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OPTIMIZATION ALGORITHMS FOR A CLASS OF SINGLE MACHINE SCHEDULING PROBLEMS USING DUE DATE DETERMINATION METHODS

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Abstract. The problem of determining optimal schedules for the static, single machine scheduling problem with the aid of CON and SLK due date determination methods is considered. The objective is to minimize the total weighted earliness and tardiness penalty in the case when weights are proportional to the processing times of the respective jobs. For each method, an optimization algorithm has been developed, by means of which the set of all optimal sequences is provided. The numerical example, presented after the theoretical foundation, confirms considerations about the structural similarity of the two methods.

Keywords: Optimal sequence, Due date, Constant flow allowance, Slack time, Scheduling

1. INTRODUCTION

Fulfilling delivery dates seems to be one of the most desirable performance criteria for the evaluation of different production schedules. The mean tardiness criterion has been a standard measure of conformance to due dates, although it ignores the consequences of jobs being completed early. The introduction of modern production systems (e.g., Flexible Manufacturing and Just-In-Time Systems) has broadened the range of the applications of the above criterion, as it discourages the tardy as well as the early completion of a job. In such environments, jobs that are completed early must be held in stock until their due date, while jobs that are completed after their due dates may cause serious problems to customers and/or the production system. Therefore, an ideal schedule is the one, in which all jobs finish exactly on their assigned due dates. If this is not possible, one may try to minimize the deviations of jobs completion times from their due dates.

The concept of penalizing both earliness and tardiness has spawned a new and rapidly developing research direction in the area of scheduling. The variety of works observed in the earliness /tardiness (E/T)/ literature stems from the diversity of assumptions related to the objective function of the problem. In most of them the objective is to optimize the due date and the job sequence simultaneously [3]. In work presented in [3], [8], [14], standing amongst the most prominent surveys of the problem, there are more than a hundred different variations of the objective function. Variations comprise different earliness and tardiness penalties, additional penalties, non-linear penalties, job dependent earliness and tardiness penalties, due date tolerances, distinct due dates and job deadlines. In addition, a variety of Due Date Determination (DDD) methods has been proposed, which refers to simple cases, dealing with the static, single machine problem with deterministic processing times [7], [10], as well as to more complicated ones, that address the problem with multiple machines [6], non-zero ready times [5], and stochastic processing times [13]. The usual objective is to minimize the total lateness.

A sace of the static single machine problem is examined in this paper, in which the weights of jobs are proportional to their processing times, and two optimization algorithms for respective DDD methods are illustrated. This case is quite realistic since a relation between the processing time of a job and the job's value is quite usual (see for example [1], [2]). So, the inventory cost (associated with the job's value) can be viewed as being proportional to the processing time of a job. Similarly, if the profit gained from a job is proportional to the processing time, the cost of a delayed sale, due to delayed completion, may be considered as being proportional to the processing time of the job. An additional tardiness penalty due to loss of goodwill may also be considered as being proportional to the processing time. In this paper, the performance criterion used is the minimization of total weighted absolute lateness (total weighted earliness and tardiness).

In practice, appropriate weights for jobs are difficult to ascertain [1]. In real world scheduling systems the weights are often set arbitrarily, in many cases by setting all of them equal to one. Typically, jobs that have longer processing times have a higher selling price and, consequently, a higher priority. In many systems both the processing time and the weight of a job are nearly proportional to the job's monetary value, resulting in weights that are nearly proportional to the processing times. Espousing the justification given in [2], this proportionality models a situation, in which the unit cost of earliness/tardiness reflects value-added cost, which is likely to be proportional to processing time.

The problem is analysed by means of two DDD methods, namely CON and SLK. For each method an optimization algorithm has been developed that leads to the set of optimal sequences. The similarity of the above methods becomes clear by using a numerical example. Both methods result in the same minimum value of total weighted absolute lateness.

2. PROBLEM FORMULATION AND ASSUMPTIONS

Let $N=\{1, 2, ..., n\}$ be the set of n independent jobs to be processed and R the set of $n!$ sequences generated out of the jobs of the set N. The assumptions for the problem are as follows:

(i) All jobs become available for machine processing simultaneously;
(ii) All processing times p_i , $i \in N$, are deterministics and known before processing starts;
(iii) Job pre-emption and job splitting are not permitted;
(iv) The nachine cannot process two or more jobs simulatneously;
(v) There will be no inserted idle time in the schedule.

