Yugoslav Journal of Operations Research
6 (1996), Number 1, 85-100

PREVENTIVE MAINTENANCE OF UNITS SUBJECT TO TWO KINDS OF WEAR FAILURE

Č. DUBOKA, Ž. ARSENIĆ, J. TODOROVIĆ
*University of Belgrade, Faculty of Mechanical Engineering,
YU-11000 Belgrade, 27. Mart 80*

Abstract. There are mechanical systems in which the most important failures originate from excessive wear. Wear failures gradually develop and can be rather easily monitored or inspected. Design of such systems usually enables one or more adjustments to be performed before a failure requiring replacement of friction elements takes place. It means that such a unit has two kinds of failures: one requiring adjustment and another requiring replacement. When preventive maintenance of a friction mechanism comprising several elements deteriorating by wear, each characterized by a specific wear intensity, i.e. by different failure rates, is considered it is a problem how to determine periods of preventive maintenance so as to provide the best satisfaction of a defined criterion, i.e. the best outputs. The paper presents an approach to the problem solution when the availability or the operational readiness are used as optimization criterion. The model is developed for braking systems of motor vehicles used in large fleets and having different brakes in the front and the rear wheels.

Keywords: Preventive maintenance, maintenance optimization, availability, operational readiness, maintenance modelling, motor vehicle brake, wear failure

1. INTRODUCTION

A procedure for optimization of preventive maintenance of units subject to two kinds of wear failure is presented in the paper. Friction brakes used in motor vehicles are observed. Wear in such mechanisms results directly from brake operation, i.e. from brake application. Wear failures take place when the brake lining wear exceeds critical value with respect to brake performance. Two different kinds of wear failure may occur: (a) a brake is worn-out partly or (b) a brake is worn-out totally. A partly worn-out brake may be renewed by wear compensation i.e. by brake adjustment while a totally worn-out brake may be renewed only by component replacement.

When preventive maintenance of a friction mechanism comprising a number of elements deteriorating by wear is considered, where each element is characterized by specific wear rate, i.e. by a different reliability function, the problem is to determine the periods of maintenance actions so as to provide the best satisfaction of a defined criterion, i.e. to minimize total maintenance costs or to maximize availability. To optimize the preventive maintenance of braking systems used in road vehicles, a maintenance engineer in a motor vehicle fleet has to find the "best" solutions for the following:

(i) the number of brake adjustments between two consecutive brake component replacements which means the relationship between the periods of brake adjustments and brake component replacements, and

(ii) the relationship between the periods of preventive maintenance for front and rear brakes.

2. ENGINEERING BACKGROUND

Braking of road vehicles and other moving machinery is an action by which driver may control vehicle speed and stop it if necessary.

Braking of vehicles is provided by a system called braking system, which is required to fulfill road safety, dependability and other requirements. From the energy point of view, braking is a process in which kinetic energy of a vehicle is transformed into other kinds of energy. It is normally realized by friction forces developed between moving and stationary parts of a brake. Wear develops during each brake application. It results directly from continuous and repeated action of friction forces developed at friction surfaces, i.e. the surfaces between moving and stationary brake parts.

A braking system consists of several subsystems and components, including a number of brakes. The number of brakes in a vehicle is related to the number of wheels. In a two-axle vehicle there are four brakes, two in the front and two in the rear axle.

Road safety requirements impose that all brakes of a vehicle must act simultaneously and into accordance with requirements. Otherwise the vehicle stability while braking would be endangered. In the engineering sense it means that a good "brake balance" between front and rear brakes has to be provided. The problem is that front and rear brakes in a vehicle usually differ by type, kind and dimensions (in one axle brakes have to be same). That is why "brake tuning" makes a rather difficult engineering task, not only with regard to the braking system design but to its maintenance as well.

A good "brake tuning" requires a good wear balance between all brakes in a vehicle, particularly between front and rear. Since the clearance between moving and stationary parts of a brake (when it is off the action) has a direct influence on vehicle stability during braking, as it is explained above, a good "brake tuning" is dependent on the wear properties of the friction pair elements, and in particular on the wear properties of elements made of friction materials - brake linings or pads. Clearance must be maintained within determined tolerance limits - otherwise a brake would not be able to fulfill its mission. Therefore, a large clearance resulting from a worn-out brake, i.e. worn-out friction element ("lining") and exceeding the predetermined threshold represents an important and typical brake failure requiring maintenance. After several adjustments, the brake lining become worn-out completely and further adjustments are not possible. In this case a brake may be repaired by replacing the worn-out lining by a new one. It is important to have in mind that from the technological point of view brake lining replacement always comprises brake adjustment, meaning that brake lining replacement is more complicated and more time consuming maintenance activity.

3. THE APPROACH TO THE PROBLEM SOLUTION

Preventive maintenance has been described in many papers and in a number of cases it was related to wear failures [3, 7, 12, 21, 22]. It is sometimes considered as the most difficult activity in the field of maintenance [6], particularly because of the difficulties in establishing the relations between the components reliability and the frequency of performing preventive maintenance. However, the benefits which preventive maintenance is providing for practically all technical systems is widely and generally accepted as an obvious fact [7, 12, 20]. That's why preventive maintenance attracts so much attention.

The traditional method for solving different preventive maintenance tasks is by modelling the most important factors affecting preventive maintenance [2, 4, 13, 18]. These models usually are the relationship between periods of preventive maintenance and the most important output characteristics: maintenance cost per unit time or components availability. Some other optimization criteria are proposed as well [13, 18, 19].

Because of complexity of the problem, modelling is always based on some assumptions or simplifications. The quality of a model depends very much on the reality of the assumptions. The same applies to the worthiness of the estimates made by a model.

To model preventive maintenance for the problem explained in the introductory section, i.e. for a braking system having two kinds of failure: partly worn-out and completely worn-our brake linings, the following questions have to be discussed:

- the nature of failures in braking systems and the significance of wear-failures,
- the relationship between two kinds of wear failure in a brake (are they dependent or not?),
- the character of wear failure occurrence or probability density functions for both kinds of wear failure, which means the character of the brake reliability function,
- the effectiveness of preventive maintenance with regard to two kinds of wear failure, i.e. whether adjustments and lining replacement renew the braking system, restoring it to the "as new" condition,

- the maintenance policy for braking systems of motor vehicles used in large fleets, regarding road safety, operational and other requirements, and
- the criterion for preventive maintenance optimization (minimization of operating costs of maximizing the fleet availability).

