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ASSEMBLY LINE BALANCING WITH SMOOTHED WORK STATION ASSIGNMENTS

Sotirios G. DIMITRADIS

and

Patroklos E. GEORGIADIS

Industrial Management Division,
Department of Mechanical Engineering,
Aristotle University of Thessaloniki,
GR 54006 Greece

Abstract. In this paper, a methodology for balancing assembly lines with smoothed work station assignments is presented. Smoothed work station assignment constitutes an additional criterion during the design and development of assembly lines characterized by a large number of tasks, a small variation of task times and a small number of precedence constraints with regard to the total number of tasks to be performed. Assembly lines having the above features are common in industrial environments. In those cases the application of the suggested methodology leads to a more efficient organization of the assembly line. This is confirmed by the successful application of the suggested methodology in redesigning the assembly line of a jeep-type vehicle in a major Greek company.

Keywords: Production/scheduling-application; Line balancing; Heuristic

1. INTRODUCTION

One of the problems encountered in organizing assembly line production is the assignment of tasks to be performed at various work stations, so as to achieve a desired production rate. The order in which the tasks can be performed is restricted by a set of precedence restrictions [5]. These precedence restrictions arise either from the technological constraints due to the structure of the product or from layout constraints, imposing the assignment of some work task groups to predetermined locations along the assembly line.

In addition to the above restrictions, each task is associated with a performance time. Thus, the sum of task times of all tasks assigned to a work station forms its work content. The maximum work content appearing among all work stations, constitutes the lower bound of the assembly line cycle time. The difference between the cycle time and the work content of any one work station is referred to as the slack time for that work station.

Usually, the criterion used for the optimum assignment of tasks to work stations is the minimization of the total slack time over all work stations of the assembly line at a given cycle time. This criterion often leads to solutions with unequal loading of the work stations. However, in an industrial environment, unequal loading of the work stations is an undesirable state, since smoothing the total slack time to all work stations must be achieved.

In this paper, we present a methodology for organizing assembly line production with smoothed total slack time assignment over all work stations. This methodology has been successfully implemented in a major vehicles assembly Greek company.

2. THE SIMPLE ASSEMBLY LINE BALANCING PROBLEM (SALB)

2.1. Mathematical formulation

Let $(J_1, J_2, \dots, J_n)$ be a set of tasks to be performed in the assembly line t_i the required time for task i (i=1,2,...,n), A the number of work stations and T the cycle time.
Given the cycle time T, we seek the assignment of the n tasks to the work stations adopting as criterion the minimization of the total slack time d, calculated by using equation (1):

$$d = A*T - \sum_{i=1}^{n} t_i \qquad (1)$$

The above function must be minimized taking into consideration the following constraints:

- The maximum task time t_{imax} over all tasks must be less than or equal to the cycle time T.
- The work content of each work station must be less than or equal to the cycle time T.
- The assignment of tasks to every work station must not violate the existing precedence restrictions.

If the task times are independent of task sequencing, then the total slack time d is minimized if the product $A * T$ is minimized. Since the cycle time T is given, the total slack time d is minimized if the number of work stations A is minimized.

2.2. Literature review

Several exact solution procedures have been suggested to solve the above problem. Most of them use Integer Programming, Dynamic Programming and Branch and Bound techniques. Using as criterion the execution time limit within which a problem can be solved [1], it appears that the prevailing exact solution procedures are those suggested by Van Assche and Herroelen (1979), Wee and Magazine (1981), Jonson (1983) and Talbot and Patterson (1984).

Unfortunately, the efficiency of such exact procedures is radically reduced with the increase in the problem order. This happens because the line balancing problem, even in this simplest form, belongs to the category of NP-complete problems [1]. All exact solution procedures developed, need a number of calculation steps which increases exponentially with the order of the problem. So, large problems are very difficult to solve. Therefore, the interest has necessarily turned to the development and use of heuristic approaches. A bibliographical survey and comparison between several heuristic methods can be found in Talbot F.B., Patterson J.H. and Gehrlein W.V. [10]. According to their conclusions, the most efficient of the heuristic methods are those suggested by Hoffman T.R. (1963), Dar-El (1973) and Dar-El and Rubinovitch (1979).