Let s denotes an arbitrary sequence in R and $J_{[i]}$ the job that occupies the i-th position of the sequence s. In the sequel, the symbol $[i]$, when used as subscript, denotes a variable of a job in this position. So, let $C_{[i]}$, $W_{[i]}$, $L_{[i]}$, $E_{[i]}$, $T_{[i]}$, $d_{[i]}$ denote the completion time, the waiting time, the lateness, the earliness, the tardiness and the assigned due date of the job $J_{[i]}$, respectively. Finally, let $a_{[i]}$ ($a_{[i]}>0$) denote the weight associated with lateness (tardiness or earliness) of $J_{[i]}$ in any sequence of R. The weights considered here are proportional to the processing times of the respective jobs, that is $a_{[i]}=\lambda\, p_{[i]}$, $\lambda>0$. The objective function $f(s)$ to be minimized is as follows:

$$f(s)=\sum_{i=1}^{n} a_{[i]}\,|\,C_{[i]}-d_{[i]}\,|=\lambda\sum_{i=1}^{n} p_{[i]}\,|\,C_{[i]}-d_{[i]}\,|. \qquad (1)$$

Treating due dates as decision variables reflects the practice of setting due dates internally, as targets to guide the progress of shop floor activities. Prescribing a common due date (CON method) might represent the case in which different items to be produced constitute a single costomer's order. Assigning identical dea dates to these items can be seen as an engagement for the simultaneous completion of the complete customer order. Salesman usually quote a common delivery date on all orders for different items of one client (or, alternatively, on all orders of different clients). Furthermore, prescribing a common due date might also reflect an assembly environment, in which the components should all be ready at the same time to avoid delays. The rationale of the SLK method is given in [12]. According to this method, the due date of a job is determined by adding a constant lead time to its processing time. If the shop status is relatively stable, all jobs may be given due dates based upon a constant lead time and their processing times, regardless of job content.

In the CON method, all jobs are assumed to have exactly the same flow allowance (due date), denoted by k. Thus, it is $d_{[i]} = k$, $\forall J_{[i]} \in s$, and (1) can be written as follows:

$$f_{CON}(s) = \lambda \sum_{i=1}^{n} p_{[i]} \mid C_{[i]} - k \mid. \tag{2}$$

The following theorems hold in this method (proofs in [4], [5]):

Theorem 1. For any specified sequence s, there exists an optimal value k^*, the common constant flow allowance for all $J_{[i]}$ in s, which coincides with the completion time of exactly one of the jobs in s.

Theorem 2. For any specified sequence s, in which the optimal flow allowance, k^*, coincides with the completion time, $C_{[r]}$, of a job $J_{[r]}$, r (the position of the optimal allowance in a sequence s) is determined by:

$$\sum_{i=1}^{r-1} a_{[i]} - \sum_{i=r}^{n} a_{[i]} < 0 \text{ and } \sum_{i=1}^{r} a_{[i]} - \sum_{i=r+1}^{n} a_{[i]} \geq 0.$$

On the other hand, according to the SLK method, all jobs are given flow allowances that reflect equal waiting time or slack, denoted by q. Thus, it is $d_{[i]} = p_{[i]} + q$, $\forall J_{[i]} \in s$, and because of $W_{[i]} = C_{[i]} - p_{[i]}$, (1) can be written as

$$f_{SLK}(s) = \lambda \sum_{i=1}^{n} p_{[i]} \mid C_{[i]} - p_{[i]} - q \mid = \lambda \sum_{i=1}^{n} p_{[i]} \mid W_{[i]} - q \mid. \tag{3}$$

Similarly to the CON method, the following theorems are valid here (proofs in [4], [5]):

Theorem 3. For any specified sequence s there exists an optimal value of q, denoted by q^*, which coincides with one of the waiting times of the jobs in s.

Theorem 4. For any specified sequence s, in which the optimal slack q^* coincides with the waiting time of a job $J_{[r]}$, r is determined by:

$$\sum_{i=1}^{r-1} a_{[i]} - \sum_{i=r}^{n} a_{[i]} < 0 \text{ and } \sum_{i=1}^{r} a_{[i]} - \sum_{i=r+1}^{n} a_{[i]} \geq 0.$$

3. OPTIMIZATION ALGORITHMS

In this section two algorithms are presented, one for each DDD method, for optimal scheduling of n jobs in the static, single machine, weighted absolute lateness problem.