3.1. The Nature of Failures in Braking Systems

It is obvious that a need for a new brake application does not depend on previous brake applications. Each next brake application depends solely on traffic conditions, traffic or other operational requirements, driver willing etc. The need for braking will be the same for a new and for an old vehicle, nevertheless on its mileage. It means that braking is a stationary, homogenous Poisson process.

On the contrary, the brake system failures depend on the previous history of the system, primarily on its age. It relates particularly to wear failures in brake linings. The failure occurrence in a brake is therefore a nonstationary, delayed process, characterized by an increasing rate of failure occurrence. It applies to both kinds of wear lining failure, requiring adjustment or replacement.

The wear failures make the most important type of all failures in a braking system, requiring much maintenance attention and effort. For friction brakes, which are normally used in all braking systems, wear failures are inevitable and they can not be avoided. Design and maintenance engineers can only control and monitor the wear failure development trying to postpone their occurrence and their negative effects.

Wear is a very complex phenomenon, attracting much attention from both design and maintenance viewpoint [9, 11, 24]. It is a typical monotonous random process, influenced by a number of deterministic and stochastic factors and characterized by a progressive rate. The rate of occurrence of wear failures is therefore also increasing with time. It was shown that the wear failure occurrence in vehicle brakes has a character of a Weibull process [8, 10, 23].

Wear in a brake is produced by each brake application, by separate "blows" [7]. After each "blow", wear is accumulated, which means that it has an additive character, a character of a cumulative process. Wear failures occur when wear exceeds a predetermined limit which represents a critical value with regard to brake operation. To reach the point at which wear has exceeded this allowed value, a large number of brake applications is usually performed. It depends, however, on many factors.

The wear output, or lining life, is very different not only for different brakes fitted with different materials but also for brakes that are considered to be "the same" and which are applied under "the same" usage conditions. There are several approaches to this problem, some being proved effective in engineering practice [16, 24, 25].

The mechanism of failures caused by wear is complex. For the purpose of these analyses, however, it is not necessary to consider the manner in which the wear failure propagate. Wear failures in this approach are treated as random and ultimate events, after which the observed brake is not able to fulfill its mission, i.e. to brake the vehicle properly and efficiently. It applies to both kinds of wear failure: partly worn-out and totally (or completely) worn-out lining failures, requiring two kinds of maintenance actions, namely adjustment and replacement.

3.2. The Relationship Between Two Kinds of Wear Failure

The preventive maintenance of systems having two or more types of failures was discussed in several papers [3, 17, 22]. The failures are considered as independent and not interrelated. However, the wear failures analyzed in this paper (partly worn-out and completely worn-out linings failure) are dependent. This results from the following:

- the mechanisms by which failures of both kinds are originated is the same, i.e. they both originate from the same deterioration process,
- the completely worn-out lining failure occurs after a number of adjustments have been done, i.e. after several occurrences of partly worn-out lining failures,
- the two kinds of wear failure are mutually exclusive: at a moment a brake can fail only because it is either partly worn-out or because it is completely worn-out; a completely worn-out brake cannot be partly worn out at the same time.

3.3. The Character of Wear Failures

The expected average number of failures $N(T)$ in a cycle of length T is a very important function for planning preventive maintenance. Function $N(T)$ depends on the effectiveness of maintenance activities. If both of preventive and corrective maintenance activities restore the component to the "as new" condition $N(T)$ represents an ordinary renewal function. If only preventive maintenance restores the component to the "as new" condition, $N(T)$ is a delayed renewal process [1].

Function $N(T)$ depends on the character of the failure rate, i.e. on the reliability function. This dependence is very complex and therefore there are various approaches to its modelling. A very convenient estimate for function $N(T)$ is given in the form [13]:

$$N(T) < F(t) \,/\, R(t) \qquad (1)$$

where $F(t)$ is unreliability function, $R(t)$ is reliability function, and t represents time.

Accepting that expression (1) provides a good estimate for the average number of failures in a cycle of length T, it is necessary to assume that reliability functions with regard to failures are analyzed in this paper. The comprehensive investigation of braking system reliability realized in laboratory and under service conditions gave a good basis for that. These investigations have shown that wear failures in vehicle brakes represent a Weibull process [8, 10, 16, 23], which can be interpreted by the Weibull distribution /14/, i.e.:

$$R_w(t) = \exp\,[-(t/\eta)^{\beta}],\ t > 0 \qquad (2)$$

where:

$R_w(t)$ - the reliability function related to brake wear failures,
η, β - the Weibull parameters.

It is worthwhile to outline that almost in all tests the values for the Weibull shape parameter β lied in the interval from 2,5 to 4, conforming to the previously explained increasing rate of wear failure occurrence and its nonstationary character.

On this basis, and having in mind that both kinds of wear failure discussed here originate from the same random process, reliability functions for the two kinds of wear failure can be expressed by means of the Weibull probability distribution having the same value of the shape parameter. It has been proved in a number of experiments and surveys. This can be expressed in the following form:

$$R_p(t) = \exp[-(t/\eta_p)^\beta] \text{ and } R_c(t) = \exp[-(t/\eta_c)^\beta] \quad (3)$$

where:

$R_p(t)$ - the reliability function related to *partly worn-out lining* failures,
$R_c(t)$ - the reliability function related to *completely worn-out lining* failures,
η_p, η_c - the scale parameters for *partly and completely worn-out lining* failures, respectively,
β - common shape parameter ($\beta = \beta_p = \beta_c$).

3.4. The Effectiveness of the Preventive Maintenance of Brakes

A very important feature of maintenance is related to the degree of a component renewal. The general attitude is that both preventive and corrective maintenance restore a component to the "as new" condition, meaning that the renewal process is an ordinary one [1]. From engineering practice, however, it is well known that maintenance in many cases does not provide a real "renewal". A component's or system's performance are poorer after maintenance then before. The decrease in reliability after maintenance is even more probable, particularly with regard to the residual lives [17]. It means that in general it is not correct to consider maintenance as action restoring a technical system to the "as new" condition.

However, it is not easy to check or test this statement in practical situations. Most probably, it is reason that in spite of all shortage this assumption is usually accepted for modelling the maintenance policy for solving practical maintenance tasks [1]. Therefore, the assumption that both brake adjustment and lining replacements restore brake to the "as new" condition is accepted for the analyses in this paper also.