3. SMOOTHING THE TOTAL SLACK TIME OVER ALL WORK STATIONS

The methodology for smoothing the total slack time over all work stations of an assembly line includes two phases.

The **first phase** deals with the solution of the SALB using an efficient heuristic procedure. The structure of this procedure is based on the heuristic procedure developed by Hoffman T.R. [6], which is considered as one of the most efficient [10], with only a small modification in step 3 (see figure 1). More specifically, the search for a subset of tasks which can be assigned to a work station is terminated when the slack time of the work station is less than or equal to a predetermined lower bound of acceptable slack time IT. This lower bound can be calculated either as a percentage of the cycle time T [7], or as a percentage of the theoretical minimum total slack time per work station $MTST$ [4]. The $MTST$ can be calculated through equation (2):

$$MTST = \frac{A*[\sum_{i=1}^{n} t_i / T]^+ - \sum_{i=1}^{n} t_i}{A} \qquad (2)$$

where $[X]^+$ denotes the smallest integer larger than or equal to x.

This modification is compulsory since Hoffman's heuristic procedure has failed to give even one feasible solution - within the admissible limits of computing time - to a problem of assembly line balancing in a specific vehicles assembly company [7]. This was due to the fact that in this problem, the multitude of feasible subsets of tasks which could be assigned to the first work station was already prohibitively large.

This case is usual when, in an industrial environment, we solve problems which are characterized by a large number of tasks to be performed, a small variation in the task times, and a relatively small number of precedence constraints in the tasks sequencing.

If the solution obtained from the first phase leads to work stations with unequal loading, then we proceed to the **second phase** of the method.

Given the minimum number of work stations A^* resulting from the first phase, the second phase deals with smoothing the total slack time d^* assignment to all the work stations. Minimization of the smooth index Sl, calculated by using the equation (3), constitutes the criterion for the equal assignment of the total slack time over all work stations:

$$Sl = \sqrt{\sum_{k=1}^{A^*} (T - S_k)^2} \tag{3}$$

where S_k is the time content of the work station k and $k=1,2,\ldots,A^*$.

The smaller the smooth index, the more smoothed the assignment of the total slack time to the work stations. It is obvious from (3) that for a given number of work stations A^* , the minimum value of the smooth index occurs when the work content of all work stations is equal, i.e.

$$S_1 = S_2 = \ldots = S_k = \ldots = S_{A^*} = \frac{\sum_{i=1}^{n} t_i}{A^*} \tag{4}$$

This work content in each work station constitutes the lower bound that the cycle time T could theoretically achieve without increasing the number of required work stations A^*. Therefore, if there is a solution to the SALB problem at a cycle time $T = \sum t_i / A^*$, with the same number of works stations A^*, then this solution is the optimum one. Otherwise, we are seeking a solution with the same number of work stations A^*, bisecting the interval between the initial cycle time T and its lower bound $\sum t_i / A^*$, according to the iterative procedure described in figure 2.

The methodology presented is embodied in a FORTRAN ver. 4.0 computer program which we have developed for this purpose. The computer program can be executed in DOS environment on any PC XT or AT IBM compatible computer.

4. AN EXAMPLE FROM INDUSTRY

The above methodology has been used within a research project of redesigning the assembly line of a jeep-type vehicle, in a major Greek company [7]. The assembly line consisted of the frame assembly and chassis assembly departments. When a specific set of tasks was completed on the chassis, then the assembled frame was transported by means of a bridge crane and placed on the chassis. These operations were followed by the final assembly, decoration, painting and final control phases. In the following paragraphs, we present the application of the methodology for the chassis assembly.

More specifically, figure 3 presents the precedence constraints diagram of the tasks to be performed for the assembly of the chassis. The total number of tasks is 99, with a total task time of $\sum t_i = 1022.29$ minutes. The multitude of precedence constraints existing among tasks amounts to 125, whereas the desired cycle time in the chassis assembly department is T=90 minutes.

