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PART AND MACHINE PARTITIONING PROBLEM IN CELLULAR MANUFACTURING: MULTIPLE MACHINE ENVIRONMENT

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Abstract. A major issue in the design of cellular manufacturing systems is the allocation of machines and parts to cells. This determines the overall structure and performance of the system in terms of part flow among cells. Several design and operational constraints such as an upper limit on the cell size, multiple units of the same machine, allocation of certain machines in the same or different cells and nonconsecutive visits of a part to the same machine during its processing, can be taken into account. The present work deals with the problem of simultaneously allocating parts and machines to cells in manufacturing systems where replicate machines and several design requirements exist. The part allocation section of the probiem, which is trivial under the single machine type hypothesis, becomes quite complicated, but more realistic, when more than one unit of each machine type exist. The problem is first defined and formulated in a mathematical programming form. Subsequently, a heuristic algorithm based on a version of simulated annealing is used to produce enhanced system configurations.

Keywords: Cellular manufacturing, multiple machines, simulated annealing

1. INTRODUCTION

Group technology (GT) is a manufacturing philosophy which organizes and uses information for grouping various parts and products with similar machining requirements into families of parts and corresponding machines into machine cells. The main objective of cellular manufacturing, which is an application of GT, is to construct machine cells, to identify part families and ultimately to allocate part families to machine cells so as to minimize interaction among different cells. This way, a number of manufacturing cells are constructed, taking into account the processing similarities of the parts. Each part (or product or subassembly), of course, should ideally be processed entirely within the machine cell to which it is assigned, imitating and taking advantage, therefore, of the benefits of small job-shops.

The part and machine partitioning problem, being a central issue of cellular manufacturing, has been tackled using a number of solution procedures. Most of the procedures suggested in the literature are based only on the machine-part incidence matrix, i.e. a zero-one matrix denoting whether a part requires processing from a machine or not. Those include the work of Mc Auley [8], Rajagopalan and Batra [9], King [4], Kumar et al. [6]. Such procedures are often incapable of accommodating design and operational constraints such as: an upper limit on the size of the cells formed, routeing information on each part, the potential for multiple nonconsecutive visits of a part on the same machine during its processing, the existence of multiple units of the same machine (replicate machines) in order to facilitate and decongest heavily machine loaded cells, etc. There exist of course procedures which incorporate some of the above design constraints. The cell size constraints, for example, are often taken into account by several techniques as in Ballakur and Steudel [2] or in Logendran and Ramakrishna [7], while other constraints, such as the machine co-location or machine separation constraints are less often taken into consideration. Multiple units of the same machine, maximum (or minimum) cell size and other important features have been taken into consideration in the work of Viswanathan [12]. Nevertheless, the major input to the model is still the machine-part incidence matrix, neglecting thus the important feature of part routeing information.

In this paper, the problem of partitioning parts and machines into cells in a multiple machine environment is considered. The realistic design and operational constraints of cellular manufacturing systems are presented and discussed in detail. A comprehensive model and a heuristic algorithm based on a version of simulated annealing, that incorporates four realistic design and operational constraints is employed, with the objective not only to produce the minimum intercell part-flow assignment of machines and parts to cells, but at the same time to determine the part routes that produce this minimum assignment. It should be mentioned here, that in the presence of several replicates of the same machine in the model, the part route of each part is no longer an input to the model, but an output to be produced along with the minimum partitioning. The applicability of the proposed model and heuristic algorithm is illustrated via a numerical example.

2. CELLULAR MANUFACTURING SYSTEM CONSTRAINTS

The most important and realistic constraints that characterize a cellular manufacturing system are the following:

2.1. Replicate (multiple) machines

Taking into account the processing times of parts in the analysis of part and machine partitioning problems, one has to ensure that the required machine capacity is always available. The number of units of each machine type, therefore, should be one or more depending on the workload of the system. This number should be treated as an input to the model. In the presence of replicate machines, i.e. when the number of units of the same machine is more than one, constraints should be imposed to ensure that the replicates of the same machine are assigned to different manufacturing cells (machine separation constraints) so that the main reason for machine replication is fulfilled, i.e. possible congestion of any cell is alleviated.

