Yugoslav Journal of Operations Research
6 (1996), Number 1, 129-136

PROBABILISTIC-POSSIBILISTIC APPROACH TO THE OPTIMAL SELECTION OF A CONTRACTOR

Živojin PRAŠČEVIĆ
Faculty of Civil Engineering, University of Belgrade, 11000 Belgrade, Yugoslavia

Sonja PETROVIĆ-LAZAREVIĆ
Welington Polytechnic, Welington, New Zealand

Abstract: This paper deals with the problem of the optimal choice among building contractor firms tendering for a construction project. The probability and possibility of such firms completing a project in a given period of time is considered. The completion of a job is treated as a fuzzy event. The procedure for ascertaining the expected time for the completion of a project at the expected cost of the project is based upon the probability and possibility theory. The selection of a contractor is made upon the minimal expected cost criterion. Following these assumptions a computer program has been created. The example in the paper illustrates the program.

Keywords: Fuzzy sets, possibility theory, bidding competition, selection of a contractor.

1. INTRODUCTION

The selection of a contractor for a construction project is one of the most important activities of an employer. The employer is defined as a firm or an institution or an individual who invests in the realization of a project. The contractor is a firm or an institution who completes a project.

The selection of a contractor is made after a tendering procedure has been finalized. The tendering procedure can be either in the form of open tendering or selective tendering. The open tendering procedure is usually used by public authorities or some other institution inviting offers to be made in the daily press or specialized technical publications. Tender documents are supplied to the contractors (tenderers) who respond to the invitation.

The selective tendering procedure is usually divided into two phases. The first phase, referred to as pre-qualification, consists of the employer's invitation to potential tenderers to submit information about their technical, technological and economic capabilities in terms of completing the project. The employer then makes the first selection and prepares a short list of contractors who are then invited to submit their offers. After receiving their tenders, the employer makes his second selection, i.e., selects one contractor for the job.

The problem of successful tendering from a tenderer point of view has been studied by several authors: Friedman [6], Gates [7], Dixie [4] and others. In our previous work [11], the utility theory was applied to solve this problem.

The problem of the optimal selection of one contractor among several tenderers was considered by Nguen [10]. His selection procedure of bid contractors is based upon the fuzzy set theory and multicriteria modeling. Namely, he takes into account several important criteria and for each criterion he aggregates k rating values of membership functions that are proposed by k estimators. Then, by applying the Bellman-Zadeh method for multicriteria decision making in a fuzzy environment [2] he may choose the most appropriate tenderer.

2. PROBABILISTIC APPPROACH

The time of project completion T_i by tenderer A_i (i=1,2,....,m) is assumed to be a stochastic variable with a normal distribution.

The probability distribution function, meaning the probability of project completion by tenderer A_i , is

$$F_i(t) = \Pr\{\ T_i \leq t\ \} = \int_0^t f_i(t)dt, \tag{1}$$

where

$$f_i(t) = \frac{1}{\sqrt{2\pi}} e^{-(t_i - t_{e,i})^2/2\sigma_i^2}, \quad (i = 1, 2, \ldots, m). \tag{2}$$

is the probability density function. Values $t_{e,i}$ and σ_i are expected time and standard deviation of project completion time by tenderer A_i. These values may be divided by the PERT method using the time schedule submitted by every tenderer.

3. POSSIBILISTIC APPROACH

The price offered by every tenderer for project completion is an important factor among other factors influencing the employer's decision. The employer is interested in the contractor's ability to execute the project in a given period of time, his technical and technological competence, financial statement, reputation, performance record, etc. Hence, the possibility of every contractor completing the project in a given time is considered to be a decision making process. The possibilistic approach takes project completion time T_i by tenderer A_i as a fuzzy variable. This variable takes the values denoted by t with the membership function value $\mu_i(t)$ that represents the degree of possibility of tenderer A_i completing the project in time t. The variable T_i (i=1,2,...,m) takes values in the universe of discourse T and denotes a subset in T, characterized by the membership function $\mu_i(t)$. T_i is a fuzzy restriction on T, and can be denoted by

$$T_i = \{ t, \mu_i(t) \}, \quad T_i \subset T, \quad t \in T, \quad (i = 1, 2, \dots, m), \tag{3}$$

where

$$0 \le \mu_i(t) \le 1, \qquad \text{or} \qquad \mu_i(t)\colon\ T \to [0,1]. \tag{4}$$