Application of the methodology

Phase 1: We solve a SALB problem with the cycle time T = 90 minutes, according to the heuristic procedure of figure 1. The results are presented in figure 4. We observe that the minimum required number of work stations is A^*=12 and the total slack time is d^* = 57.71 minutes. This total slack time is unequally assigned to the work stations with a smooth index SI = 53.32. In this case, the smooth index value is very large because 92.20% of the total slack time is assigned to the last work station No. 12.

Phase 2:

- We determine the lower bound T_A of the cycle time for A^*=12 work stations, using the formula $T_A = \sum t_i / A^* = 1022.29 / 12 = 85.20$ minu tes. We solve the SALB problem with the cycle time T = T_A = 85.20 minutes, according to the heuristic procedure of figure 1. The results are presented in figure 5. We observe that the minimum required number of work stations has increased to 13.

- We search for a solution to the SALB problem with the same number of work stations A^*=12, bisecting the interval between the initial cycle T=90 minutes and its lower bound T_A=85.20 minutes according to the iterative procedure described in figure 2. The optimal solution with a cycle time T_C=85.31 minutes, is presented in figure 6.

Comparing the initial solution shown in figure 4 and the optimum in figure 6, we observe a significant improvement in the smoothing assignment of total slack time over all work stations.

5. CONCLUSIONS

In this paper, the methodology for balancing assembly lines with smoothed work station assignment has been presented. Smoothed work station assignment constitutes an additional criterion during the design and development of assembly lines characterized by a large number of tasks, small variation of task times and a small number of precedence constraints with regard to the total number of tasks to be performed. Assembly lines having the above features are very common in an industrial environment. In those cases, the application of the suggested methodology leads to an efficient organization of the assembly line. This last point is confirmed by the successful application of this methodology in redesigning the assembly line of jeep-type vehicles in a major Greek company.

REFERENCES

[1] Baybars I., A. "Survey of Exact Algorithms for the Simple Assembly Line Balancing Problem", *Management Science*, Vol. 32, No.8, pp. 909-932, 1986.

[2] Dar-El E.M. (Mansoor), "MALB - A Heuristic Technique for Balancing Large Single-Model Assembly Lines", *AIIE Trans.*, Vol. 5, No.4, pp 343-356, 1973.

[3] Dar-El E.M. (Mansoor), Rubinovitch Y., "MUST - A Multiple Solutions Technique for Balancing Single Model Assembly Lines", *Management Science*, Vol. 25, No.11, pp 1105-1115, 1979.

[4] Gehrlein, Patterson J.H., "Sequencing for Assembly Lines with Integer Task Times", *Management Science*, Vol. 21, No. 9, pp 1064-1070, 1975.

[5] Ghosh S., Gagnon R.J., "A Comprehensive literature review and analysis of the design, balancing and scheduling of assembly systems", *International Journal of Production Research*, Vol. 27, No. 4, pp. 637-670, 1989.

[6] Hoffman T.R., "Assembly Line Balancing with a Precedence Matrix", *Management Science*, Vol. 9, pp. 551-562, 1963.

[7] Industrial Management Division, Aristotle University of Thessaloniki, Organizing the production line of military jeep-type vehicles (Mercedes 240 GD) of Hellenic Vehicle Industry S.A. (EL.B.O.), *Research report*, 1987 (In Greek).

[8] Johnson R.V., "A Branch and Bound Algorithm for Assembly Line Balancing Problems with Formulation Irregularities", *Management Science*, Vol. 29, No. 11, pp. 1309-1324, 1983.

[9] Talbot F.B., Patterson J.H., "An Integer Programming Algorithm with Networks Cuts for Solving the Assembly Line Balancing Problem", *Management Science*, Vol. 30, No. 1, pp. 85-99, 1984.

[10] Talbot F.B., Patterson J.H., Gehrlein W.V., "A Comparative Evaluation of Heuristic Line Balancing Techniques", *Management Science*, Vol. 32, No. 4, pp. 430-454, 1986.