The target of the analysis presented here, i.e. the development of a procedure which will provide improved and more efficient maintenance planning for front and rear brakes in motor vehicles, supports this assumption. Namely, for the problem observed in this paper it is primarily important to achieve the "best" relationship between necessary maintenance actions, so as to satisfy the relevant criterion. The provisions for providing a "higher" reliability or longer component life are not in the focus of our attention. The differences in residual life and reliability after subsequent adjustments may be therefore ignored.

3.5. The Maintenance Policy For Braking System

It is not a question whether the maintenance of motor vehicles and their components has to be preventive. This policy is the only acceptable and obvious. The question is, however, which type of preventive maintenance is the most convenient for determined vehicle types and components and for specific usage conditions.

The "on-condition" preventive maintenance has proven very effective in many cases, including maintenance of motor vehicles, particularly if used in large fleets [6]. It means that maintenance actions depend on the state of the unit, established by continuous monitoring or periodical checking by means of appropriate instruments, sensors or similar.

This type of preventive maintenance is probably the best one for maintenance of vehicle braking systems and brakes also. However, since brakes have an exceptional impact on road safety, the preventive maintenance according to the "hard-time limit" concept has to be applied as well. It means that adjustments and replacements of friction linings have to be done not only on the basis of the "condition" established by periodical checking, but also after certain intervals of time. The "hard-time limit" maintenance means that adjustments and replacements of nonfailed elements have to be done before failure occurs, i.e. before a poor condition of an element is established. This maintenance concept is very convenient from the technological viewpoint also, offering possibilities for higher operational readiness and higher availability. It is obvious that for an effective maintenance it would be convenient if time or mileage period for "hard-time" replacement represents a multiple of the time or mileage period for "hard-time" brake adjustment. This will be explained later.

3.6. The Criterion of Optimization

The time or mileage periods of preventive maintenance have to be determined so as to satisfy the requirements coming out from the defined target or goal function. Maintenance models are the best tool for solving this problem. Maintenance models give "the best" solution for the accepted optimization criterion.

This typical optimization problem usually boils down to the minimization of total maintenance costs or to the maximization of availability [1, 2, 13, 18, 20]. It may be expressed by the following:

$$C(T) = [C_{PM} + N(T)\, C_{CM}]/T \quad (4)$$

and

$$A(T) = [T - t_{PM} - N(T)\, t_{CM}]/T \quad (5)$$

where:

$C(T)$ - the total maintenance cost of component per unit time over time interval T,
C_{PM} - the average cost of preventive maintenance,
$N(T)$ - the expected average number of component failures over time interval T.

C_{CM} - the average cost of corrective maintenance,
T - time interval,
$A(T)$ - the component availability per unit time over time interval T,
t_{PM} - the average down time related to preventive maintenance over time interval T,
t_{CM} - the average down time related to corrective maintenance over time interval T,

The cost optimization criterion is used almost generally but the availability optimization criterion is often more convenient for maintenance of various mechanical components and systems. In particular, it is the case with utility and public transport systems which have to provide the highest possible quality of service meaning a very high availability. That's why this criterion is accepted for the task which is elaborated in this paper.

4. THE PREVENTIVE MAINTENANCE MODEL

Although the availability of a technical system over a period of time T can be expressed in the following general form:

$$A(T) = t_u/(t_u + t_d) \tag{6}$$

where t_u is *up time* and t_d is *down time* over a time period T in further analyses a more convenient form as given by expression (5) will be applied.

Dealing with two kinds of failure, namely partly and completely worn-out linings (using indexes p and c, respectively, as marked before) availability can be expressed in two forms, each one related to one kind of failure. Hence:

$$A_p(T_p) = [T_p - t_{PM_p} - N_p(T_p)\, t_{CM_p}]/T_p \tag{7'}$$

and

$$A_c(T_c) = [T_c - t_{PM_c} - N_c(T_c)\, t_{CM_c}]/T_c \tag{7''}$$

where:

$A_p(T_p)$ and $A_c(T_c)$ - availability related to partly (p) and completely (c) worn-out linings over the time intervals T_p and T_c, respectively,
t_{PM_p} and t_{PM_c} - the average down time related to two kinds of preventive maintenance (PM) actions, namely: adjustment caused by partly (p) worn-out linings and replacement caused by completely (c) worn-out linings, respectively,
t_{CM_p} and t_{CM_c} - the average down time related to two kinds of corrective maintenance (CM) actions, namely: adjustment caused by partly (p) worn-out linings and replacement caused by completely (c) worn-out linings, respectively.

Using expression (1), formulae (7') and (7'') become:

$$A_p(T_p) = \{T_p - t_{PM_p} - [F_p(T_p)/R_p(T_p)]\, t_{CM_p}\}/T_p \tag{8'}$$

and

$$A_c(T_c) = \{T_c - t_{PM_c} - [F_p(T_c)/R_p(T_p)]\, t_{CM_c}\}/T_c \tag{8''}$$

It means that the optimal periods for preventive adjustment and replacement are determined by the minimizing of the functions (8') and (8''), i.e.:

$$\delta A_p(T_p)/\,\delta T_p = 0 \qquad (9')$$

and

$$\delta A_c(T_c)/\,\delta T_c = 0 \qquad (9'')$$

Hence, for a set of brake linings (in one brake) there are two optimums for preventive maintenance, one for preventive adjustment and another for preventive replacement. The same applies for each brake. It means that if four brakes in a vehicle are considered separately, the expressions (9') and (9'') will give eight optimums, i.e. eight preventive maintenance periods providing maximum availability with regard to both kinds of wear failure. Such a maintenance schedule would obviously be very inefficient and expensive.

In reality, this situation is simplified by the fact that two brakes fitted at one axle may be considered to be similar. It means that for a two-axle vehicle there are only four optimums: two for preventive adjustment of both front and rear brakes and two for preventive replacements of both front and rear brakes. These optimums giving the highest availability with regard to preventive adjustment and replacement of brakes fitted to front (f) and rear (r) wheels in such a vehicle may be expressed by the following notations:

- $A_{fp}(T_{fp})$ = the maximum availability of front brakes (f) with respect to their preventive adjustment over the optimum period of preventive adjustment T_{fp} associated with partly (p) worn-out brakes,
- $A_{fc}(T_{fc})$ = the maximum availability of front brakes (f) with respect to their preventive replacement over the optimum period of preventive replacement T_{fc} associated with completely (c) worn-out brakes,
- $A_{rp}(T_{rp})$ = the maximum availability of rear brakes (r) with respect to their preventive adjustment over the optimum period of preventive adjustment T_{rp} associated with partly (p) worn-out brakes,
- $A_{rc}(T_{rc})$ = the maximum availability of rear brakes (r) with respect to their preventive replacement over the optimum period of preventive replacement T_{rc} associated with completely (c) worn-out brakes.

