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A FUZZY MODEL FOR INVENTORY ALLOCATION IN A SERIAL PRODUCTION/INVENTORY SYSTEM *)

Dobrila PETROVIĆ

Mihajlo Pupin Institute, Yugoslavia,
Visiting Ph. D. student at University of Warwick,
Department of Engineering, Manufacturing Group, UK

Rajat ROY

University of Warwick, Department of Engineering,
Manufacturing Group, UK

Radivoj PETROVIĆ

University of Belgrade, Traffic and Transportation Faculty,
Yugoslavia

Abstract: This paper considers the production/inventory model of a system in which inventory facilities are in series and between them are production/assembly units. There are inventory stocks for raw material, in-process products and end-product. External demand for an end-product is placed at the end-stock point in the system. It is assumed that uncertainty appears in external demand. Uncertainty is propagated along the production/inventory system. Modelling of the external discrete imprecise demand is based on fuzzy sets.

The fuzzy model developed enables a search for the stock allocation that gives the minimum possible inventory cost of the whole serial system. The backtrack searching procedure for the most appropriate inventory stocks allocation is illustrated by an example.

Keywords: Production/inventory system, inventory stocks allocation, fuzzy set.

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1. INTRODUCTION

In recent years there has been an acceleration of interest in supply chain management and control. Numerous complex problems have arisen on how a supply chain should be controlled in a modern manufacturing ambient in such a way that some customer services are realised and the total costs involved are as small as possible [1-3].

Generally, a supply chain is viewed as a complex system which consists of material suppliers, production facilities, distribution services and customers. They are interconnected by the feedforward flow of materials and intensive flow of information. Each of the facilities in a supply chain either adds value to the end-product or represents intermediate buffer storage.

In this paper we shall consider a supply chain with inventories in series: a raw material inventory, in-process inventories and end-product inventory. It is assumed that there is uncertainty in customer demand at the end-stock point. Demand may be the sum of the firm orders and forecasts. The forecasts are expressed using linguistic terms such as "*about* d units per given time period T", or "*much more* (*less*) than d units per T", etc. Such linguistic terms are modelled by fuzzy sets [4]. In each inventory in the chain the stock of an item, expressed in end-product units, is determined to guarantee reliable delivery under reasonable surplus cost. The incurrence of shortage costs arises from external demand's uncertainty. Inventories in the chain are not independent cost centres, but they are "under one roof". The objective is to minimise the possible total cost of the chain as a whole. The possible total cost is the sum of purchase cost, and either possible shortage cost or possible surplus cost over a given time period.

The paper is organised in the following way. Section 2 gives a formal statement of the problem and a fuzzy model for a supply chain with inventories in series. An algorithm for inventory allocation is described in Section 3 and illustrated by an example in Section 4.

Notation

T	–	fixed time period,
I	–	number of inventory facilities,
$\widetilde{D}_1$	–	fuzzy external demand,
$\widetilde{D}_i$	–	fuzzy internal demand imposed from facility i-1 to facility i, $i=2,...,I$,
$\mathcal{D}$	=	$\{d_1,...,d_J\}$ - discretised support of $\widetilde{D}_i$, $i=1,...,I$,
$\mu_{\widetilde{D}_i}(d_j)$	–	membership function of $\widetilde{D}_i$, $i=1,...,I$, $j=1,...,J$,
p_i	–	unit shortage-penalty cost for facility i, $i=1,...,I$,
h_i	–	unit surplus cost for facility i, $i=1,...,I$,
c	–	unit purchase cost,

S_i – inventory stock of facility i, $i=1,...,I$,
$\mathcal{S}$ – domain of S_i ,
$F_i(S_i)$ – possible total cost of facility i, $i=1,...,I$,
F_{tot} – possible total cost of the serial system as a whole.

2. MODEL AND PROBLEM STATEMENT

A schematic view of the supply chain considered is given in Fig. 1.