[11] Van Assche F., Herroelen W.S., "An optimal procedure for the single-model deterministic assembly line balancing problem", *European Journal of Operations Research*, Vol. 3, pp. 142-149, 1978.

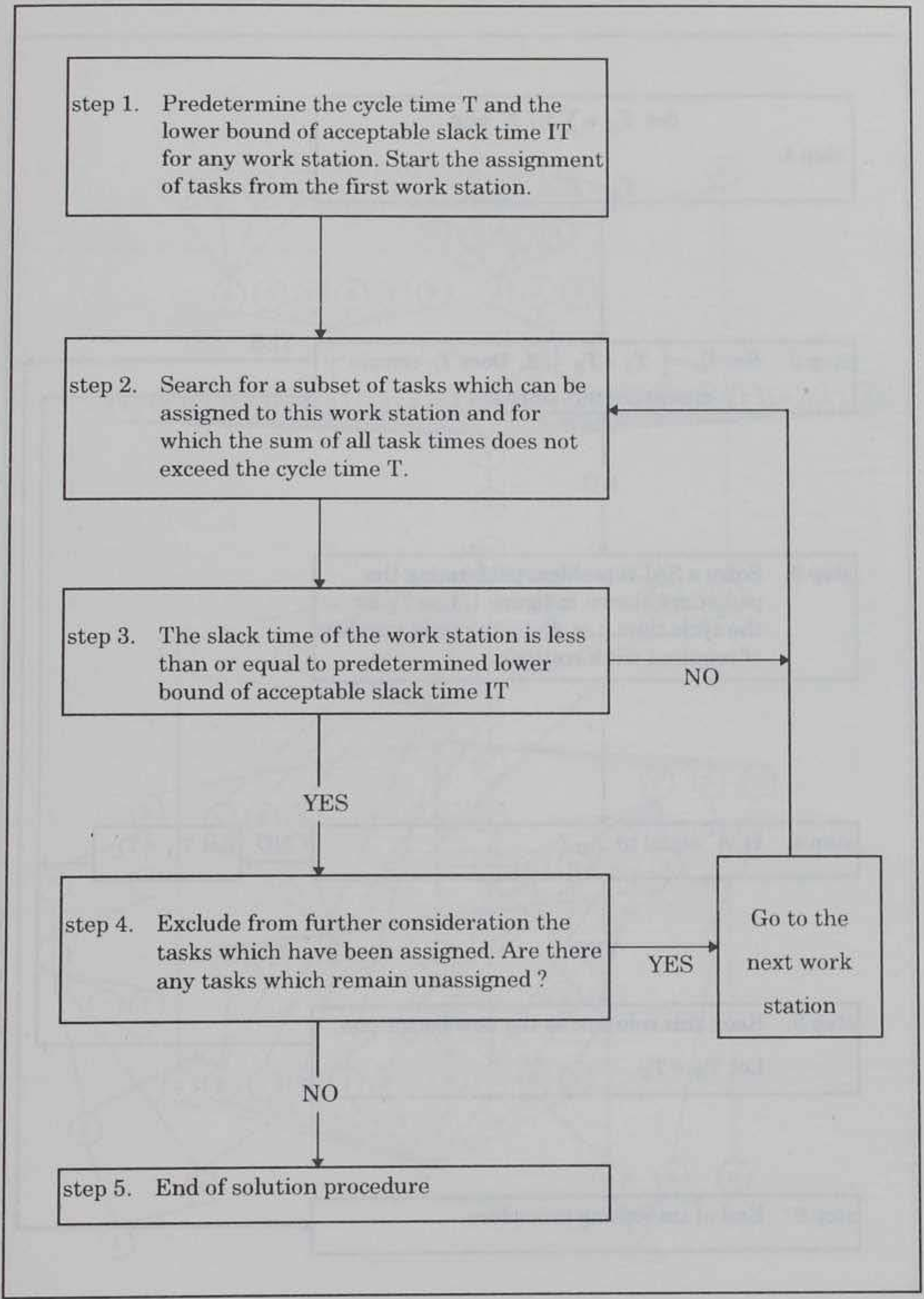

Figure 1. Flow chart diagram for the solution procedure of the SALB problem.