Such a maintenance schedule will be still complicated, inefficient and expensive. It is logical therefore to combine some of the maintenance activities so as to perform them simultaneously in the course of the same down time. In general, it would contribute to more effective maintenance control and management and will decrease operational costs.

However, since the rates of failure occurrence for front and rear brakes are not the same, simultaneous maintenance would mean that at brakes having more favorable reliability the maintenance action would be performed earlier then needed. This may increase the maintenance costs. However, in the case of brake adjustment actions, this increase in costs will be negligible. Therefore, simultaneous adjustment of all brakes in a vehicle is a general practice in all maintenance schedules.

If replacement of brakes is considered, the premature replacement may cause important additional expenses. Simultaneous replacement of all brakes in a vehicle may be rational only if reliability of the front brake is similar to that of the rear brake, giving rather small difference between their mean lives. Since it usually is not the case, maintenance schedules generally prescribe the replacement of brakes at each vehicle axle to be realized separately. If such an approach is applied in the case of a two-axle vehicle there will be three optimum maintenance periods, one for simultaneous adjustment of all brakes and two for separate replacement of front and rear brakes, respectively.

This problem may be simplified if certain relations between three optimal maintenance periods would be established. It may be achieved by putting the relationship between time intervals for replacements and for adjustments to be expressed by integer, as explained in Figure 1 and by means of the following expression:

$$T_{fc}/T_p = M \quad \text{and} \quad T_{rc}/T_p = N \tag{10}$$

where:

T_p - the optimum period of simultaneous adjustment of all front and rear brakes,
M and N - integers.

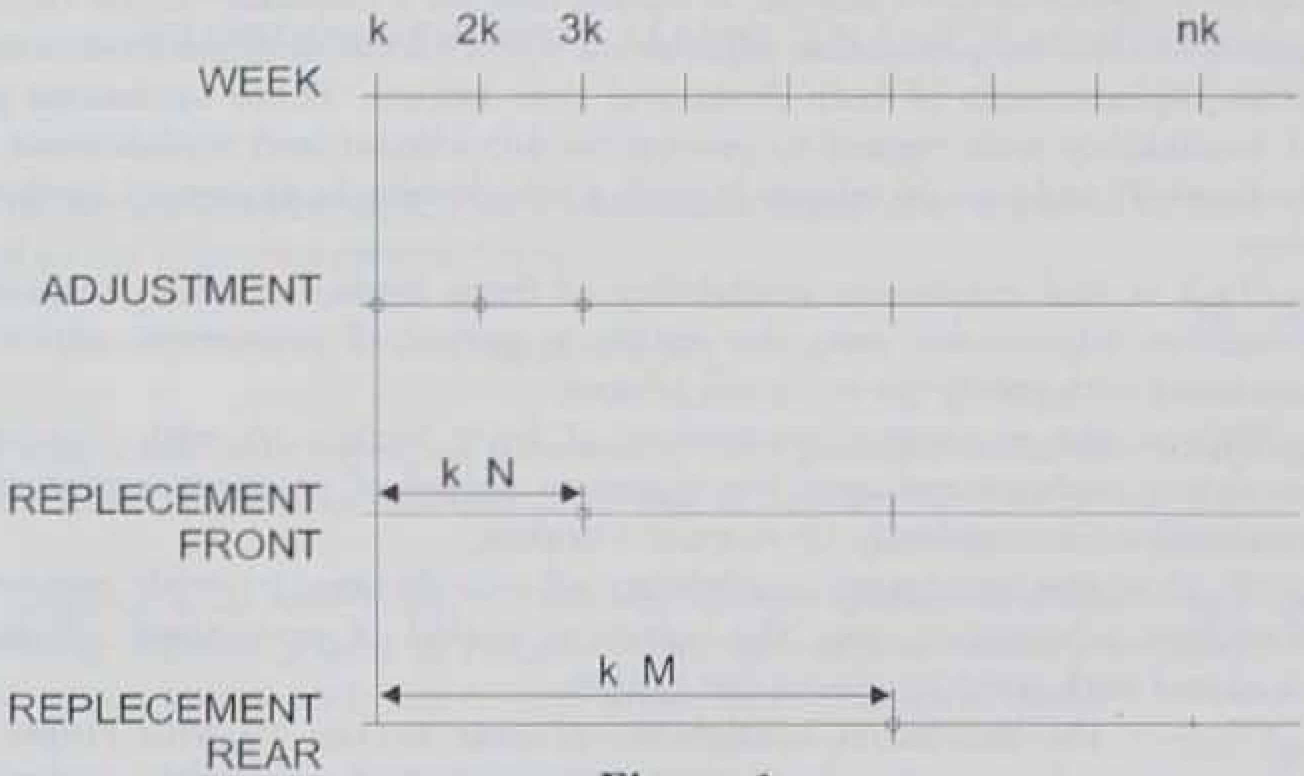

Figure 1

An additional convenience may be achieved if the ratio between figures M and N may also be made a whole number, i.e. if the wear balance between front and rear brakes may be expressed as an integer Q, or:

$$M/N = Q \tag{11}$$

5. AN EXAMPLE

To illustrate the models an example of braking systems of two-axle public transport vehicles used within a large fleet will be demonstrated. The vehicles have to provide the highest possible quality of service or availability. It means that maintenance policy explained above, representing a combination of "on-condition" and "hard-time limit" concepts, has to be optimized with regard to fleet availability. From the previous investigations under usage conditions, characterized by an average vehicle usage intensity of about 15 km per hour or about 250 km per day [8,10, 23] it may be assumed that reliability of the front (f) and rear (r) brakes can be expressed by the following functions:

- Front brake adjustment: $R_{fp}(t) = \exp[-(t/300)^3]$
- Front brake replacement: $R_{fc}(t) = \exp[-(t/1000)^3]$
- Rear brake adjustment: $R_{rp}(t) = \exp[-(t/600)^4]$
- Rear brake replacement: $R_{rc}(t) = \exp[-(t/2500)^4]$

Besides, it will be assumed that the average time for preventive adjustment is t_{pm_p} = 5h (for all brakes in the vehicle) and the average time for preventive replacement is t_{pm_c} = 50h (for one brake), while corresponding average times for corrective maintenance of brakes are t_{cm_p} = 15h and t_{cm_c} = 150h.

