative phenomena can be described by expert evaluation. Consequently, the "vague" presentation of a phenomenon or a process is determined by fuzzy numedrical input data. The main problem of this approach lies in the subjectivity of the expert's estimation. In fuzzy linear programming each obtained solution is characterized by its own affiliation level being of importance during the decision making process. Ćirović (1991) used fuzzy linear programming to solve the optimum qualification structure of task forces in mining. These experiences together with new knowledge within the domain of fuzzy linear programming have been of significant importance for our further studies. Part of this research is presented in this paper.

2. PROBLEM: OPTIMUM PRODUCTION PLAN

Setting up the linear problem

Using fuzzy linear programming in mining production planning is illustrated by a bauxite basen that includes five mines with different bauxite quality. The price of bauxite at the plate for homogenization, conditioned by quality, transport and exploatation costs, considerably varies. The required bauxite production amounts to 3000 (t/shift). Numerous data referring to the relative problem are presented in Table1 and the disposition of the system's facilities is given in Fig. 1.

PROBLEM: Determine the production of each mine separately, under the condition that maximum income is realized and that bauxite at the plate for homogenization is characterized by a module=8 (the module represents the Al_2O_3 / SiO_2 ratio).

Table 1

Mine	Quality of bauxite			Capacity	Price of
	Al_2O_3 (%)	SiO_2 (%)	module Al_2O_3 / SiO_2	(t/sm.)	bauxite (din/t)
1	58	4	14,5	1000	230
2	58	6	9,7	500	200
3	53	12	4,4	750	170
4	58	5	11,6	500	240
5	52	12	4,3	800	160

3. LP MODEL FOR THE PROBLEM

Mathematical model for the problem

Selection variables

x1 productio of bauxite in the first mine
x2 productio of bauxite in the second mine
x3 productio of bauxite in the third mine
x4 productio of bauxite in the fourth mine
x5 productio of bauxite in the fifth mine