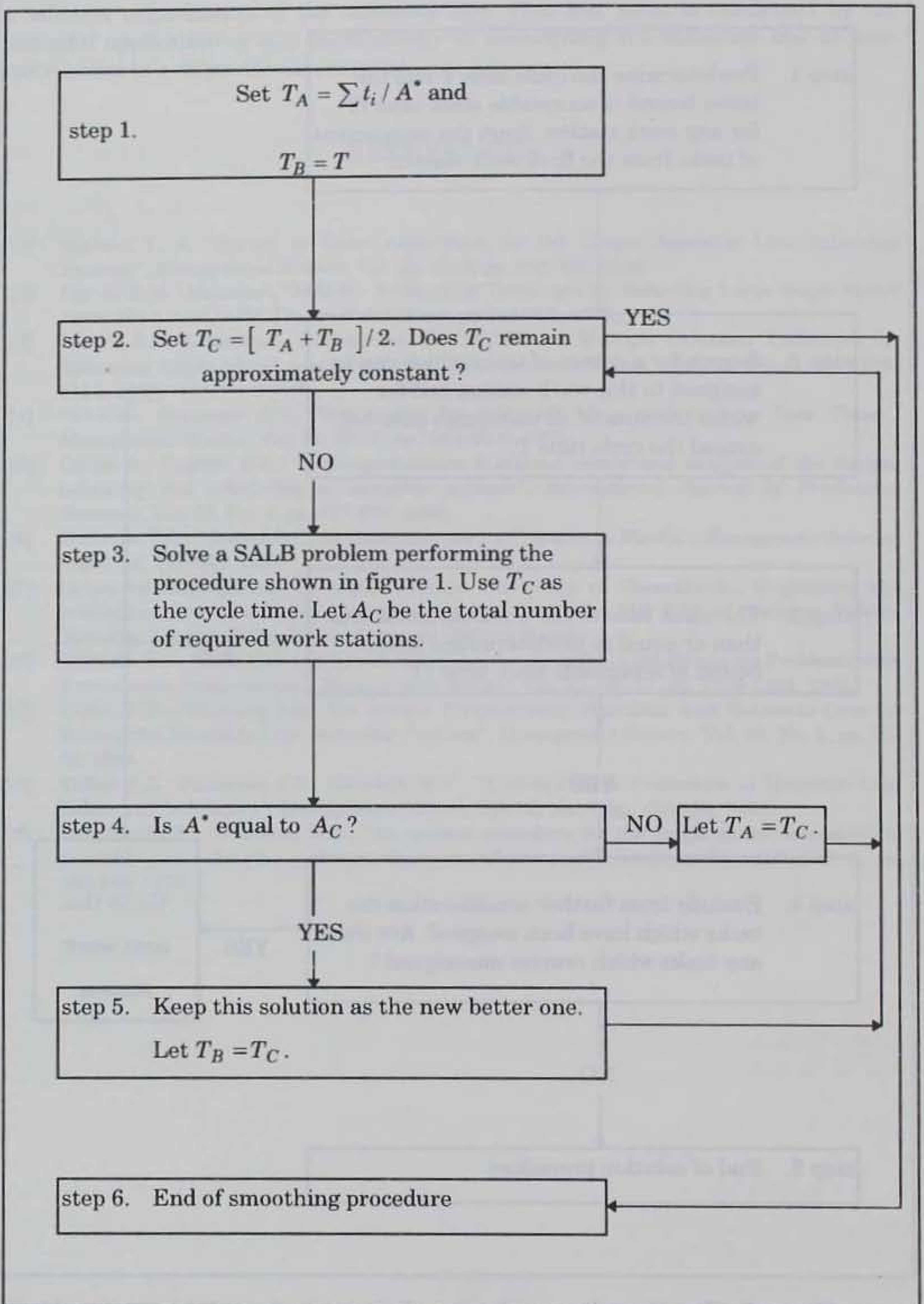

Figure 2. Flow chart diagram for the smoothing procedure.