With these figures, expressions (8') and (8'') will give the availability functions with regard to all four maintenance actions. These functions are plotted in Figure 2. It can be seen that the maximums of all four curves are achieved for very different times T, taking values from 156 to 1402 hours. It proves that it is difficult to design such maintenance programme which would link all four maintenance actions so as to obtain a common optimum.

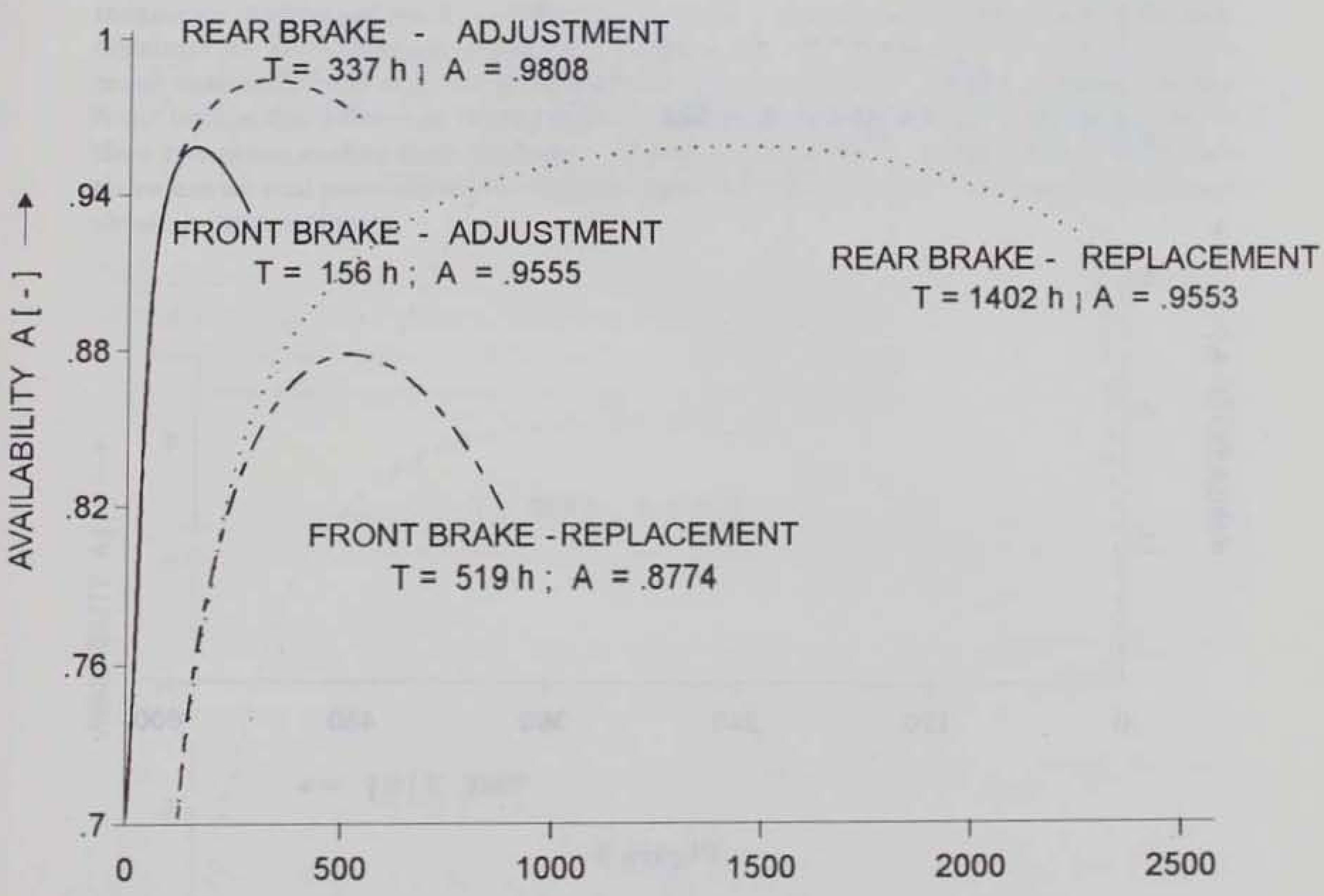

Figure 2

The picture is a little bit better if only adjustments of brakes on the front and the rear wheels are considered: the optimums correspond to 156 and 337 hours. In this case it seems reasonable to perform simultaneous adjustments of both front and rear brakes, after a period of time which will be the maximum of the multiple of two availability functions. The plot of this new function is presented in Figure 3., pointing out that the common maximum for both adjustment functions is obtained for 186 hours, or about 30 hours above the optimum obtained for only the front brake adjustment, but even 150 hours below the optimum for the rear brake adjustment. It is important to notice, however, that the differences in availability by performing separate adjustment of each brake are not very high: availability of the front brake adjustment for T = 186 hours decrease to 0.9537 or for only 0.2%, while corresponding decrease for rear brake adjustment availability reaches about 10.5%.