3.1. Using the CON method

A primary *optimal* sequence as well as all *alternative optima* can be found by the following algorithm:

Algorithm 1

Step 1: From a given sequence s in R construct the sequence s_{LPT}, by arranging the set of jobs in non-increasing order of their processing times.

Step 2: Use Theorem 2 to obtain the position r (r is the position of a job $J_{[i]}$ whose completion time $C_{[r]}$ is equal to the optimal flow allowance k^*).

Step 3: Let A be the set of jobs occupying the places 1 to r in the sequence s_{LPT}, and B the set of jobs that occupy the places $(r+1)$ to n in the same sequence. Obtain all sequences $s_{opt}(CON)$ derived from all possible combinations of job seqauencing **in each** of the above sets.

Theorem 5. For any specified sequence s, assuming that the weights of jobs are proportional to their processing times, the minimization of the total weighted absolute lateness is achieved by the s_{opt}(CON) sequences determined using the above algorithm.

Proof: Because of Theorems 1 and 2, (2) can be written as follows:

$$
\begin{aligned}
f_{CON}(s) &= \lambda \sum_{i=1}^{r-1} p_{[i]}(k^* - C_{[i]}) + \lambda \sum_{i=r+1}^{n} p_{[i]}(C_{[i]} - k^*) = \\
&= \lambda \{\, p_{[1]}(k^* - p_{[1]}) + p_{[2]}(k^* - p_{[1]} - p_{[2]}) + \ldots + p_{[r-1]}(k^* - p_{[1]} - p_{[2]} - \ldots - p_{[r-1]} \,\} + \\
&+ \lambda \{\, p_{[r+1]}(p_{[1]} + p_{[2]} + \ldots + p_{[r+1]} - k^*) + p_{[r+2]}(p_{[1]} + p_{[2]} + \ldots + p_{[r+1]} + p_{[r+2]} - k^*) + \\
&+ \ldots + p_{[n]}(p_{[1]} + p_{[2]} + \ldots + p_{[n-1]} + p_{[n]} - k^*) \,\}.
\end{aligned}
$$

It holds that $k^* = C_{[r]} = p_{[1]} + p_{[2]} + \ldots + p_{[r]}$ and consequently,

$$
\begin{aligned}
f_{CON}(s) &= \lambda \{\, p_{[1]}(p_{[2]} + p_{[3]} + \ldots + p_{[r]}) + p_{[2]}(p_{[3]} + \ldots + p_{[r]}) + \ldots + p_{[r-1]}p_{[r]} + \\
&+ \lambda \{\, p_{[r+1]}p_{[r+1]} + p_{[r+2]}(p_{[r+1]} + p_{[r+2]} + \ldots + p_{[n]}(p_{[r+1]} + p_{[r+2]} + \ldots + p_{[n]}) \,\} = \\
&= \lambda \{\, p_{[1]}(p_{[2]} + p_{[3]} + \ldots + p_{[r]}) + p_{[2]}(p_{[3]} + \ldots + p_{[r]}) + \ldots + p_{[r-1]}p_{[r]} \,\} + \\
&+ \{\, p_{[r+2]}p_{[r+1]} + p_{[r+3]}(p_{[r+1]} + p_{[r+2]} + \ldots + p_{[n]}(p_{[r+1]} + p_{[r+2]} + \ldots + p_{[n-1]}) \,\} + \\
&+ (p^2_{[r+1]} + p^2_{[r+2]} + \ldots + p^2_{[n]}) \,\}.
\end{aligned}
$$

Let $f_{CON}(s,1) = \sum_{i=1}^{r-1} p_{[i]} \sum_{j=i+1}^{r} p_{[j]}$, $f_{CON}(s,2) = \sum_{i=r+2}^{n} p_{[i]} \sum_{j=i+1}^{i-1} p_{[j]}$, and $f_{CON}(s,3) = \sum_{i=r+1}^{n} p^2_{[i]}$.

Then, $f_{CON}(s) = \lambda\,(f_{CON}(s,1) + f_{CON}(s,2) + f_{CON}(s,3))$ holds. The following lemmas hold (we omit the proofs, since they are quite obvious):

Lemma 1. $f_{CON}(s,1)$ is independent of the sequence of jobs occupying the places 1 to r, and constant for a given subsequence of s.