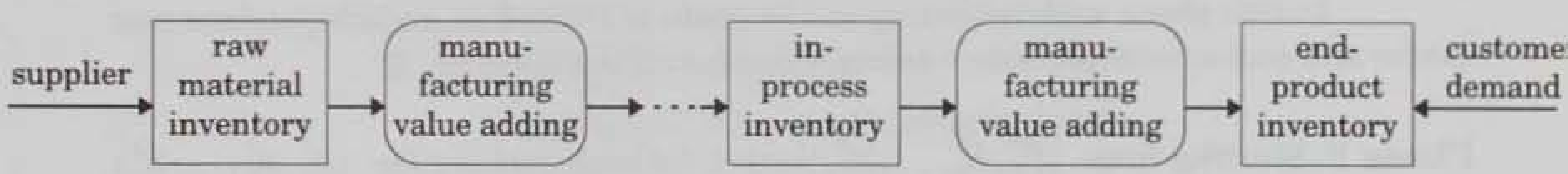

Figure 1. Schematic view of the supply chain

There are I inventory facilities connected in a serial system. Uncertain external demand for the product considered, for a given period T, is placed at the end facility in the series, indexed by 1. If inventory 1 cannot meet the demand, it makes a request for the remaining un-met quantity to inventory 2. Inventory 2 repeats the procedure and makes a request to inventory 3, and so on to inventory I. Shortages are allowed at every facility. They produce a linear shortage-penalty cost because the demand requirements are not fulfilled promptly. Besides, each facility has a linear stock surplus cost. Due to adding value along the chain the relation: $h_1>h_2>...>h_I$ exists.

Uncertain external demand described by a fuzzy set causes uncertain cost at each facility. The total cost is the sum of purchase, shortage or surplus costs. The possible total cost for the serial system as a whole is the sum of the possible costs recorded at each facility.

$$F_{tot} = \sum_{i=1}^{I} F_i(S_i) \tag{1}$$

Calculation of $F_i(S_i)$ is explained in Appendix.

The problem is: given $\widetilde{D}_1$ over T, c, p_i, h_i, $i=1,...,I$, determine S_i^*, $i=1,...,I$, that minimise F_{tot}.

3. ALGORITHM

Determining S_i^*, $i=1,...,I$, by direct enumeration, i.e., by generating all the inventory stock combinations and selecting one that gives the minimum possible cost F_{tot} in (1) is not computationally effective. Even for $I = 3$ the search among 3-tuples integers that minimise (1) is not a simple numerical task.

The algorithm for determination of S_i^*, $i=1,...,I$, proposed in this paper has two phases.

Phase 1: Determine the initial inventory allocation S_i^0, $i=1,...,I$.

In this phase each inventory in the chain is treated as an independent cost centre and cost interdependancy among inventories is neglected.

Phase 2: Starting from $(S_1^0, S_2^0, \ldots, S_I^0)$ search by backtracking for $(S_1^*, S_2^*, \ldots, S_I^*)$ that leads to the smallest possible total cost.

Phase 1:

Step 1:

Calculate $F_1(s)$, $s \in \mathcal{S}$

Find S_1^0 that minimises $F_1(s)$: $F_1(S_1^0) = \min_{s \in \mathcal{S}} F_1(s)$

Step 2:

$\widetilde{D}_2 = \widetilde{D}_1 - S_1^0$

Calculate $F_2(s)$, $s \in \mathcal{S}$

Find S_2^0 that minimises $F_2(s)$: $F_2(S_2^0) = \min_{s \in \mathcal{S}} F_2(s)$

Continue for $i = 3,...,I$

Step i, i = 3,...,I:

$\widetilde{D}_i = \widetilde{D}_1 - (S_1^0 + ... + S_{i-1}^0)$

Calculate $F_i(s)$, $s \in \mathcal{S}$

Find S_i^0 that minimises $F_i(s)$: $F_i(S_i^0) = \min_{s \in \mathcal{S}} F_i(s)$