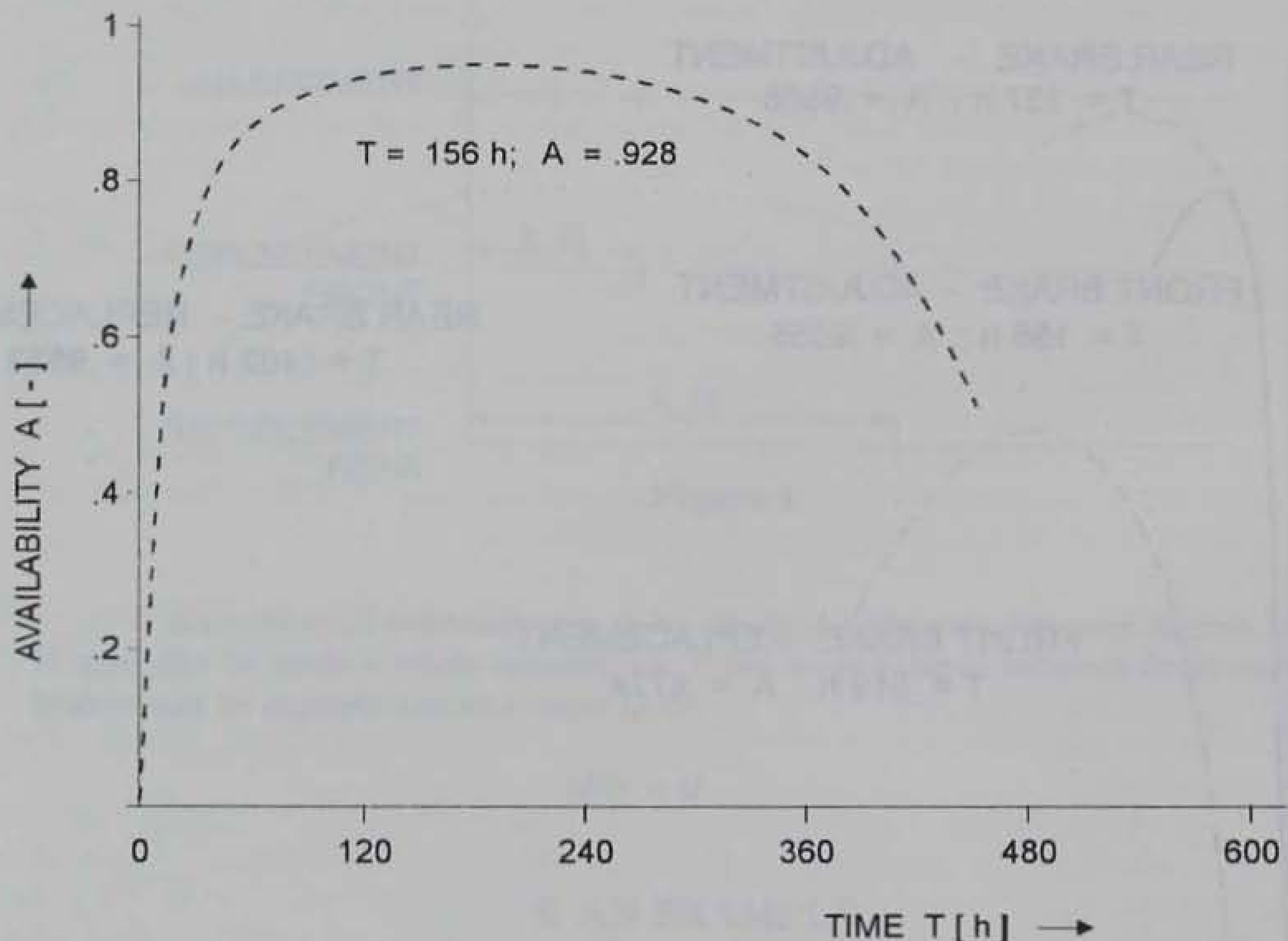

Figure 3

A good reason for accepting such a "joint optimization" of brake adjustment comes out also from technological reasons. It is an usual practice in motor vehicle fleets similar to the one under observation to organize preventive maintenance in segments which correspond to certain time intervals, like one day, one week, a month or so. Having in mind the usage intensity mentioned above (250 km per day in average) the optimum period for front brake adjustment corresponds to app. 2400 km and for the rear to app. 5000 km. In other words, front brakes have to be adjusted every 10 days and rear brakes every 20 days, or about every two and every three weeks. For a joint optimum of 186 hours or 2800 km the adjustment has to be done every 11 days. Since in a public transport a week means in average 6 working days or 1500 km, brake adjustments have to be performed approximately every 2 weeks or after 3000 km or every 200 hours of operation. It is evidently not too far from the optimums obtained for the front and the rear brake separately.

The situation would be quite different if "joint optimization" of preventive replacement of the front and rear brakes is considered. The optimums obtained for these actions are rather far-off: 519 and 1402 hours, as given in Figure 2. It means that the replacement of the front brakes has to be performed roughly every 8.000 km or after one month of service, while these figures for the rear brakes are 20.000 km or three months. It results from significant wear unbalance between front and rear brakes, which originates from design reasons and different brake loading. This can be seen even more clearly if the joint maximum of two derived availability functions are considered, as it is done above. The diagram plotted in Figure 4 shows that the joint maximum is obtained for $T = 629$ hours, what is rather far away from both optimums obtained for each function separately. If this joint optimum would be applied, it will mean that both front and rear brakes have to be replaced after roughly 40 days. For the front brakes the difference is not too large, but the rear brakes would be replaced more then two times earlier then necessary. This is obviously quite uneconomical. Therefore there are no real possibilities to combine these two preventive actions together, so as to obtain a joint optimum.