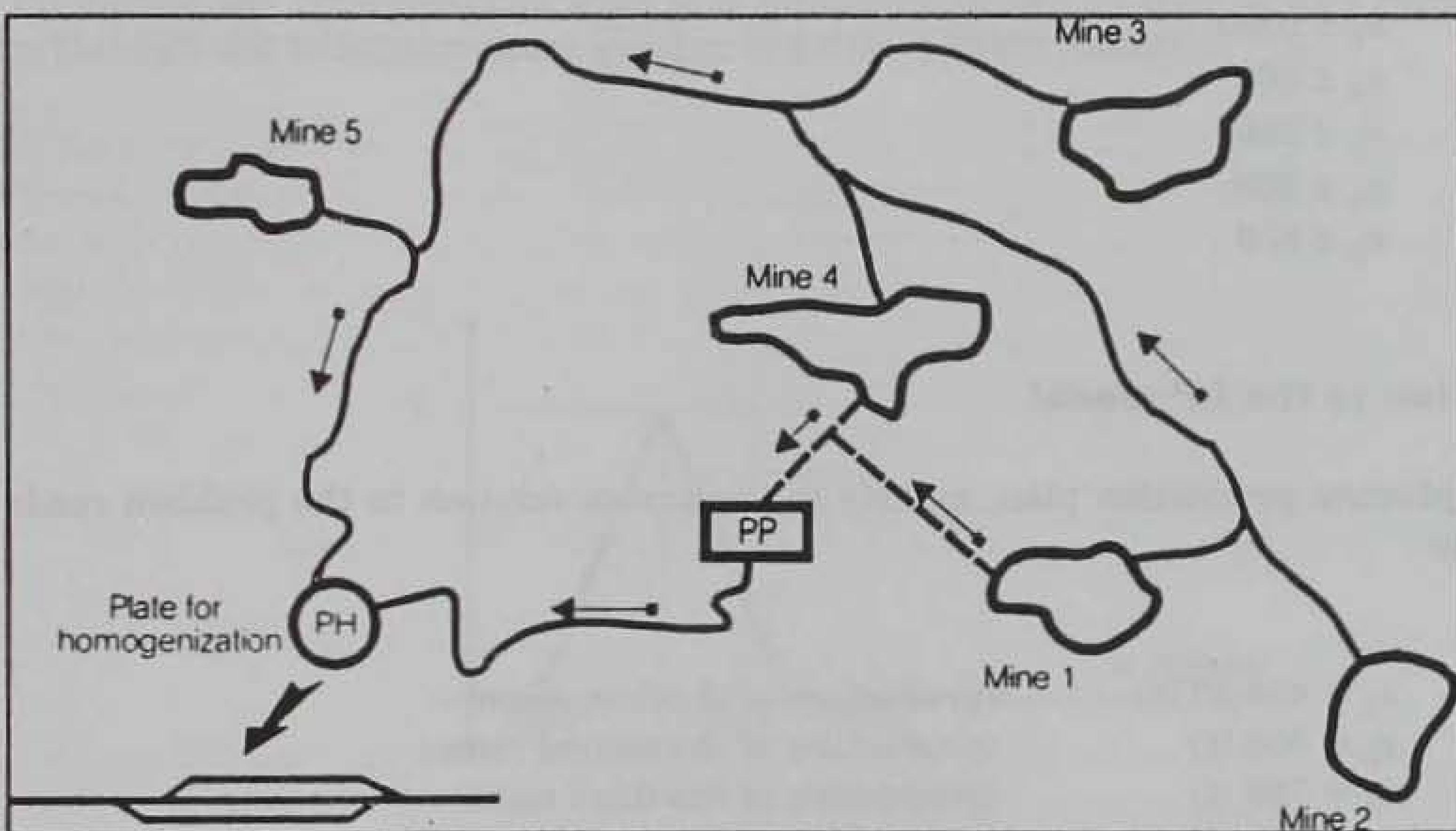

Figure 1: Schematic presentation of the disposition of the system's facilities

Selection of optimum criterion

MAXIMUM INCOME is taken as an optimum criterion

$$Z \to (MAX)$$

Setting the objective function

$$(\max) Z = 230x_1 + 200x_2 + 170x_3 + 240x_4 + 160x_5 \quad (1)$$

$$\text{for: } x_i \geq 0 \qquad (i = 1, 2, 3, 4, 5) \quad (1a)$$

Mathematical formulation of constraints

Bauxite production from the whole system should amount to 3000 (t/shift) and it therefore follows

$$x_1 + x_2 + x_3 + x_4 + x_5 = 3000 \tag{2}$$

As the bauxite module should be m=8 at the outflow of the plate for homogenization, the second constraint follows:

$$14,5x_1 + 9,7x_2 + 4,4x_3 + 11,6x_4 + 4,3x_5 = 24000 \tag{3}$$

The constraints according to production possibilities of certain mines will be as follows:

$$x_1 \leq 1000 \tag{4}$$
$$x_2 \leq 500 \tag{5}$$
$$x_3 \leq 750 \tag{6}$$
$$x_4 \leq 500 \tag{7}$$
$$x_5 \leq 800 \tag{8}$$

Solution to the LP model

The optimum production plan, namely the optimum solution to the problem reads as follows:

x_1 = 458,33 (t) (production of the first mine)
x_2 = 500 (t) (production of the second mine)
x_3 = 750 (t) (production of the third mine)
x_4 = 500 (t) (production of the fourth mine)
x_5 = 791,66 (t) (production of the fifth mine)

The optimum value for the objective constraint function is

$$Z_{max} = 579583{,}33 \text{ din (for 3000 t of bauxite/shift)}$$

namely,

$$Z_{max} = 193{,}19 \text{ (din/t bauxite)}$$

Thus, on the basis of the above we can conclude that the realization of the maximum income amounting to 193,19 (din/t) requires the quality of bauxite at the outflow of plate of homogenization to be m = 8 as well as the fulfilment of other restrictions: the first mine should produce 458,33 (t) bauxite in a shift, tre second mine 500 (t), the third mine 750 (t), the fourth mine 500 (t) and fifth mine 791,66 (t).