Lemma 2. $f_{CON}(s,2)$ is independent of the sequence of jobs occupying the places $(r+1)$ to n, and constant for a given subsequence of s.

Keeping in mind that the position r defines a set early and a set of tardy jobs [3], [9] one can observe that, because of Lemmas 1 and 2 and the form of the term $f_{CON}(s,3)$, that is a sum of squared processing times of jobs $J_{[r+1]}$ to $J_{[n]}$, the minimization of $f_{CON}(s)$ can be achieved by sequencing the shortest $(n\text{-}r)$ jobs in the last positions of the optimal sequence. Furthermore, Lemmas 1 and 2 guarantee that any possible permutation of the jobs associated with the term $f_{CON}(s,1)$ and/or $f_{CON}(s,2)$ also leads to an optimal sequence. The number of the alternative optimal sequences of a sequence s with n jobs is $r!\,(n\text{-}r)!$.

3.2. Using the SLK method

Working similarly, a primary *optimal* sequence as well as all *alternative optima* can be found by the following algorithm:

Algorithm 2

Step 1: From a given sequence s in R construct the sequence s_{SPT}, by arranging the set of jobs in non-decreasing order of their processing times.

Step 2: Use Theorem 4 to obtain the position r (r is the position of a job $J_{[i]}$ whose waiting time $W_{[r]}$ is equal to the optimal slack q^*).

Step 3: Let C be the set of jobs occupying the places 1 to $(r\text{-}1)$ in the sequence s_{SPT}, and D the set of jobs that occupy the places r to n in the same sequence. Obtain all sequences s_{opt}(SLK) derived from all possible combinations of job sequencing **in each** of the above sets.

Theorem 6: For any specified sequence s, assuming that the weights of jobs are proportional to their processing times, the minimization of the total weighted absolute lateness is achieved by the s_{opt}(SLK) sequence determined using the above algorithm.

P r o o f: $f_{SLK}(s)$ has essentially the same structure as $f_{CON}(s)$. Therefore (because of Theorems 3 and 4, and $q^* = W_{[r]} = p_{[1]} + p_{[2]} + \ldots + p_{[r-1]}$, (3) can be written as follows:

$$f_{SLK}(s) = \lambda \sum_{i=1}^{r-1} p_{[i]}\,(q^* - W_{[i]}) + \lambda \sum_{i=r+1}^{n} p_{[i]}\,(W_{[i]} - q^*) =$$
$$= \lambda\{\, p_{[1]}(p_{[2]} + p_{[3]} + \ldots + p_{[r-1]}) + p_{[2]}(p_{[3]} + \ldots + p_{[r-1]}) + \ldots + p_{[r-2]}p_{[r-1]} \,\} +$$

$$+\{ p_{[r+1]}p_{[r]} + p_{[r+2]}(p_{[r]} + p_{[r+1]}) + \ldots + p_{[n]}(p_{[r]} + p_{[r+1]} + \ldots + p_{[n-1]}) \} +$$
$$+ (p_{[1]}^2 + p_{[2]}^2 + \ldots + p_{[r-1]}^2) \}$$

As above, let $f_{SLK}(s,1) = \sum_{i=1}^{r-2} p_{[i]} \sum_{j=i+1}^{r-1} p_{[j]}$, $f_{SLK}(s,2) = \sum_{i=r+1}^{n} p_{[i]} \sum_{j=r}^{i-1} p_{[j]}$, and $f_{SLK}(s,3) =$ $= \sum_{i=1}^{r-1} p_{[i]}^2$.

It holds that $f_{SLK}(s) = \lambda\,(f_{SLK}(s,1) + f_{SLK}(s,2) + f_{SLK}(s,3))$. The following lemmas hold in this case:

Lemma 3. $f_{SLK}(s,1)$ is independent of the sequence of jobs occupying the places 1 to $(r$-$1)$ and is constant for a given subsequence of s.

Lemma 4. $f_{SLK}(s,2)$ is independent of the sequence of jobs occupying the places r to n and is constant for a given subsequence of s.

