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A MULTI-CRITERIA MULTI-PERIOD SCHEDULING MODEL OF MEDICAL RESOURCES

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Abstract: A modelling framework is proposed to study the allocation of doctors during specific time blocks to individual clinics of a major multi-branch medical practice in Hong Kong. This problem is motivated by our previous simulation work in one of the typical clinics that has identified bottlenecks in doctor's availability at various times. This paper suggests a non-linear integer multi-criteria formulation with four prioritized goals: minimum loss of demand, minimum average waiting time, minimum doctors idleness and maximum use of consultation rooms. Our computational results from such a modelling framework (or a simplified model structure) show it can generate allocation decision support predicting possible operational improvement as well as helpful insight into the existence and explanation of system bottlenecks arising from a combination of more than one factors.

Keywords: Scheduling, medical resources, goal programming, average service rate.

1. INTRODUCTION

The background of this study is the operation management of the multiple branches (or clinics) of a major medical practice in Hong Kong [8,13]. Medical consultations at its individual clinics are provided by different numbers of doctors (from a common pool of full - and part - time physicians) for different hours of the day and different days of the week. This scheduling variation is necessary for primarily two reasons: patient demand is not constant over time and the idle time of doctor's is very expensive.

In a previous study of ours [9], work was carried out to build an extensive and detailed simulation model with a process-tracing (or patient-following) technique [6,11] for a typical clinic.The simulation adopts the usual model of a complex queueing network [4,12]. Statistical findings and analyses from our simulation project [9] have identified one of the dominant factors causing bottlenecks in the system's operational efficiency to be doctor availability at various times and locations. This discovery led to our present work on proposing a mathematical modelling framework to generate decision support to optimize the use of doctors time as a scarce resource. The objectives are to, firstly, alleviate the bottleneck effect as much as possible by optimal doctor allocation; and secondly, further pinpoint any subsequent limitations that may result from such improved doctor schedules. Our experience with the system and its simulator has suggested four prioritized goals: minimum loss of customer demand, minimum average patient waiting time, minimum doctor idleness and maximum use of consultation rooms. These factors, also commonly adopted in the literature [1,7], taken together translate into maximizing profit, utilization of resources and perceived systems efficiency.

Putting these multiple objectives into mathematical statements gives rise naturally to a goal programming problem with integer decision variables [10] representing the schedule of allocating doctors across all branches for specific periods. The structural (or non-goal) constraints include capacity restrictions, being the number of doctors and rooms at various times and places. Our model differs from any conventional integer goal program of resource allocation in that our goal constraints resulting from the target average of patient waiting time are highly non-linear (as they consist of queueing formulae [3]). In this paper, where our intention is to put together a framework for more extensive computation to follow, we consider a simplified non-linear but inversely proportional relationship between waiting time and the number of doctors allocated. Also the numerical results are obtained by an iterative rather than a direct solution approach to circumvent the non-linearity of the model [5]. This has enabled us to exemplify the overall modelling idea and its solution technique as such, which may then incorporate estimates of model parameters from detailed simulation of the actual system performance [9]. This integration of simulation and allocation modelling will be our future direction of work. Another insight gained from this pilot study is the important interplay between the availability of space (or consultation rooms) and manpower (or medical doctors) in the face of external patient demand. Numerical results are reported as an illustration to this modelling approach.

2. ALLOCATION MODEL

The whole of the medical practice consists of I clinics, indexed by $i = 1, \dots, I$. The hours of a clinic work week are divided into time blocks of morning and afternoon sessions, indexed by $t = 1, \dots, T$. The medical practice employs a total of N doctors. They are the general practitioners (GP's), as opposed to a (smaller) number of specialized physicians whose assignments are pre-determined because they work on a strictly appointment basis and are available only at particular clinics at specific times. Hence our allocation refers only to the GP's (whom we shall simply call the doctors).

Parameters and Variables

For a given time block t, the maximum number of consultation rooms available at clinic i is given by $R_{it} = Size(i) - Spec(i,t)$, where $Size(i)$ is the number of rooms at clinic i and $Spec(i,t)$ is the number of specialists scheduled to work there at time t.