Step I+1

Calculate $F_{tot}^0 = \sum_{i=1}^{I} F_i(S_i^0)$

Phase 2:

Step 1: Determine the list of elements $\mathcal{S}_1 \subset \mathcal{S}$: $F_1(s_1) < F_{tot}^0$ for all $s_1 \in \mathcal{S}_1$

Step 2: Select the first/next element s_1 of the list $\mathcal{S}_1$ if it exists and let $S_1 = s_1$

If there are no more elements $s_1 \in \mathcal{S}_1$ then $(S_1^*, S_2^*, \ldots, S_I^*)$ is the generated I-tuple $(S_1, S_2, \ldots, S_I)$ and stop the procedure

$\widetilde{D}_2 = \widetilde{D}_1 - S_1$

Calculate $F_2(s)$, $s \in \mathcal{S}$

Find S_2 that minimises $F_2(s)$: $F_2(S_2) = \min_{s \in \mathcal{S}} F_2(s)$

If $F_1(S_1) + F_2(S_2) < F_{tot}^0$ then

  Determine the list of elements $\mathcal{S}_2 \subset \mathcal{S}$:

  $F_1(S_1) + F_2(s_2) < F_{tot}^0$ for all $s_2 \in \mathcal{S}_2$

  Continue the procedure for $i = 3$, *Step 3*

else

  Repeat *Step 2*

end if

Step i, i = 3,...,I:

Select the first/next element $s_{i-1} \in \mathcal{S}_{i-1}$ if it exists and let $S_{i-1} = s_{i-1}$, $i = 3,\ldots,I$

If there are no more elements $s_{i-1} \in \mathcal{S}_{i-1}$ then move to facility i-2 and repeat *Step i-2*.

$\widetilde{D}_i = \widetilde{D}_1 - (S_1 + \ldots + S_{i-1})$

Calculate $F_i(s)$, $s \in \mathcal{S}$

Find S_i that minimises $F_i(s)$: $F_i(S_i) = \min_{s \in \mathcal{S}} F_i(s)$

If $F_1(S_1) + \ldots + F_i(S_i) < F_{tot}^0$ then

  if $i = I$ then

    A new I-tuple $S_i, i = 1, \ldots, I$ is generated that leads to a smaller F_{tot}; F_{tot} is used as a new reference cost

    Continue to *Step I+1*

  else

Determine the list of elements $\delta_i \subset \delta$:

$F_1(S_1)+\ldots+F_{i-1}(S_{i-1})+F_i(s_i) < F_{tot}^0$ for all $s_i \in \delta_i$

Continue the procedure for $i = i+1$, *Step* $i+1$

end if

else

Repeat *Step* i

end if

Step $I+1$ Determine the new list of elements δ_{i-1} such that

$F_1(S_1)+\ldots+F_{I-2}(S_{I-2})+F_{I-1}(s_{I-1}) < F_{tot}^0,\ s_{I-1} \in \delta_{I-1}$

Select the first element s_{i-1} of the list δ_{I-1}

Repeat the computation starting from *Step I* using the new reference cost F_{tot}.

4. EXAMPLE

Consider an example with I=3 facilities. Unit costs are: $p_1 = 12$, $h_1 = 8$, $p_2 = 15$, $h_2 = 6$, $p_3 = 20$ and $h_3 = 2$. Unit purchase cost c is neglected.

Demand is described by "*about* 10 products". Imprecise demand is modelled by $\tilde{D}_1$ with the following membership degrees:

demand:	7	8	9	10	11	12	13
possibility:	0.25	0.5	0.75	1	0.75	0.5	0.25

First, apply Phase 1 of the algorithm described in Section 3.

The possible costs $F_1(S_1)$ incurred by different inventory levels S_1 at inventory 1, considered as an isolated facility, are given in Table 1.

Table 1: The possible costs of inventory 1

S_1	$F_1(S_1)$
7	36.00
8	25.52
9	17.00
10	11.73
11	12.27
12	17.25
13	24.00

Stock $S_1^0 = 10$ will cause the minimum cost of inventory 1.