2.2. Cell size

A trivial solution to the part and machine partitioning problem would be to cluster the entire set of parts and machines into a single cell. This of course would eliminate any intercell movements among parts. However, this is far beyond what is desired. Reasonable upper bounds on the size of the cells are usually assumed, in terms of machines. This limit on the number of machines per cell is often imposed in practice based on previous experience. A lower bound on the number of machines per cell is sometimes incorporated in some of the models presented in the literature (Askin and Chiu [1], Viswanathan [12]), but in our model the nature of the objective function itself inherently prevents the generation of small-sized cells and thus the inclusion of a lower bound on cell size has not been considered.

2.3. Machine co-location and separation constraints

As already mentioned above, it is sometimes necessary to impose constraints on the co-location of some machines. For example, in the case of replicate machines, care should be taken not to allow multiple units of the same machine to be assigned to the same cell. Apart from the case of machine replication (where machine separation constraints are required) other reasons may dictate the placement of certain machines in different cells. Heragu [3], for example, reports that for safety reasons sometimes two or more machines have to be assigned to different cells. Other reasons may dictate the opposite, i.e. placing two or more machines in the same cell (machine co-location constraints).

2.4. Part routeing information

In order to describe real intercell traffic among parts in a cellular manufacturing system part operation sequences are considered. When part routeing sheets are taken into account, one can easily calculate the total number of consecutive operations, i.e. total intermachine flow, based on all parts, between any two machines for all machines. Thus, the total intermachine flow which forms the objective function to be minimized in our model, describes the exact total number of intermachine movements of all parts. This objective function is sought to be minimized under the constraints of operation sequences, taking also into account the possibility for a part to undergo two or more nonconsecutive operations on the same machine, regardless of the direction of move. For example, if the routeing of a part is from machine M1 to machine M2 and back to machine M1 again, then this particular part would contribute with two intracell or two intercell movements, depending on whether machines M1 and M2 are located in the same or different cells respectively.

3. THE MODEL

3.1. Notation

Before we present and discuss our model we introduce the notation employed.

$i,j = 1,...,M$ machines
$p = 1,...,P$ parts
$r = 1,...,R_p$ alternative routeings (due to the presence of replicate machines) for part p
$s = 1,...,S_p$ operation sequence number for part p
N = maximum number of machines allowed in any cell
c_{prsi}= a 0-1 coefficient indicating whether the s^{th} operation of part's p r^{th} alternative routeing requires processing from machine i (c_{prsi}=1) or not (c_{prsi}=0).

It should be mentioned at this point that although there is a single process plan for each part, alternative routeings are generated for a part if this requires processing from one or more types of machines with multiple units (given that the number of units of each machine type is known *a priori*). For example, if the routeing of a part is M1, M2, M4, M5 and machine M2 has one replicate, say machine M3, then a second (alternative) routeing is generated for this part which is M1, M3, M4, M5.

3.2. Mathematical model

Given an allocation of machines into cells and noting that for each part p there are R_p alternative routeings, one wishes to choose the routeing which generates the minimum amount of intercell movements during the processing of a particular part. Therefore, let

$$c_p^* = \min_r \sum_{s=1}^{S_p} \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} c_{prsi}\, c_{pr(s+1)j}\, (1 - X_{ij})$$

where X_{ij} is a 0-1 decision variable indicating whether machines i and j are located in the same cell or not. Thus, given a solution X_{ij} for all i and j, c_p^* represents the minimum amount of traffic generated during the processing of part p when this follows a particular routeing (over all alternative routeings). In the presence, therefore, of multiple (replicate) machines, the cell formation problem is formulated as follows:

$$\min \sum_{p=1}^{P} c_p^* \tag{1}$$

s.t.

$$\sum_{i=1}^{k-1} X_{ik} + \sum_{j=k+1}^{N} X_{kj} \le N-1 \qquad k = 1,\dots,M \tag{2}$$

$$X_{ij} + X_{ik} - X_{jk} \le 1$$

$$i = 1,\ldots,M-2$$

$$X_{ij} - X_{ik} + X_{jk} \le 1 \tag{3}$$

$$j = i+1,\ldots,M-1, \quad k = j+1,\ldots,M$$

$$-X_{ij} + X_{ik} + X_{jk} \le 1$$

$$X_{ij} = 0,1 \qquad i = 1,\ldots,M-1, \quad j = i+1,\ldots,M \tag{4}$$

In the above formulation, constraints (2) require that the maximum size of each cell is N. Constraints (3) are imposed to preserve the connectivity of the above formulation, i.e. they ensure that machines j and k are grouped together ($X_{jk} = 1$) if machines i and j as well as machines i and k are both allocated to the same cell ($X_{ij}=1$ and $X_{ik}=1$ respectively).