The possibility that tenderer A_i will complete the project in time t may be written

$$Poss\ \{T_i = t_i\} = \mu_i(t), \qquad t \in T. \tag{5}$$

According to Zadeh [15] the membership function $\mu_i(t)$ is the possibility distribution function associated with T_i. It defines the possibility that T_i could assume any specified value of t in T.

To determine the possibility distribution function several factors have to be assumed for every tenderer. These factors are:

- technical and technological ability,
- financial ability,
- resource supply ability,
- reputation and excellent performance record and others.

The criteria are fuzzy or linguistic variables and influence the possibility distribution function $\mu_i(t)$. Therefore, it is possible to introduce a new fuzzy subset T_{ij} in the universe of discourse. This subset with a membership function μ_{ij} is associated with tenderer A_i and criterion K_j,

$$T_{ij} = \{ t, \mu_{ij}(t) \}, \qquad T_{ij} \subset T, \qquad t \in T, \tag{6}$$

$$0 \le \mu_{ij}(t) \le 1, \qquad (i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n). \tag{7}$$

The resulting values of the possibility distribution function $\mu_i(t)$ can be derived by the aggregation of corresponding factors.

For a pessimistic aggregation, the fuzzy set T_i is assumed to be the intersection of the fuzzy set T_{ij}.

$$T_i = T_{i1} \cap T_{i2} \cap \dots\dots\dots \cap T_{in}, \qquad (i = 1, 2, \dots, m). \tag{8}$$

$$\mu_i(t) = \min\{ \mu_{i1}(t), \mu_{i2}(t), \dots\dots, \mu_{in}(t) \} \tag{9}$$

and for an optimistic aggregation

$$T_i = T_{i1} \cup T_{i2} \cup \dots\dots\dots \cup T_{in}. \tag{10}$$

According to Hipel [8] a pessimistic aggregation attempts to minimize risk, while an optimistic aggregation may present the best case viewpoint between the interest groups. In this paper the optimal choice is considered to be more realistic than the optimistic one.

A more complex and realistic method to calculate the possibility distribution function $\mu_i(t)$ for project realization is described by Praščević in [12]. This method is based on the project network plan, where the duration of every partial activity is analyzed as a fuzzy variable. Taking into account Zadeh's extension principle, the correspon- ding function of project completion $\mu_i(t)$ is calculated. All values $\mu_i(t)$ and $\mu_{ij}(t)$ have to be calculated by the employer's consultants or consulting firms.

4. PROBABILITY OF PROJECT COMPLETION AS A FUZZY EVENT

Project completion time T_i has a probabilistic and possibilistic character. Hence, completion of the project can be assumed to be a fuzzy event. For determination of the probability density function $f_i^*(t)$ and probability distribution function $F_i^*(t)$ of this event, Zadeh's possibility/probability consistency principle, can be used [15], [16]. According to this principle, the probability of project completion as a fuzzy event by tenderer A_i within time t, as described in [15] is

$$Poss\{T_i \le t\} = F_i^*(t) = \int_0^t f_i^*(t)\,dt, \qquad (i = 1, 2, \ldots, m), \tag{11}$$

where

$$f_i^*(t) = \frac{1}{\alpha_i}\int_0^t f_i(t)\,\mu_i(t)\,dt, \qquad \alpha_i = \int_0^\infty f_i(t)\,\mu_i(t)\,dt. \tag{12}$$

The expected time of project completion by tenderer A_i calculated by the probabilistic $t_{e,i}$ and probabilistic-possibilistic procedure $t_{e,i}^*$ is

$$t_{e,i} = \int_0^\infty f_i(t)\,t\,dt, \qquad t_{e,i}^* = \int_0^\infty f_i^*(t)\;t\,dt. \tag{13}$$

These functions are shown on Fig. 1.