4. FUZZY LINEAR MODEL

Mathematical model of a fuzzy linear problem

When production planning is concerned, some uncertainties exist regarding

a) the possibilities of open pit capacity realization
b) the possibility of income realization
c) the possibility of realizing the capacity of a shift

This is why the linear programming problem should be transformed into a fuzzy linear programming model. Intead of maximizing the income the concept of satisfactory income has been introduced and presented by a triangular fuzzy number (TFN) having the corresponding level of h affiliation ($0<h<1$). Satisfactory income is given through the triangular fuzzy number (540000, 570000, 600000).

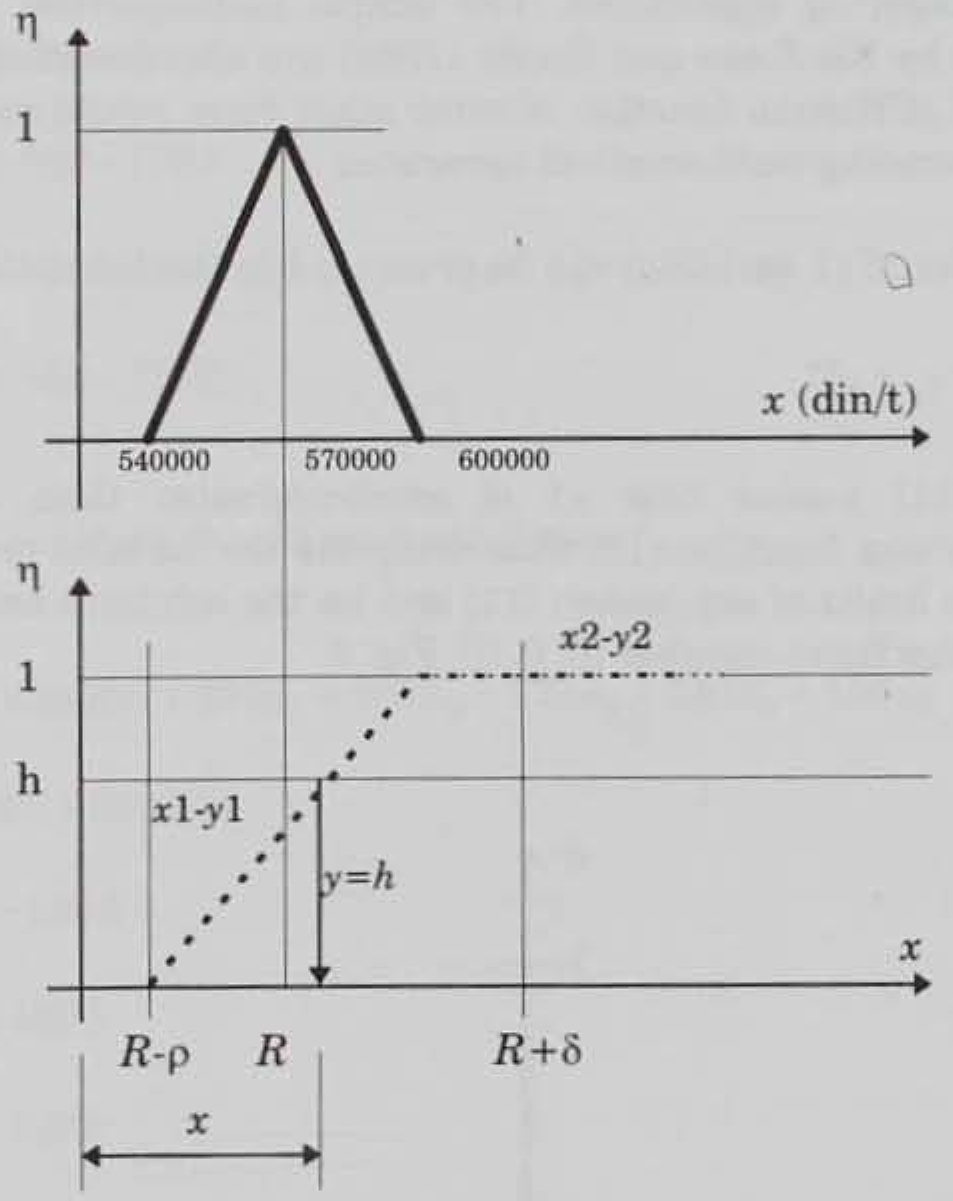

Figure 2: Fuzzy numbers characterizing "satisfactory income" and "higher tha satisfactory income "

The fuzzy number presented in Fig. 2 is analogous to the fuzzy number "less than satisfactory costs" introduced by Perinchery and Kikuchi (1990) with the aim of solving similar fuzzy linear problem. It is obvious from Fig. 2 that the value of parameter h represents a field "exceeding satisfactory income". The mathematical definition of this field (using the equation of a straight line through two points) enables a somewhat closer formulation of aim function. For h max, namely for the highest level that should be satisfied when criterium function and restrictions are concerned, the criterion function (1) will have the following form:

$$R+\rho(2h-1)\geq 230x_1+200x_2+170x_3+240x_4+160x_5 \tag{9}$$