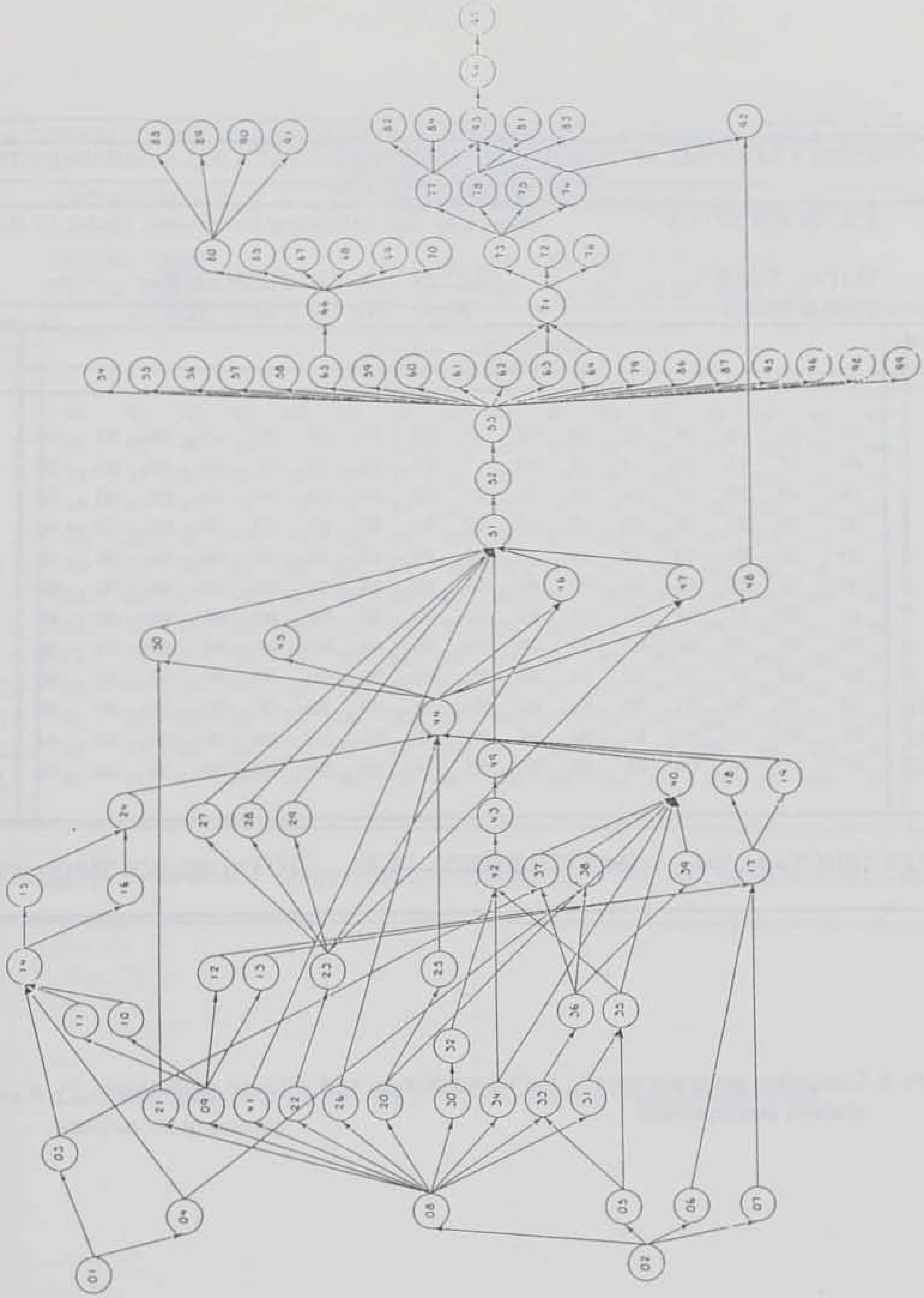

Figure 3. Precedence diagram for chassis assembly

ASSEMBLY LINE BALANCING WITH SMOOTHED STATIONS ASSIGNMENTS
HEURISTIC METHOD

PROBLEM TITLE : Assembly line balancing for chassis [date: 15 Nov 93]