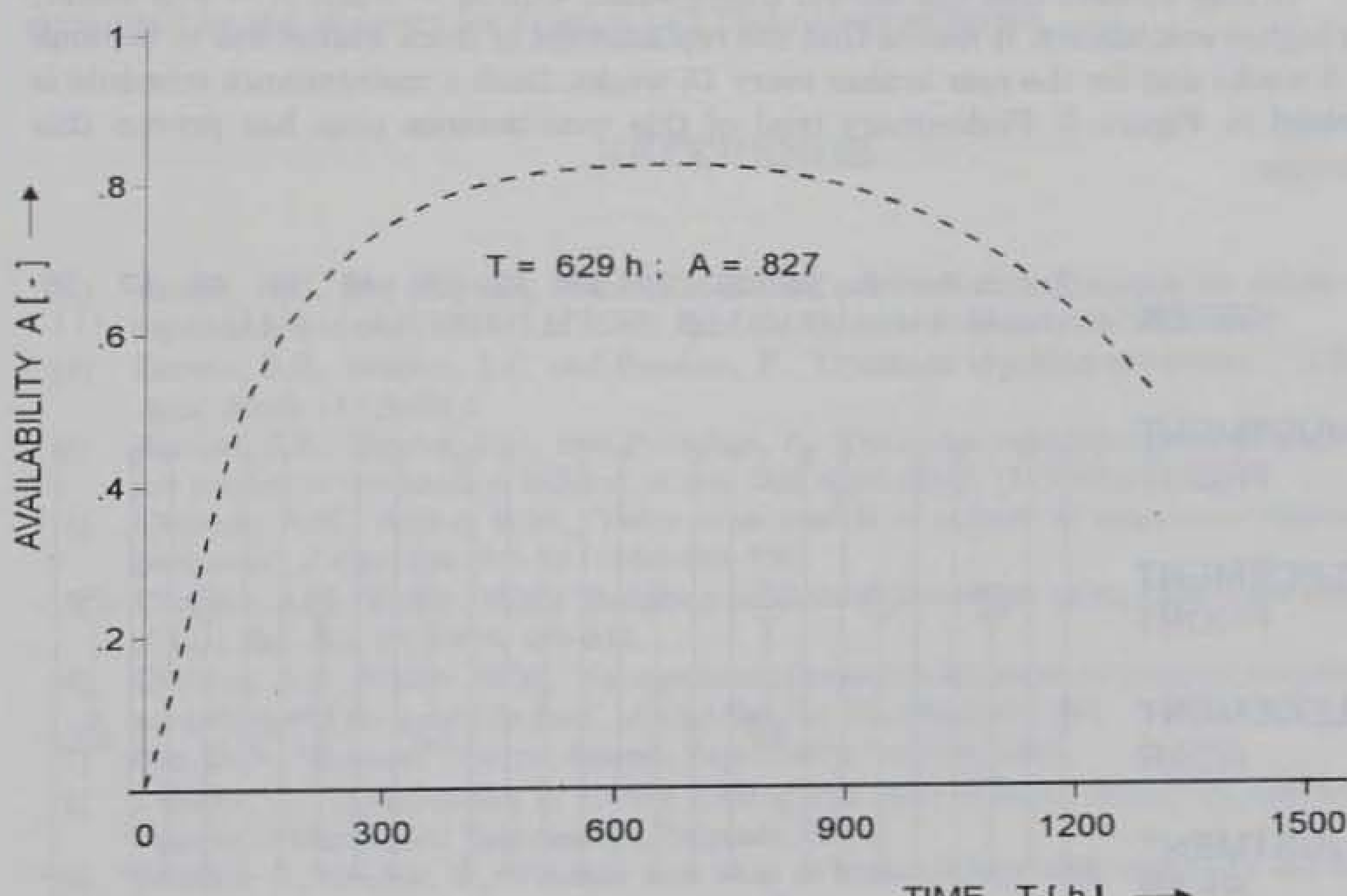

Figure 4

However, a very positive effect can be achieved if the time periods for preventive replacements are put in the relation to corresponding intervals for brake adjustments, as it is explained by expressions (10). Namely, the comparison of the optimum obtained gives:

$$T_{fc}/T_p = 519/200 = 2{,}60 = M$$

$$T_{rc}/T_p = 1402/200 = 7{,}01 = N$$

The figures *2,60* and *7,01* have to be rounded up so as to become integers and to give the ratio between them as the integer Q as well (expression 11).

It is easy to conclude that there are two solutions: M can be rounded up to 2 or 3 and N to 8 or 9, giving Q = 3 or 4. In such a way the periods of preventive replacement are T_{fc} = 400 or 600 hours and T_{rc} = 1600 or 1800 hours.

Using the diagrams shown in Figures 2 and 3., these two cases are compared in Table 1.

Table 1

T_{fc}	T_{rc}	$A_{fc}(T_{fc})$	$A_{rc}(T_{rc})$	$A_{fc}(T_{fc})xA_{rc}(T_{rc})$
400	1.600	0.86973	0.95386	0.912
600	1.800	0.87442	0.94924	0.912

It may be seen that the second combination with M = 3 and N = 9 is better, giving higher availability. It means that the replacement of front brakes has to be done every 6 weeks and for the rear brakes every 18 weeks. Such a maintenance schedule is illustrated in Figure 5. Preliminary trial of this maintenance plan has proven this programme.