5. COSTS

The employer's costs are considered by several authors: Cassimats [2], Ferry and Brandon [5], Raftery [13], Lavender [9], Stone [14], Ashworth [1]. More or less all of them treat the client's cost as a budgeted building cost. In this paper the client's cost is not only the budgeted cost, but also the penalty or prize of the contractor depending upon the moment the obligation to the client is fulfilled. That is to say, the employer's function cost is expressed in time t. It may increase or decrease. If the construction project is built before the fixed time the employer's costs rise, and vice versa. If there is a delay in construction, $C_i(t)$ decreases.

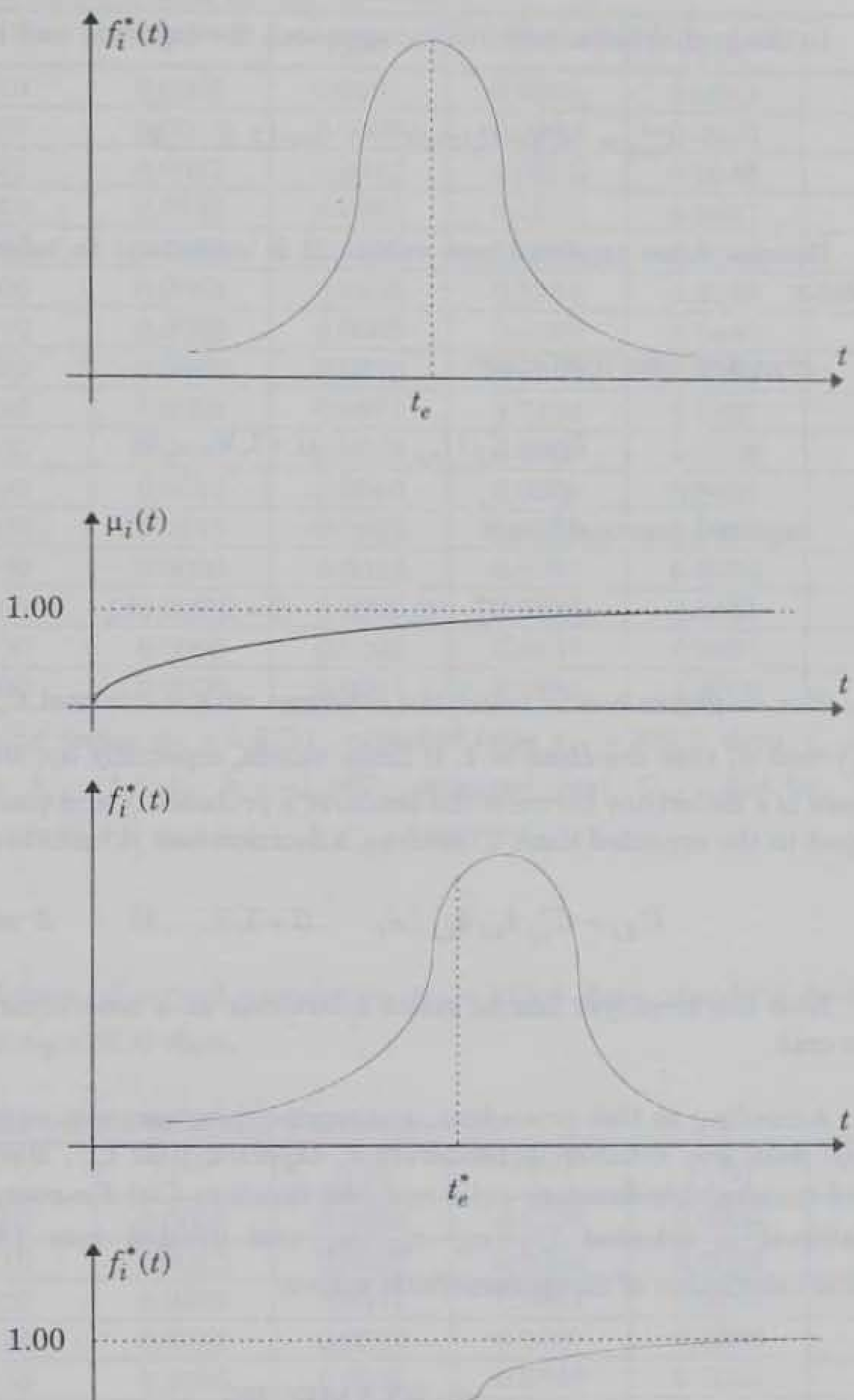

Figure 1

For known cost function $C_i(t)$ of tenderer A_i the expected cost $C_{e,i}(t)$ in the probabilistic approach is:

$$C_{e,i} = \int_0^\infty f_i(t)\, C_i(t)\, dt, \qquad (i = 1, 2, \ldots, k). \tag{14}$$

In the probabilistic-possibilistic approach the expected cost is:

$$C^{*}_{e,i} = \int_{0}^{\infty} f_i^{*}(t)\, C_i(t)\, dt, \qquad (i = 11, 2, \dots, k). \tag{15}$$

Besides these expected cost values, it is important to calculate the following coefficients:

- expected time coefficient

$$k_{t,i} = t^{*}_{e,i} \,/\, t_{e,i} \qquad (i = 1, 2, \dots, k), \tag{16}$$

- expected cost coefficient

$$k_{c,i} = C^{*}_{e,i} \,/\, C_{e,i} \qquad (i = 1, 2, \dots, k), \tag{17}$$