The above model can also be applied for the case of separation constraints, i.e. constraints which prevent two or more machines from being allocated to the same cell as happens in the case of replicate machines. This is a distinct feature of the particular cell formation problem we are dealing with. In the context of the above formulation, these constraints are of the form:

$$X_{kl} \neq 1 \qquad k<l \tag{5}$$

which indicate that machines k and l cannot be allocated to the same cell. Similarly, when operational or other reasons require that machines q and t be placed in the same cell, the following constraints

$$X_{qt} = 1 \qquad q<t \tag{6}$$

have to be considered.

Once the best machine-to-cell allocation has been produced via the solution of model (1)-(6), one has to consider the corresponding assignment of parts to cells. This can be achieved by solving a simple linear assignment problem [11].

4. HEURISTIC ALGORITHM

The cell formation problem itself is a NP-complete problem. This rules out the possibility of finding the optimum solution using an exact algorithm. The heuristic proposed in this work is based on a version of simulated annealing proposed by Kirkpatrick et al. [5]. The algorithm can be applied to the case of machine replicates as well as to problems with machine separation and co-location requirements. It starts with a random initial feasible assignment of machines to cells. Each time the assignment is changed by accepting solutions which not only improve but also worsen the objective function. The latter assignments are accepted in order to prevent problem from the beirg trapped in local optima. Any implementation of the simulated annealing heuristic requires the setting of the following features (Sofianopoulou [10], [11]):

1. A perturbation scheme to generate new neighbouring solutions.
2. An annealing schedule which includes *a*) an initial (T_0) and a final (T_f) value for the temperature parameter T, both empirically set by a number of pilot runs, *b*) a rate of cooling α, set equal to 0.90 and *c*) a rate of change $\in$ in the number of solutions attempted at each temperature value which was set equal to 0.10.

The heuristic consists of the following modules.

4.1. Initial assignment

The initial assignment of machines to cells is random. Machines are randomly chosen and if they do not violate any separation or co-location constraints, are grouped together until the cell size constraint is violated, and a new cell starts forming. The objective function value corresponding to this initial assignment is also calculated.

4.2. Change of machine assignment

A machine is randomly chosen and its (cell) assignment is again randomly changed. If the machine violates any of the constraints in its new position, another assignment is randomly chosen until no constraint is violated. If this reassignment procedure repeatedly fails, the machine that has been picked starts the formation of a new cell. Then the change in the objective function Δf is calculated. In particular, for each new candidate solution, the change in the amount of intercellular moves is calculated for all alternative routeings of each part, and the most advantageous one is determined. The sum of all those partial differences forms the total change Δf in the objective function. If $\Delta f < 0$, then the new solution is accepted, otherwise the case is treated probabilistically. The candidate solution is accepted only if $e^{-\Delta f / T} \leq y$, a random variable $y \sim U(0,1)$. The probability of accepting such a solution, which augments the objective function value, is reduced as the value of T - the simulated annealing control parameter- decreases (by a rate $\alpha = 0.90$) and the change Δf in the objective function increases. The algorithm continues with a certain number of iterations (reduced by $\in$) at each temperature level until the total prescribed number of iterations has been reached or when no solution has been obtained.

5. ILLUSTRATIVE EXAMPLE

The present implementation of the simulated annealing algorithm was coded in Fortran 77 and run on a RISC technology DEC 5810 computer. In order to demonstrate the applicability of the algorithm to this class of the cell formation problem, an instance of the problem with 13 machines and 30 parts is considered. The particular data set that was used to run this application was randomly generated. Data input to the model include the number of machines (M=13), the number of parts (P=30), the maximum cell size (N=7), the number of groups of replicate machines and the number of multiple units, replicates, in each group. Random sequences of machines were produced to construct the part routes for each part, which are presented in Table 1. In this example, two groups of replicate machines are present. The first one is comprised of machines M2 and M3 and the second one of machines M8, M9 and M10. In the data set of Table 1 it can be seen that some parts visit the same machine (or one of its replicates) nonconsecutively, more than once, e.g. part P3 visits machine M8 at the second stage of its processing and again visits the same machine (or one of its replicates) at the sixth stage.