Demand D_{it} is defined as the mean number of patient arrivals; while the *current* number of doctors assigned is denoted by n_{it}. The average waiting time W_{it} hence reflects the *current* setup of the level of service and demand. A useful quantity for our allocation model is the *average service rate* per doctor, given by

$$S_{it} = \sum_{j \in J_{it}} \frac{a_j h_{itj}}{n_{it}}, \quad i = 1, \ldots, I; \quad t = 1, \ldots, T \tag{1}$$

where J_{it} indicates the set of doctors on duty, a_j is the unit consultation rate of the j^{th} doctor, and h_{itj} is his amount of time on duty during time block t at clinic i. Note that these quantities are calculated with respect to the current allocation n_{it} and schedule J_{it} of the doctors. Also $1/S_{it}$ then gives the average requirement rate (i.e. the number) of doctors per patient. The decision variables of our model are the allocations denoted by x_{it}, which represent the number of doctors assigned to clinic i for time block t.

Goals and Constraints

There are four prioritized performance goals represented by the deviation variables d_{it}^-, being the loss of patient demand; w_{it}^+ being the patient's average waiting time; d_{it}^+, being proportional to doctor idleness; and c_{it}^-, being the number of unused consultation rooms. We adopt non-preemptive priorities [10] of weighting factors $P_1, \ldots, P_4$, respectively for the four goals.

The units of these deviation variables also reflect our modelling concept: d_{it}^- in number of patients, w_{it}^+ in minutes of waiting time, d_{it}^+/S_{it} in number of doctors, and c_{it}^- in number of rooms.

The first three criteria give rise to the following goal constraints

$$p(x_{it}) + d_{it}^- - d_{it}^+ = D_{it}, \qquad i = 1, \ldots, I; \quad t = 1, \ldots, T \tag{2}$$

$$q(x_{it}) + w_{it}^- - w_{it}^+ = W_{it}, \qquad i = 1, \ldots, I; \quad t = 1, \ldots, T \tag{3}$$

where $p(x_{it})$ and $q(x_{it})$ denote the number of patients served and the average waiting time of patients for service, respectively, as functions of the number of doctors allocated x_{it}. The last criterion leads to structural constraints of the form

$$x_{it} + c_{it}^- = R_{it}, \qquad i = 1, \ldots, I, \quad t = 1, \ldots, T \tag{4}$$

since it is impossible to have over-achievement of the consultation rooms used.

Optimization Formulation

From (2), (3) and (4) above, the overall allocation model is described by a set of T goal programming problems given for each t and non-negative deviation variables by,

$$\text{Min } P_1 \sum_{i=1}^{I} d_{it}^- + P_2 \sum_{i=1}^{I} w_{it}^+ + P_3 \sum_{i=1}^{I} d_{it}^+ + P_4 \sum_{i=1}^{I} c_{it}^- \tag{5}$$

Subject to

$$S_{it}x_{it} + d_{it}^- - d_{it}^+ = D_{it}, \qquad i = 1,\dots,I \tag{6}$$

$$W_{it}\left(\frac{n_{it}}{x_{it}}\right) + w_{it}^- - w_{it}^+ = W_{it}, \qquad i = 1,\dots,I \tag{7}$$

$$x_{it} + c_{it}^- = R_{it}, \qquad i = 1,\dots,I \tag{8}$$

$$\sum_{i=1}^{I} x_{it} \le N \tag{9}$$

$$x_{it} = 1, 2, \dots, N - I + 1, \qquad i = 1,\dots,I \tag{10}$$

Here (6) is obtained from (2) by substituting $S_{it}x_{it}$ by $p(x_{it})$, thus assuming a proportional service rate for the doctors; and (7) is derived from (3) by postulating an inversely proportional waiting time for the patients in the form $W_{it}n_{it} / x_{it}$. Note that constraints (10) enforce the required presence of at least one doctor.

A comment is needed here to explain the assumptions on the functional forms of $p(\bullet)$ and $q(\bullet)$. We are suggesting (5)-(10) here as a modelling framework for our doctor allocation problem, rather than putting these forth as the concluding model assumptions of the actual case study. The latter will require more empirical and validated estimates for the parameter functions $p(\bullet)$ and $q(\bullet)$. This we shall pursue later with more extensive computations and integration of our previous work on the clinic simulation study [9] to produce more realistic functional dependencies of p and q on x_{it}.