Demand $\tilde{D}_2$ that could be imposed on inventory 2 is uncertain, too. For example, when $S_1 = 10$ then, if the external demand is 7, 8, 9 or 10, demand made on inventory 2 will be 0. The possibility of this event is equal to the maximum of the possibilities of external demand taking these values, i.e., the possibility is equal to 1. Similarly, if external demand is 11, demand on inventory 2 will be 1 with possibility 0.75, and so on:

demand placed on inventory 2:	0	1	2	3
possibility:	1	0.75	0.5	0.25

The possible costs of inventory 2, $F_2(S_2)$, are given in Table 2.

Table 2. Possible costs of inventory 2 when imposed demand is 0, 1, 2 or 3

S_2	$F_2(S_2)$
0	15.00
1	8.40
2	8.10
3	12.00

Stock $S_2^0 = 2$ will cause the minimum cost of inventory 2. If $S_1=10$, $S_2=2$ then demand that will be made on inventory 3 is uncertain, also:

demand made on inventory 3:	0	1
possibility:	1	0.25

The possible costs of inventory 3, $F_3(S_3)$, are given in Table 3.
Stock $S_3^0 = 1$ gives the minimum cost of inventory 3.

The initial inventory allocation is then $(S_1^0, S_2^0, S_3^0) = (10, 2, 1)$. The possible total cost caused by this inventory allocation is $F_{tot}^0 = 21.43$.

Now, apply Phase 2 of the algorithm described in Section 3.

Table 3. Possible costs of inventory 3 when imposed demand is 0 or 1

S_3	$F_3(S_3)$
0	4.00
1	1.60

According to Table 1, the possible inventory stocks of inventory 1 that might cause possible total cost to be lower than $F_{tot}^{0} = 21.43$ are 9, 10 and 12, i.e. δ_1={9, 10, 11, 12}. These inventory levels will be examined sequentially.

If S_1=9, then the possible total cost of inventory 1 is 17. The possible demands on inventory 2 are:

demand placed on inventory 2:	0	1	2	3	4
possibility:	0.75	1	0.75	0.5	0.25

The possible costs of inventory 2, F_2 (S_2), are given in Table 4.

Table 4. Possible costs of inventory 2 when imposed demand is 0, 1, 2, 3 or 4

S_2	$F_2(S_2)$
0	23.08
1	12.92
2	9.23
3	10.38
4	14.77

The minimum cost will be obtained by S_2=2. However, the sum of total costs incurred in inventories 1 and 2 is already greater than $F_{tot}^{0} = 21.43$.

If S_1=10 the minimum cost of inventory 2 is obtained by S_2=2. Also, S_2 can be reduced to one product (see Table 2), because the sum of the total possible costs of inventories 1 and 2 is still less than $F_{tot}^{0} = 21.43$, i.e. δ_2={1, 2}. In the case when S_2=1, the possible demands that could be made on inventory 3 are:

demand made on inventory 3:	0	1	2
possibility:	1	0.5	0.25

The possible costs of inventory 3, $F_3(S_3)$, are given in Table 5.

Table 5. Possible costs of inventory 3 when imposed demand is 0, 1 or 2

S_3	$F_3(S_3)$
0	11.43
1	4.00
2	2.86

If S_3=2, then this inventory allocation will lead to the possible total cost of 22.99, which greater than the already achieved cost of $F_{tot}^0 = 21.43$.

If S_1=11, then the possible cost of inventory 1 is $F_1(11)$=12.27. The possible demands on inventory 2 are:

demand made on inventory 2:	0	1	2
possibility:	1	0.5	0.25

The possible costs of inventory 2, F_2 (S_2), are given in Table 6.

Table 6. Possible costs of inventory 2 when imposed demand is 0, 1 or 2

S_2	$F_2(S_2)$
0	8.57
1	5.57
2	8.57

The minimum possible cost will be obtained by S_2=1. Then the possible demands on inventory 3 are:

demand placed on inventory 3:	0	1
possibility:	1	0.25

The possible costs $F_3(S_3)$ are given in Table 3.