Due to Lemmas 3 and 4 and to the term $f_{SLK}(s,3)$, that is a sum of squared processing times of jobs $J_{[1]}$ to $J_{[r-1]}$, the minimization $f_{SLK}(s)$ can be achieved by sequencing the longest $(n$-$r+1)$ jobs in the last positions of the optimal sequence. Any possible permutation in the former and/or in the latter subsequences defined above is an alternative optimal sequence. The number of the alternative optimal sequences of a sequence s with n jobs is $(n$-$r+1)!(r$-$1)!=r!(n$-$r)!$.

4. A NUMERICAL EXPERIMENT

A set of five jobs is given with processing times as shown in the first row of Table 1. In Step 1, a primary optimal sequence is obtained and after the computation of the position r (Step 2) all the possible alternative optimal sequences are found (Step 3). All the above optimal sequences for the CON as well as for the SLK method result in $f_{opt}(s) = 363\,\lambda$, while for the former method it is $k^* = 22$, and for the latter $q^* = 19$.

It is easy to observe that both algorithms, by means of which the set of all optimal sequences is provided, have a similar structure. Additionally, there is a form similarity between the alternative optima of the above methods. Each one of the optimal sequences can be partitioned into two subsequences *subseq1* and *subseq2*, in respect to the position of r. For example, with reference to the CON method, an optimal sequence can be partitioned into *subseq1*, with job $J_{[1]}$ to $J_{[r]}$, and *subseq2*, with jobs $J_{[r+1]}$ to $J_{[n]}$. By interchanging the positions of *subseq1* and *subseq2*, an optimal sequence for the SLK method is provided. Consequently, if the alternative optima derived from the application of one method are known, then those derived from the application of the other can be easily found. More details on the form similarity of the above methods can be found in [11].

Table 1: A numerical example

initially	(5, 12, 10, 8, 6)	
	CON method	**SLK method**
Step 1	(12, 10, 8, 6, 5)	(5, 6, 8, 10, 12)
Step 2	r = 2	r = 4
Step 3	(12, 10, 5, 6, 8)	(5, 6, 8, 12, 10)
	(12, 10, 6, 5, 8)	(6, 5, 8, 12, 10)
	(12, 10, 6, 8, 5)	(6, 8, 5, 12, 10)
	(12, 10, 8, 5, 6)	(8, 5, 6, 12, 10)
	(12, 10, 5, 8, 6)	(5, 8, 6, 12, 10)
	(10, 12, 8, 6, 5)	(8, 6, 5, 10, 12)
	(10, 12, 5, 6, 8)	(8, 6, 5, 12, 10)
	(10, 12, 6, 5, 8)	(6, 5, 8, 10, 12)
	(10, 12, 6, 8, 5)	(6, 8, 5, 10, 12)
	(10, 12, 8, 5, 6)	(8, 5, 6, 10, 12)
	(10, 12, 5, 8, 6)	(5, 8, 6, 10, 12)

5. REMARKS AND CONCLUSION

This paper has examined the static, single machine scheduling problem of optimizing the due date determination and sequencing of n jobs, when the weights of jobs are proportional to their processing times. The performance criterion has been the minimization of the total weighted absolute lateness. Both CON and SLK methods have been used for the assignment of the appropriate due dates. Two algorithms, one for each method, presented, by means of which the complete set of optimal sequences can be determined, while the structural similarity of the two methods has also been indicated.

The above work is an extension of that on the non-weighted problem [10], which, in comparison with previous methodologies presented so far [9], has been proven to give the optimal solutions much faster. Considering computational complexity measures, the superiority of the algorithm of Karacapilidis & Pappis [10] has been illustrated, since it requires only $nlogn$ MADs (i.e., Multiplications and Additions) and n assignments, while the algorithm of Gupta et al. [9] requires $nlogn+7n+4$ MADs, $2n$ assignments and $n+3n^2/4$ comparisons.

The algorithms presented in this paper have the same computational complexity, as a consequence of their similar structure. Regarding the CON method, its computational complexity is:

Step 1: requires $nlogn$ MADs for the construction of s_{LPT};
Step 2: requires $2n^2$ MADs (in the worst case). Our algorithm performs n times $2n$ MADs in order to determine the position r (see Theorem 2);
Step 3: requires n assignments for each optimal sequence.

Therefore, in the worst case, both algorithms require $nlogn+2n^2$MADs and n assignments for the determination of an optimal sequence. In order to extract each alternative optimal sequence, n assignments should be additionally charged.

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