where ρ represents the left/right boundary of the triangular fuzzy number of satisfactory income, i.e., in our case

$$0\geq -540000-60000+230x_1+200x_2+170x_3+240x_4+160x_5 \tag{10}$$

The boundaries of TFN variables are given in aim function (10). The variables $x1$ are treated as linguistic variables and they are also TFN. The TFN have been sekected as both the presentation and solution to the problem are elaborated to a level that enables efficient engineering application. The simple mathematical rules presented through fuzzy algebra by Kaufman and Gupte (1988) are also designated factors. The study on application of affiliation function of some other form would require further elaboration of the corresponding mathematical apparatus.

The boundaries of $x1$ variables can be presented in the following way:

$$x_i^{*}\leq x_i\leq x_i^{**} \tag{11}$$

Expression (11) means that $x1$ is smaller/greater than x/x approximately. The coefficients of aim function (10) that designate the bauxite price are constants. Let us determine the limits of expression (11) and let the left limit be, analogous to expression (1a) a triangular fuzzy number (0, 0, 0), Fig. 3.

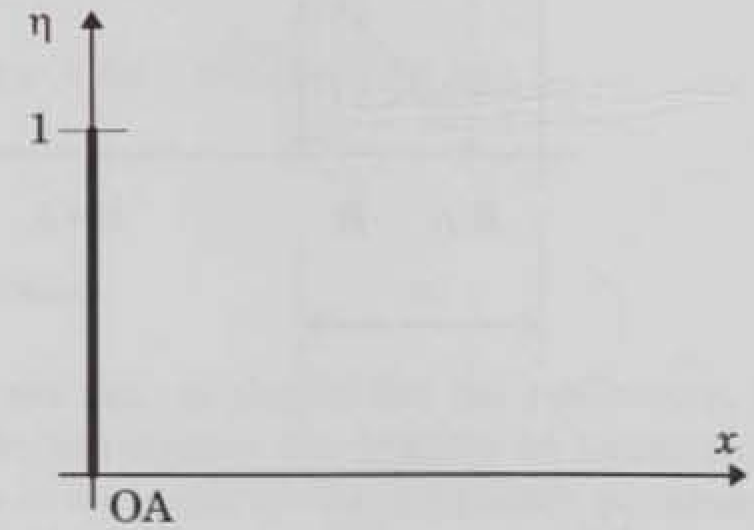

Figure 3: A triangular fuzzy number - TFN

In that case we shall have the following : $(0, 0, 0) < x_1$ (12)

and ythe right boundary of limitation: $x_1 < x_1^{**}$ (13)

will be determined as TFN "less than approximately 1000" which corresponds to the value (4). This number is given in Fig. 4.