3. NUMERICAL RESULTS

We provide in this section some simple computational results to illustrate our goal programming formulation idea given in (5)-(10). The model parameters used are for the case of N=28 doctors in I=12 clinics for T=11 time blocks (five and one half days of morning or afternoon sessions), and W_{it}=18 minutes of target average waiting time. Also the original assignments of doctors n_{it} are given in Table A.1 of the Appendix. The weightings $P_1,\dots,P_4$ in (5) are specified by fixing P_1 to be 1 and then setting $P_2 = W_{it}P_1$ since P_2 / P_1 should represent the average amount of waiting W_{it} of one patient. Finally, $P_3 = 10^{-2} P_1$ and $P_4 = 10^{-4} P_1$ are chosen to reflect the difference in terms of order of magnitude of the remaining two goals of lower priorities.

Goal Programming Solution

Non-linearity (7) and integrality (10) necessitate the solution approach to be iterative rather than direct [5] (treating in this case the function $q(x_{it})$ as variable coefficients [3]). The solution procedure is then as follows. Initially x_{it} are set to n_{it} in (7). This results in a simple integer goal program with deviation variables $d_{it}^-, d_{it}^+, w_{it}^-,$

w_{it}^{+} and c_{it}^{-}; plus our allocation variables x_{it}. The values of all these are calculated, for each t there is an integer program of 72 variables and 49 constraints, made by a user-friendly package QSB+[2]. This procedure is then repeated with the newly computed (integral) values of x_{it}. The computation is completed when the values of x_{it} stabilize, which, together with the deviation variables, are then taken to be solutions from the allocation model. For our sample problem here, only one iteration is required.

Improvement

The suggested allocations x_{it} as a result of the above computations are shown in Table A.1 of the Appendix, alongside the original n_{it}. We can see that there are only relatively moderate changes being recommended. These mild modifications, however, can lead to improvement in terms of the predicted overall performance measures of our model. As shown in Table 1 below, $\sum_{i,t} d_{it}^{-}$ refers to the total estimated demand loss, $\sum_{i,t} d_{it}^{+}$ gives the total over-achievement of demand hence the measure of doctor idleness and $\sum_{i,t} c_{it}^{-}$ indicates the under-usage (in number) of consultation rooms, which is directly related to the total number of doctors assigned over all clinics and times, $\sum_{i,t} x_{it}^{-}$. (Note that the under-usage of consultation rooms actually increases because of the bottleneck effect discussed below).

Table 1. Improvement in Model Performance Measures over Current Schedule.

	Model	Original	Improvement
$\sum_{i,t} d_{it}^{-}$	798.22	857.87	6.95%
$\sum_{i,t} d_{it}^{+}$	833.52	1289.84	35.38%
$\sum_{i,t} c_{it}^{-}$	68.00	50.00	–36.00%
$\sum_{i,t} x_{it}^{-}$	259.00	278.00	6.83%

These improvements, however, must be interpreted against the background of service quality, the over- (respectively, under-) achievement of average patient waiting time w_{it}^{+} (respectively, w_{it}^{-}). It can be seen from Table A.4 of the Appendix that, with respect to the target value $W_{it}=18$, we have a maximum $w_{it}^{+}=21.6$, with an average (over positive values) of 7.96, which is quite acceptable when there is a maximum of $w_{it}^{-}=14.6$, with an average (again over positive values) of 6.05. The individual values for all these deviation variables are summarized as Tables A.2-A.4 in the Appendix.

Bottleneck

One may notice that a major additional finding of our work is that even though the maximum numbe r of doctors available is N=28, the actual recommended allocation $\sum_i x_{it}^-$ for each t ranges only from 21 to 27. This at first sight may seem rather odd, particularly in the presence of (albeit small) demand loss as indicated by $d_{it}^- > 0$ for some i,t. This phenomenon is explained nicely by our model solution of an entry $c_{it}^-(= R_{it} - x_{it})$ being zero for that combination of i,t such that $x_{it} = R_{it}$ and $d_{it}^- > 0$. Physically, this means that there are occasions when the availabilities of doctors (x_{it}) and rooms (R_{it}) are not synchronized with higher demand (D_{it}). On the other hand, the matching of lower (respectively, higher) demand with more (respectively, fewer) rooms will still render doctor idleness ($x_{it}<R_{it}$ or $\sum_i x_{it} <N$) regardless of their availability.