Table 1. Random data set.

PARTS / MACHINES	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1				2		3						1	4	5		
2			7	2		6	1			3	4	5				
3						5	4	6		2		1	7	3		
4				3	2		1		4							
5											1	2				
6	2				3					1					4	
7		1				3	4		5		2					
8						1			2			3				
9		3				2										1
10			4		1				3		5		2			
11		1							4		5		2	3		
12						2	1	4		3	5					
13			2	3			4					5			1	
14		1							3		4			5		2
15					3			5		1	2					4
16		4							2		1					3
17			2			3	5	4						1		
18			2	5					1				3	4		
19	3								5	1			4		2	
20		2		4	5					3		1				
21		4	3					2								1
22	5					3	1	4					2			
23			1					4				3	2			
24						1		3							2	
25				1	2			3	4					6		5
26			3			1				2						
27	2		3								1					
28						2			5	1	3		4			
29	7		4				5	3	2	1			6			
30				4		7		1		3			2	5		6

Table 2 presents the solution of this part and machine partitioning problem. The algorithm was run for 2000 iterations (attempted but not necessarily accepted solutions). Three cells were produced. Two of the cells are loaded up to their size limit while the rest of the machines are placed in a third cell. The best total number of intercellular moves among all parts is 30.

Table 2. Parts and machines partitioning.

Cell	Machines	Parts
1	1,3,5,8,11,14,16	5,6,9-11,14-16,21,25,27
2	2,4,6,7,10,12,13	1-4,7,8,12,13,17,18,20, 22-24,26,28-30
3	9, 15	19

Part routeings	
P 1 :	12-4-6-13-14
P 2 :	7-4-8-11-12-6-2 <u>or</u> 7-4-10-11-12-6-2
P 3 :	12-8-14-7-6-10-13 <u>or</u> 12-10-14-7-6-10-13
P 4 :	7-5-4-10
P 5 :	11-12
P 6 :	8-1-5-15
P 7 :	3-11-6-7-10
P 8 :	6-10-12
P 9 :	16-6-2
P10 :	5-13-10-2-11 <u>or</u> 5-13-8-3-11 <u>or</u> 5-13-10-3-11
P11 :	2-13-14-8-11
P12 :	7-6-8-8-11 <u>or</u> 7-6-10-8-11 <u>or</u> 7-6-10-10-11
P13 :	15-2-4-7-12
P14 :	3-16-8-11-14
P15 :	8-11-5-16-8
P16 :	11-8-16-3
P17 :	14-2-6-10-7 <u>or</u> 14-3-6-10-7
P18 :	10-2-13-14-4
P19 :	9-15-1-13-10
P20 :	12-2-10-4-5
P21 :	16-8-2-2 <u>or</u> 16-10-2-2 <u>or</u> 16-8-3-2
P22 :	7-13-6-8-1 <u>or</u> 7-13-6-10-1
P23 :	2-13-12-10
P24 :	6-15-9
P25 :	4-5-8-8-16-14
P26 :	6-10-2
P27 :	11-1-3
P28 :	10-6-11-13-10
P29 :	10-4-6-2-7-13-1
P30 :	10-13-10-4-14-16-6

6. CONCLUSIONS

In the present work the part and machine allocation problem was examined with the aim of minimizing the amount of intercellular moves. Four important design and operational constraints were discussed. A mathematical model that incorporates these constraints was developed. The model applies to the case where multiple units of the same machine are present. Care was taken to allocate them to different manufacturing cells. Machine co-location and/or machine separation constraints were also included in order to prevent and/or to impose machine co-location. Finally, part routeing information was taken into account in order to determine the exact amount of intercellular moves. A heuristic based on simulated annealing was employed to solve the problem. Results for a test-case problem were also included.

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