The employer has to select the tenderer with a minimal $C^{*}_{e,i}$ and with values $k^{*}_{c,i}$, $k^{*}_{t,i}$ and α_i that are close to 1. If these values, especially α_i, are different from 1, then there is a difference between the tenderer's probability and possibility to complete the project in the expected time. Therefore, a decision cost is introduced:

$$C_{d,i} = C^{*}_{e,i}\, k_{t,i}\, k_{c,i} \,/\, \alpha_i \qquad (i = 1, 2, \dots, k). \tag{18}$$

Now the employer has to select a tenderer as a contractor with the minimal decision cost.

According to this procedure, a computer program was written in FORTRAN 77. Input data are: number of tenderers k, expected time $t_{e,i}$, standard deviation σ_i, values of membership function $\mu_i(t)$ and cost function $C(t)$ for every tenderer A_i. The time interval is selected $t_{e,i} - \sigma_i$, $t_{e,i} + \sigma_i$ and divided into 18 subintervals for numerical calculation of all characteristic values.

6. EXAMPLES

Two tenderers have submitted their tenders and upon the employer's request have made additional data available to calculate all characteristic input values, which together with the output values are shown in the following tables (Tab. 1., and Tab. 2.).

Tenderer 1

Expected time of project completion $t_{e,1} = 200.00$ days, standard deviation of project completion $\sigma_1 = 60.00$ days.