Stated mathematically, the solution always achieves the goal programming optimality conditions that $w_{it}^- w_{it}^+ = 0$ and $d_{it}^- d_{it}^+ = 0$. It further satisfies either $c_{it}^- d_{it}^- = 0$ or $d_{it}^-(N - \sum_i x_{it}) = 0$. That is, $c_{it}^- d_{it}^-(N - \sum_i x_{it}) = 0$. For the case of $d_{it}^+ > 0$, it satisfies either $c_{it}^- > 0$ or $N - \sum_i x_{it} > 0$. That is, $c_{it}^- + N - \sum_i x_{it} > 0$, for $d_{it}^+ > 0$.

4. CONCLUSIONS

We would thus like to conclude this paper of our pilot study by offering the following insight. On the basis of a detailed study of the daily operation of a clinic, doctors time is scarce and is normally the performance bottleneck in comparison with other resources. On the basis of multi-branch planning, however, it is often the variation of system parameters (doctors, patients and rooms) over time and location that combine with demand into the unavoidable bottleneck of the system's operation. It is indeed these factors when taken and planned together (by a modelling approach) that may translate into maximum profit, utilization of resources and perceived efficiency.

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APPENDIX

Table A.1. Doctor allocations from model (x_{it}) and current schedule (n_{it})

(x_{it}, n_{it})	1	2	3	4	5	6	7	8	9	10	11	Total
1	(5,5)	(4,5)	(4,5)	(4,5)	(4,4)	(4,4)	(4,5)	(4,6)	(5,6)	(4,5)	(4,4)	(46,54)
2	(3,3)	(1,2)	(3,3)	(2,2)	(3,3)	(2,2)	(3,3)	(2,2)	(3,3)	(2,2)	(3,3)	(27,28)
3	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(11,11)
4	(3,2)	(3,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(3,2)	(25,22)
5	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(11,11)
6	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)	(2,3)	(3,3)	(4,2)	(3,3)	(33,32)
7	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(1,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(21,22)
8	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(12,11)
9	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,2)	(2,3)	(3,3)	(3,3)	(3,3)	(4,3)	(27,32)
10	(2,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(2,2)	(13,22)
11	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(11,11)
12	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(22,22)
Total	(27,26)	(22,25)	(23,26)	(22,25)	(23,25)	(21,23)	(23,26)	22,26)	(25,27)	(24,24)	(27,25)	(259,278)

Table A.2. Unoccupied consultation rooms from model (c_{it}^-) and current schedule (r_{it}^-)

(c_{it}^-, r_{it}^-)	1	2	3	4	5	6	7	8	9	10	11	Total
1	(0,0)	(1,0)	(1,0)	(2,1)	(0,0)	(1,1)	(2,1)	(3,1)	(2,1)	(3,2)	(2,2)	(17,9)
2	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
3	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
4	(0,1)	(0,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(0,1)	(8,11)
5	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
6	(0,0)	(2,2)	(0,0)	(1,1)	(1,1)	(2,2)	(0,0)	(4,3)	(0,0)	(0,2)	(0,0)	(10,11)
7	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(1,1)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(1,1)
8	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
9	(2,1)	(3,2)	(3,2)	(3,2)	(3,2)	(3,3)	(2,1)	(2,2)	(1,1)	(1,1)	(0,1)	(23,18)
10	(0,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(0,0)	(9,0)
11	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
12	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
Total	(2,2)	(7,5)	(6,3)	(8,5)	(6,4)	(9,8)	(6,3)	(11,7)	(5,3)	(6,6)	(2,4)	(68,50)

Table A.3. Under-achievement (d_{it}^-) and over-achievement (d_{it}^+) of patients demands