For max h the right boundary of limitation (13) is as below:

$$x_1 \le B + b(1-2h) \tag{14}$$

namely

$$x_1 \le 1050 - 100h \tag{15}$$

All other boundaries of variables can be obtained analogously:

$$(0,0,0) \le x_2 \le 550 - 100h \tag{16}$$

$$(0,0,0) \le x_3 \le 800 - 100h \tag{17}$$

$$(0,0,0) \le x_4 \le 550 - 100h \tag{18}$$

$$(0,0,0) \le x_5 \le 800 - 100h \tag{19}$$

Thus, the model of the problem should read as follows:

$$(\max h)\quad 0 \ge -540000 - 60000h + 230x_1 + 200x_2 + 170x_3 + 240x_4 + 160x_5 \tag{10'}$$

$$\max Z = 540000 + 60000h \tag{21}$$

$$0 \le x_1 \le 1050 - 100h \tag{15'}$$

$$0 \le x_2 \le 550 - 100h \tag{16'}$$

$$0 \le x_3 \le 800 - 100h \tag{17'}$$

$$0 \le x_4 \le 550 - 100h \tag{18'}$$

$$0 \le x_5 \le 800 - 100h \tag{19'}$$

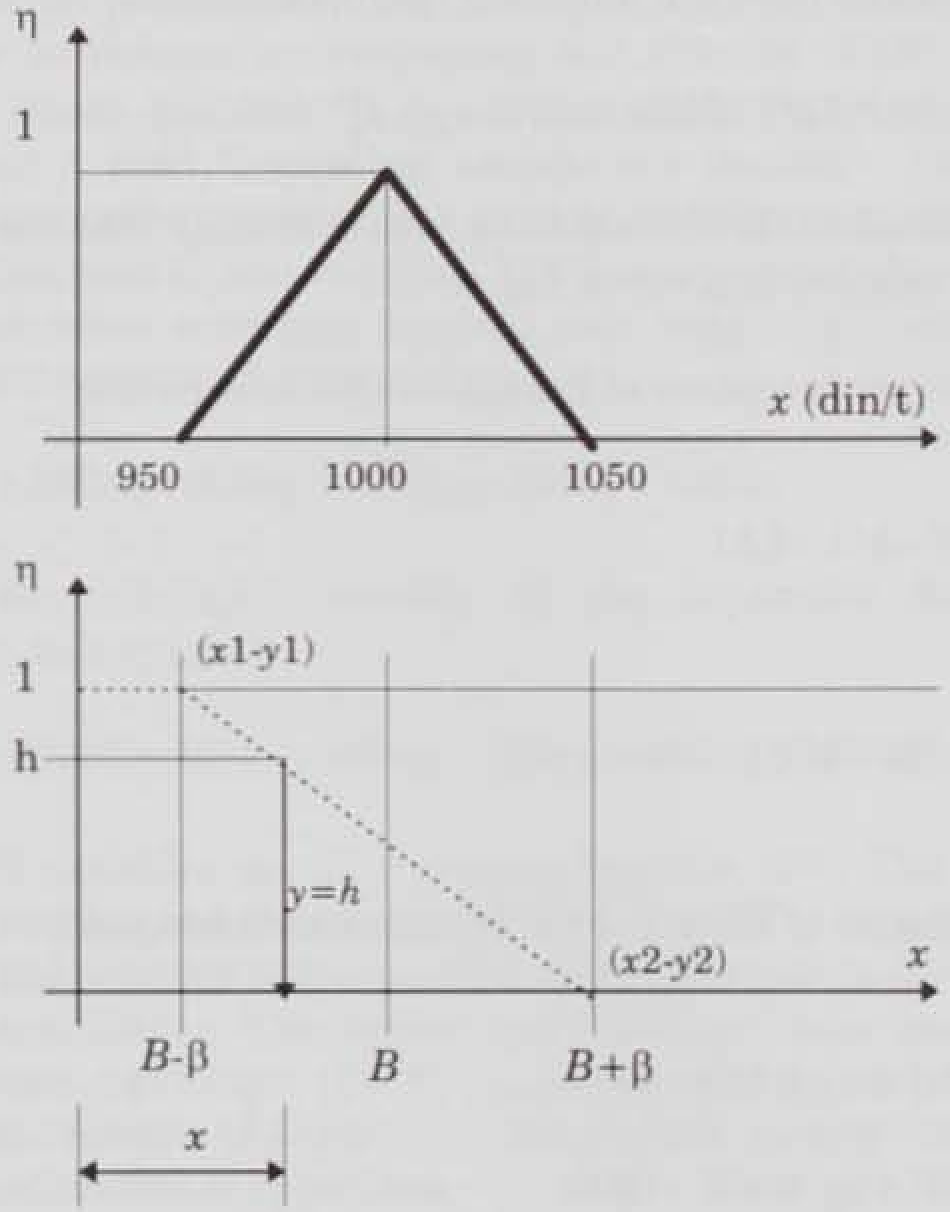

Figure 4: TFN (950, 1000, 1050)

bearing in mind the already defined limitations.

$$x_1 + x_2 + x_3 + x_4 + x_5 = 3000 \quad (2')$$

$$14,5x_1 + 9,7x_2 + 4,4x_3 + 11,6x_4 + 4,3x_5 = 24000 \quad (3')$$

Expression (10′) represents the aim function (in concrete case max should be determined) and (15′)-(19′), (2′), (3′) are limitations. It is obvious that the FLP problem has been transformed into a LP task by the procedure of introducing parameter h as a level that fulfills the criterium function and limitations.

Solution to the FLP model

Here is the solution to the problem:

$h = 0,53$
$x_1 = 462,14$
$x_2 = 496,96$
$x_3 = 746,96$
$x_4 = 496,96$
$x_5 = 796,96$

Z max = 571800 din (for 3000 t bauxite/shift)
namely: Z max = 190, 6 din/t.

Discussion of the FLP model

Selection of a satisfactory income

It is obvious that the obtained Z max is less than in the case of LP even with a small satisfactory level of parameter h. When selecting a satisfactory income as TFN (550000, 580000, 610000) i.e. for aim function.