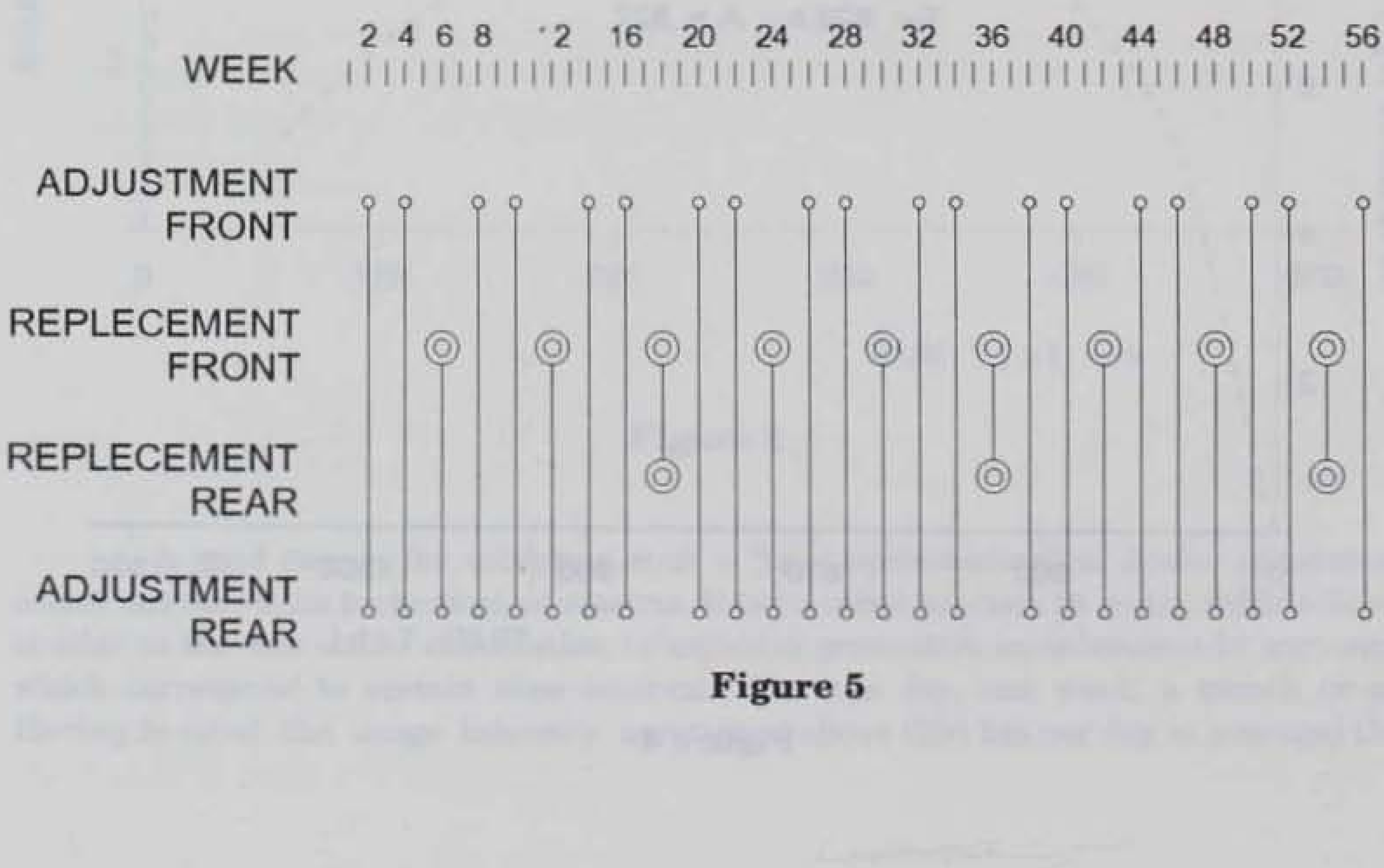

Figure 5

7. CONCLUSIONS

1. The mechanical systems in which the most important failures originate from excessive wear are specific in several aspects. Firstly, failures originating from wear may be of two kinds, one requiring adjustment and another requiring replacement. Secondly, different kinds of wear failure have a direct impact on the maintenance policy, creating specific problems with regard to preventive maintenance schedules. Therefore, the optimization of maintenance of this type of engineering systems requires a thorough understanding of all relevant factors.
2. Since two kinds of failure in friction mechanisms used in automotive brakes originate from the same wear mechanism, it is possible to combine corresponding preventive maintenance actions. One of the solutions is to design such a braking system so as to obtain the wear balance between front and rear brakes providing that intervals for preventive adjustment and preventive replacement of each brakes can be expressed also as integers. Such a maintenance schedule has proved quite acceptable and satisfactory.
3. If the optimization criterion of optimality is availability, as it is usually accepted for public services and utility systems, the optimization has to be achieved by successive approximation, comparing different possible combinations.

Acknowledgment. This paper was prepared under a research project financially supported by the Ministry for Science and Technology of Serbia.

REFERENCE

[1] Ascher, H.E., and Kobbacy, K.A.H., "Modelling preventive maintenance for deteriorating repairable systems", *IMA J. of Math. Appl. in Buisness & Industry* 6(1995)85-99.

[2] Barlow, R.E., Hunter, L.C. and Proshan, F., "Optimum checking procedure ", *J.Soc.Ind. Appl. Math.* 11 (1963) 4.

[3] Barlow, R.E., Hunter, L.C., and Proschan, F., "Optimum redundancy when components are subject to two kinds of failure", *J. Soc. Ind. Appl. Math.* 11 (1963)1.

[4] Christer, A.H., Waller, W.M., "Delay time models of industrial inspection maintenance problems", *J. Opl. Res. Soc.* 35 (1984) 401-406.

[5] Christer, A.H., Waller, W.M., "Reducing production downtime using delay-time analyses", *J. Opl. Res. Soc.* 35 (1984) 499-512.

[6] Christer, A.H., Waller, W.M., "An operational research approach to planned maintenance: modelling PM for a vehicle fleet", *J. Opl.Res.Soc.* 35 (1984) 967-984.

[7] Cox, D. R., *"Renewal Theory"*, Science Paperbacks, London, 1967.

[8] Duboka, Č., "Investigation of service loading and wear of brake lining" (in Serbo-Croat), Faculty of Mechanical Engineering, Belgrade, 1977.

[9] Duboka, Č., Arsenić, Ž., "Friction and wear of brake linings with regard to the bedding procedure", *SAE Paper* 865130, Pitsburg, 1986.

[10] Duboka, Č., Arsenić, Ž., "Distribution of time between failures in friction mechanisms", *3rd Balkan Conference on OR*, Thessaloniki, October 1995.

[11] Esary, J.D., Marshall, A.W., and Proschan F., "Shock models and wear processes", *The Annals of Probability* 1 (1973) 627-649.

[12] Giglmayr, J., "An age-wear dependent model of failure", *IEEE Transactions on Reliability* R-36(1987)5.

[13] Gertsbakh, I. B., *Models of Preventive Maintenance*, North-Holland, New York, 1967.

[14] Gnedenko, B.V., Belyayev, Yu.K., and Solovyev, A.D., *Mathematical Methods of Reliability Theory*, Academic Press, New York, 1969.

[15] Handlarsky, J., "Mathematical analysis of preventive maintenance schemes", *J. Opl. Res. Soc.* 31(1980)227-237.

[16] Harding, P.R.J., "An investigation into the operational life of disc brake pads", *Proc. IMechE.* C11/83, London, 1983.

[17] Joe, H., and Proschan, F., "Comparison of two life distributions on the basis of their percentile residual life functions", *The Canadian Journal of Statistics* 12(1984)2, 91-97.

[18] Jorgenson, D.W., McCall, J., and Radner, R., *Optimal Replacement Policy*, North-Holland, New York, 1974.

[19] Kay, E., "The effectiveness of preventive maintenance", *Int. J. Prod. Res.*13(1976)329-344

[20] *Optimale Instandhaltungs-methoden - ein praktischer Leifaden*, VEB Verlag Technik, Berlin, 1983.

[21] Park, K.S., "Optimal wear-limit replacement with wear-dependent failures", *IEEE Transactions on Reliability* 37 (1988) 3.

[22] Tillman, F.A., "Optimization by integer programming of constrained reliability problems with several modes of failure", *IEEE Transactions on Reliability* R-18(1969)2.

[23] Todorović, J., "Investigation of brake lining reliability under usage conditions" (in Serbo-Croat), Faculty of Mechanical Engineering, Belgrade, 1972.

[24] Todorović, J., Duboka, Č., and Arsenić, Ž., "Operational life expectancy of rubbing elements in automotive brakes", accepted for publication in *Tribology International*, London, 1995.

[25] Van Heck, J.G., "Prediction of wear and operational life of drum brake linings", *SAE Paper* 885141, Pitsburg, 1988.