(d_{it}^-, d_{it}^+)	1	2	3	4	5	6
1	(0.0,1.7)	(0.0,7.8)	(0.0,2.8)	(0.0,8.4)	(0.0,16.1)	(0.0,11.8)
2	(6.8,0.0)	(20.4,0.0)	(0.0,0.0)	(3.1,0.0)	(0.0,12.8)	(0.0,0.9)
3	(4.6,0.0)	(1.5,0.0)	(0.0,0.0)	(0.0,1.5)	(1.5,0.0)	(1.6,0.0)
4	(0.0,39.1)	(0.0,25.1)	(0.0,7.5)	(0.0,5.0)	(0.0,14.1)	(0.0,8.9)
5	(8.0,0.0)	(1.4,0.0)	(0.0,0.9)	(0.0,1.2)	(0.0,2.3)	(0.0,3.1)
6	(0.0,0.2)	(0.0,8.8)	(0.0,10.3)	(0.0,2.0)	(0.0,12.4)	(0.0,7.6)
7	(0.0,11.0)	(0.0,3.6)	(0.0,9.6)	(0.0,9.9)	(0.0,11.8)	(0.0,0.4)
8	(93.1,0.0)	(78.3,0.0)	(86.9,0.0)	(58.3,0.0)	(79.8,0.0)	(53.4,0.0)
9	(0.0,0.1)	(0.0,10.7)	(0.0,17.2)	(0.0,17.7)	(0.0,20.2)	(0.0,23.4)
10	(0.0,29.5)	(0.0,1.5)	(0.0,6.3)	(0.0,5.4)	(0.0,7.8)	(0.0,6.2)
11	(24.0,0.0)	(11.1,0.0)	(13.2,0.0)	(6.1,0.0)	(12.9,0.0)	(5.6,0.0)
12	(15.3,0.0)	(11.9,0.0)	(0.0,0.4)	(3.3,0.0)	(0.0,4.5)	(0.0,1.0)
Total	(125.0,81.7)	(125.0,57.4)	(100.0,55.0)	(70.7,51.0)	(94.1,102.0)	(60.5,63.3)

7	8	9	10	11	Total
(0.0,9.3)	(0.0,0.2)	(0.0,9.9)	(0.0,9.8)	(0.0,19.9)	(0.0,97.6)
(0.0,10.5)	(0.0,0.9)	(0.0,10.4)	(0.0,2.5)	(0.0,8.4)	(30.2,46.3)
(0.0,3.2)	(0.8,0.0)	(0.2,0.0)	(0.2,0.0)	(1.0,0.0)	(11.4,4.6)
(0.0,15.8)	(0.0,12.4)	(0.0,9.0)	(0.0,1.6)	(0.0,47.7)	(0.0,186.1)
(0.0,3.9)	(0.0,3.8)	(0.0,1.9)	(0.0,3.1)	(12.0,0.0)	(21.4,20.3)
(0.0,19.5)	(0.0,2.9)	(0.0,8.2)	(0.0,11.2)	(10.8,0.0)	(10.8,83.0)
(0.0,16.4)	(0.0,10.6)	(0.0,11.6)	(0.0,12.0)	(0.0,11.4)	(0.0,108.6)
(82.2,0.0)	(0.8,0.0)	(0.0,2.3)	(0.0,0.1)	(6.6,0.0)	(539.2,2.3)
(0.0,19.6)	(0.0,9.0)	(0.0,12.0)	(0.0,9.3)	(0.0,19.9)	(0.0,159.0)
(0.0,6.2)	(0.0,11.4)	(0.0,5.0)	(0.0,6.7)	(0.0,26.0)	(0.0,111.9)
(11.2,0.0)	(5.1,0.0)	(12.4,0.0)	(5.4,0.0)	(24.9,0.0)	(131.7,0.0)
(0.0,3.4)	(0.0,0.2)	(0.0,3.2)	(0.0,1.1)	(23.1,0.0)	(53.5,13.8)
(93.3,108.0)	(6.7,51.6)	(12.6,73.4)	(5.6,57.4)	(78.4,133.0)	(798.2,833.5)

Table A.4. Under-achievement (w_{it}^{-}) and over-achievement (w_{it}^{+}) of patients waiting time