Table 1: Input and output values for the tenderer 1

Time t	$\mu_1(t)$	$f_1(t)$	$f_1^*(t)$	$F_1(t)$	$F_1^*(t)$	$C_1(t)$
80	0.10	0.0009	0.0001	0.0236	0.0013	570
100	0.20	0.0017	0.0005	0.0492	0.0076	560
120	0.30	0.0027	0.0012	0.0931	0.0246	555
140	0.40	0.0040	0.0024	0.1607	0.0607	550
160	0.50	0.0053	0.0039	0.2543	0.1240	545
180	0.60	0.0063	0.0056	0.3704	0.2193	540
200	0.70	0.0066	0.0069	0.4998	0.3442	550
220	0.80	0.0063	0.0075	0.6292	0.4876	555
240	0.90	0.0053	0.0071	0.7454	0.6331	560
260	0.95	0.0040	0.0060	0.8389	0.7638	565
280	1.00	0.0027	0.0040	0.9066	0.8641	575
300	1.00	0.0017	0.0025	0.9505	0.9291	585
320	1.00	0.0009	0.0013	0.9761	0.9670	595
340	1.00	0.0004	0.0006	0.9894	0.9868	600
360	1.00	0.0002	0.0003	0.9957	0.9961	610
380	1.00	0.0001	0.0001	0.9984	1.0000	620

Normalizing factor $\alpha_1 = 0.6751$, expected time $t_{e,1} = 200.0$ days, $t^*_{e,2} = 222.2$ days, coefficients $k_{t,1} = 1.111$, $k_{c,1} = 1.007$, expected cost $C_{e,1} = 554.98$, possibly expected cost $C^*_{d,1} = 558.72$, decision cost $C_{d,1} = 925.73$.

Tenderer 2

Expected time of project completion $t_{e,2} = 210.0$ days, standard deviation of project completion $\sigma_2 = 57.0$ days.

Table 2: Input and output values for the tenderer 2

Time t	$\mu_2(t)$	$f_2(t)$	$f_2^*(t)$	$F_2(t)$	$F_2^*(t)$	$C_2(t)$
96	0.00	0.0009	0.0000	0.0236	0.0000	600
115	0.10	0.0017	0.0003	0.0492	0.0032	595
134	0.20	0.0029	0.0011	0.0931	0.0169	580
153	0.30	0.0042	0.0024	0.1607	0.0506	560
172	0.35	0.0056	0.0038	0.2543	0.1096	550
191	0.40	0.0066	0.0051	0.3704	0.1937	540
210	0.50	0.0070	0.0067	0.4998	0.3059	535
229	0.60	0.0066	0.0076	0.6292	0.4422	545
248	0.70	0.0056	0.0075	0.7454	0.5863	555
267	0.80	0.0042	0.0065	0.8389	0.7198	570
286	0.90	0.0029	0.0050	0.9066	0.8290	580
305	1.00	0.0017	0.0034	0.9505	0.9081	590
324	1.00	0.0009	0.0018	0.9761	0.9672	605
343	1.00	0.0005	0.0009	0.9894	0.9829	620
362	1.00	0.0002	0.0004	0.9957	0.9949	640
381	1.00	0.0001	0.0001	0.9984	1.0000	650

Normalizing factor $\alpha_2 = 0.5207$, expected time $t_{e,2} = 210.0$ days, $t = 237.1$ days, coefficients $k_{t,2} = 1.129$, $k_{c,2} = 1.005$, expected cost $C_{e,2} = 557.23$, possibly expected cost $C^{*}_{e,2} = 560.23$, decision cost $C_{d,2} = 1221.44$.

It is obvious from these results that the expected possible costs are similar for both tenderers. Tenderer 2 has normalizing factor a_2 which is more unfavorable, a greater difference between the possibility and probability of project completion, and much a higher decision value $C_{d,2}$ than tenderer 1. Hence, the employer has to select tenderer 1.

7. CONCLUSION

The methodology and the procedure proposed in this paper may be used by employers to calculate the expected costs of project execution, taking into account the tenderer's possibility and probability to complete the project in a given period of time. This procedure is based both on the theory of probability and the theory of possibility, and provides more complex data about the tenderer's ability to complete the project. However, the main problem from the employer's point of view is how to assess the tenderer's probability and possibility of project completion. To overcome this, the employer has to collect all relevant information about the tenderers. The proposed procedure for the optimal choice of a contractor among several tenderers is rather simple and gives more complete data about the expected costs compared to the procedure based on the PERT method.

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