$$0 \geq -550000 - 60000h + 230x_1 + 200x_2 + 170x_3 + 240x_4 + 160x_5 \qquad (22)$$

the obtained solution is: h = 0,494

x_1 = 457,51
x_2 = 500,64
x_3 = 750,64
x_4 = 500,64
x_5 = 790,53
Z max = 579640 (din);

while when TFN (600000, 630000, 660000) the solution of satisfactory income is unfeasible.

This means that the possibility of higher satisfactory income is reduced (h is reduced at higher income requirements). Both x, x and x are characterized by values somewhat higher 500, 700 namely 500 representing a transgression in the LP model (formulae (5), (6), (7) but in FLP model this is in complete agreement with TFN formulation such as: "approximately 500", "approximately 700" and "approximately 500".

Selection of capacity per pit

It logically follows that by narrowing the boundaries of TFN designating the capacity of single pits, the uncertainty in connection with capacity is reduced. When these capatities are respectively (975, 1000, 1025), (475, 500, 525), (725, 750, 775), (475, 500, 525) and (775, 800, 825) parameter h increases and h = 0,652 and Zmax = 579120 (din).

The modules of bauxite

When m = 7, the problem is unfeasible and for m = 9 it is 1. In further elaborations the idea that the modulus of bauxite as a fuzzy number "approximately 8" will certainly remain. The problem is even more complex when m is considered as the quotient of a fuzzy number denoting bauxite quality espressed as a percentage od Al_2O_3 and a fuzzy number denoting the percentage of SiO_2 in accordance with fuzzy algebra presented by Kaufman and Gupte (1988).

Selection of production capacity per shift

Constraint (2′) considered from the physical aspect can be read as "a daily capacity of approximately 3000". Let it be TFN (2800, 3000, 3200) as given in Fig. 5.