The inventory allocation S_1=11, S_2=1 and S_3=1 leads to the possible total cost equal to F_{tot}=19.44. It is less than with the initial allocation where $S_1^0 = 10$, $S_2^0 = 2$ and $S_3^0 = 1$.

The new 3-tuple (11,1,1) is generated. The cost F_{tot}=19.44 is now used as the reference cost. Computation is going backward to inventory 2.

The new $\mathcal{S}_2$= ∅ because in the case S_1=11, any S_2 different from 1 will cause the possible total cost greater than 19.44. Computation is moved back to the inventory 1.

Step 1 of the algorithm is repeated for S_1=12. The possible total cost of inventory 1 is F_1 (12)=17.25. The possible demands on inventory 2 are:

demand placed on inventory 2:	0	1
possibility:	1	0.5

Both demands at inventory 2, 0 and 1, cause the sum of the possible total cost of inventories 1 and 2 greater than F_{tot}=19.44.

It means that inventory stocks S_1=11, S_2=1 and S_3=1 cause the minimum possible costs F_{tot}=19.44.

5. CONCLUSION

In the fuzzy model of a supply chain with I inventories in series the propagation of uncertainty along the chain caused by uncertain external demand at end-stock point is demonstrated. The developed algorithm for the allocation of stocks to the inventories in series which minimises the possible total inventory costs is computationally efficient even for a large I. The algorithm could be extended to solve the stock allocations for the cases when external fuzzy demands appear along the chain. Also, the extension of the algorithm is possible to cover the stock allocation problem with continuous fuzzy external demand.

Appendix: Calculation of the possible total cost for the isolated inventory *i* when demand is fuzzy [5]

Fuzzy $\widetilde{D}_i$ causes an uncertain shortage cost $\widetilde{P}_i$ and uncertain surplus cost $\widetilde{H}_i$. The possible shortage costs are $P_i(S_i, d_j)=p_i \bullet \max(d_j\text{-}S_i, 0)$ and possible surplus costs are $H_i(S_i, d_j)=h_i \bullet \max(S_i\text{-}d_j, 0), j=1,\dots,J$. The possibilities of $\widetilde{P}_i$ and $\widetilde{H}_i$ taking these values are equal to the possibilities of demand being d_j :

$$\mu_{\widetilde{P}_i}(P_i(S_i,d_j)) = \mu_{\widetilde{D}_i}(d_j), \quad j=1,\dots,J$$
$$\mu_{\widetilde{H}_i}(H_i(S_i,d_j)) = \mu_{\widetilde{D}_i}(d_j), \quad j=1,\dots,J \tag{2}$$

The sum of $\widetilde{P}_i$ and $\widetilde{H}_i$ can take the values $P_i(S_i,d_j)+H_i(S_i,d_j) \in \{f_1, f_2, \dots, f_K\}$ with the possibilities:

$$\mu_{\widetilde{P}_i+\widetilde{H}_i}(f_k) = \max_{j=1,\dots,J} \mu_{\widetilde{D}_i}(d_j), \quad k=1,\dots,K$$
$$f_j = P_i(S_i,d_j)+H_i(S_i,d_j) \tag{3}$$

The possible total cost of facility i is:

$$F_i(S_i) = c \cdot S_i + \text{defuzz}\,(\widetilde{P}_i + \widetilde{H}_i) \tag{4}$$

where the operator "defuzz" denotes arithmetic defuzzification:

$$\text{defuz}\,(\widetilde{P}_i + \widetilde{H}_i) = \frac{\sum_{k=1}^{K} f_k \cdot \mu_{\widetilde{P}_i+\widetilde{H}_i}(f_k)}{\sum_{k=1}^{K} \mu_{\widetilde{P}_i+\widetilde{H}_i}(f_k)} \tag{5}$$

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