(w_{it}^{-}, w_{it}^{+})	1	2	3	4	5	6
1	(0.0,8.2)	(0.0,9.0)	(0.0,1.4)	(0.0,3.0)	(0.0,4.4)	(0.0,3.5)
2	(0.0,1.7)	(10.1,0.0)	(0.0,0.9)	(4.4,0.0)	(0.0,2.0)	(4.1,0.0)
3	(0.0,10.2)	(0.0,10.9)	(0.0,11.6)	(0.0,13.4)	(0.0,7.8)	(0.0,3.2)
4	(4.9,0.0)	(6.4,0.0)	(9.4,0.0)	(9.7,0.0)	(9.4,0.0)	(9.9,0.0)
5	(4.2,0.0)	(3.3,0.0)	(3.2,0.0)	(3.3,0.0)	(3.5,0.0)	(2.9,0.0)
6	(0.0,7.9)	(0.0,11.7)	(0.0,7.5)	(0.0,3.3)	(0.0,6.7)	(0.0,5.0)
7	(0.0,21.6)	(0.0,11.1)	(0.0,12.7)	(0.0,18.5)	(0.0,13.0)	(0.0,2.7)
8	(14.0,0.0)	(14.6,0.0)	(13.8,0.0)	(14.0,0.0)	(14.0,0.0)	(14.2,0.0)
9	(0.0,1.8)	(3.1,0.0)	(4.1,0.0)	(2.2,0.0)	(4.4,0.0)	(0.5,0.0)
10	(0.0,12.6)	(2.4,0.0)	(1.4,0.0)	(0.9,0.0)	(1.7,0.0)	(1.4,0.0)
11	(7.9,0.0)	(7.5,0.0)	(7.6,0.0)	(6.9,0.0)	(8.3,0.0)	(7.6,0.0)
12	(4.0,0.0)	(5.9,0.0)	(3.4,0.0)	(5.2,0.0)	(3.4,0.0)	(4.8,0.0)
Total	(35.0,64.0	(53.3,42.7)	(42.8,34.2)	(46.5,38.2)	(44.7,33.9)	(45.3,14.3)

(w_{it}^{-}, w_{it}^{+})	7	8	9	10	11	Total
1	(0.0,1.3)	(1.3,0.0)	(0.0,3.1)	(0.0,1.7)	(0.0,10.1)	(1.3,45.7)
2	(0.0,1.0)	(4.9,0.0)	(0.0,1.6)	(3.3,0.0)	(0.0,3.9)	(26.8,11.1)
3	(0.0,14.1)	(0.0,3.4)	(0.0,10.9)	(0.0,8.7)	(0.0,10.7)	(0.0,104.9)
4	(9.3,0.0)	(9.9,0.0)	(9.7,0.0)	(10.8,0.0)	(5.3,0.0)	(94.7,0.0)
5	(2.7,0.0)	(3.2,0.0)	(3.4,0.0)	(2.8,0.0)	(6.9,0.0)	(39.3,0.0)
6	(0.0,10.8)	(0.0,1.6)	(0.0,5.2)	(0.0,5.7)	(0.0,1.5)	(0.0,67.0)
7	(0.0,19.6)	(0.0,15.2)	(0.0,13.4)	(0.0,21.4)	(0.0,18.1)	(0.0,167.3)
8	(14.0,0.0)	(0.0,8.2)	(2.1,0.0)	(0.0,7.7)	(0.0,6.4)	(100.7,22.3)
9	(4.7,0.0)	(7.9,0.0)	(6.6,0.0)	(7.1,0.0)	(4.4,0.0)	(44.9,1.8)
10	(2.9,0.0)	(0.0,6.1)	(3.1,0.0)	(0.8,0.0)	(0.0,6.8)	(14.6,25.5)
11	(8.0,0.0)	(8.0,0.0)	(7.9,0.0)	(7.4,0.0)	(9.2,0.0)	(86.2,0.0)
12	(3.8,0.0)	(5.9,0.0)	(3.4,0.0)	(4.7,0.0)	(6.7,0.0)	(51.0,0.0)
Total	(45.5,46.7)	(41.0,34.5)	(36.2,34.2)	(36.9,45.3)	(32.4,57.5)	(459.5,445.5)

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