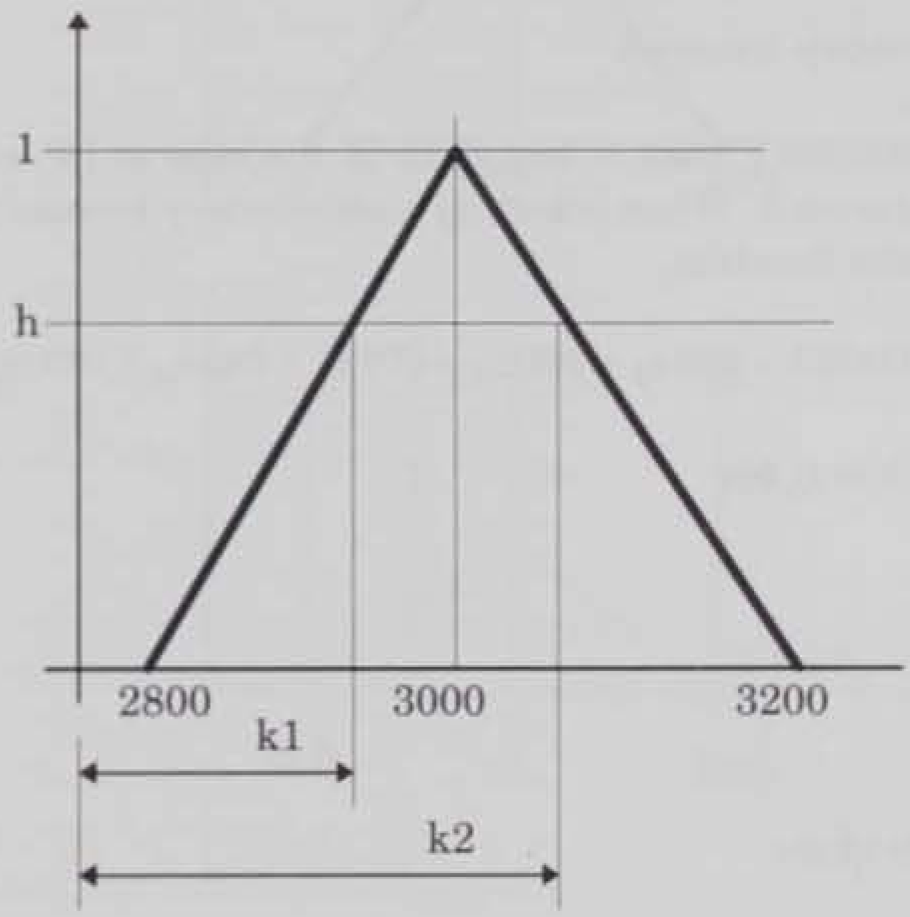

Figure 5

In this case

$$k_1 = 2800 + 200h \quad (23)$$
$$k_2 = 3200 - 200h \quad (24)$$

where the constraint (2′) can be changed to the two following limitations, the solution is as follows:

$$x_1 + x_2 + x_3 + x_4 + x_5 \geq 2800 + 200h \quad (25)$$
$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 3200 - 200h \quad (26)$$

In for satisfactory income (540000, 750000, 600000)

$h = 0{,}657$
$x_1 = 462{,}36$
$x_2 = 496{,}85$
$x_3 = 746{,}85$
$x_4 = 496{,}85$
$x_5 = 796{,}85$
$Z\max = 579420$ (din)

5. CONCLUSION

Modern approaches to the study of mining production systems are based on mathematical-model tools and computer technologies. Engineering analysis can be realized only when taking into consideration all the factors that influence a system as well as the connections between attributes. The method that adequately responds to the uncertainty of monitoring occurences and processes, impreciseness, polysemy and subjectivity when presented, is considered to be a successful method. The example illustrates how particular problems of real production systems can be treated by the theory on fuzzy sets. Thus, one real system is considered as a fuzzy system which is of essential importance due to our frequent impossibility description of both production systems and processes.

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