TOTAL TIME : 1022.29 No. OF ELEMENTS : 99
CYCLE TIME : 90.00 No. OF PREC. RESTR. : 137

station number	WORK ELEMENTS																slack time
1	01	02	03	04	06	08	09	10	12	31	00	00	00	00	00	00	0.00
2	05	07	11	13	14	15	16	17	22	23	34	41	00	00	00	00	0.00
3	18	19	20	21	24	25	26	27	28	33	00	00	00	00	00	00	0.00
4	30	32	35	36	42	43	00	00	00	00	00	00	00	00	00	00	0.45
5	29	37	38	39	40	44	45	46	47	48	49	00	00	00	00	00	3.48
6	50	51	52	53	55	59	62	65	66	68	00	00	00	00	00	00	0.00
7	54	56	57	58	64	67	98	00	00	00	00	00	00	00	00	00	0.00
8	60	61	79	99	00	00	00	00	00	00	00	00	00	00	00	00	0.00
9	63	69	70	71	73	74	80	92	00	00	00	00	00	00	00	00	0.03
10	72	75	76	77	78	83	84	86	88	89	93	00	00	00	00	00	0.00
11	81	82	85	90	94	96	97	00	00	00	00	00	00	00	00	00	0.55
12	87	91	95	00	00	00	00	00	00	00	00	00	00	00	00	00	53.20

EFFICIENCY=94.35% SMOOTH INDEX=53.32 TOTAL SLACK TIME=57.71

Figure 4. Computer program results for T=90.00 min and without smoothed station assignments

ASSEMBLY LINE BALANCING WITH SMOOTHED STATIONS ASSIGNMENTS
HEURISTIC METHOD

PROBLEM TITLE : Assembly line balancing for chassis [date: 15 Nov 93]

TOTAL TIME : 1022.29 No. OF ELEMENTS : 99
CYCLE TIME : 85.20 No. OF PREC. RESTR. : 137

station number	WORK ELEMENTS																slack time
1	01	02	03	04	06	08	26	34	00	00	00	00	00	00	00	00	0.00
2	05	07	09	10	11	12	13	14	15	16	20	21	22	25	00	00	0.00
3	17	18	23	24	27	28	29	31	35	00	00	00	00	00	00	00	0.05
4	19	30	32	33	36	37	42	00	00	00	00	00	00	00	00	00	0.50
5	38	39	40	41	44	45	46	48	50	00	00	00	00	00	00	00	0.08
6	43	47	49	51	52	53	59	65	96	98	00	00	00	00	00	00	0.00
7	55	56	57	64	66	68	69	00	00	00	00	00	00	00	00	00	0.00
8	54	62	63	67	70	71	76	86	00	00	00	00	00	00	00	00	0.01
9	72	73	75	77	78	79	99	00	00	00	00	00	00	00	00	00	0.00
10	58	60	74	81	83	84	85	87	00	00	00	00	00	00	00	00	0.03
11	61	80	82	92	95	00	00	00	00	00	00	00	00	00	00	00	0.09
12	88	89	90	93	94	97	00	00	00	00	00	00	00	00	00	00	3.95
13	91	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	80.60

EFFICIENCY=91.66% SMOOTH INDEX=80.70 TOTAL SLACK TIME=85.31

Figure 5. Computer program results for T=85.20 min and without smoothed station assignments

ASSEMBLY LINE BALANCING WITH SMOOTHED STATIONS ASSIGNMENTS
HEURISTIC METHOD

PROBLEM TITLE : Assembly line balancing for chassis [date: 15 Nov 93]

TOTAL TIME : 1022.29 No. OF ELEMENTS : 99
CYCLE TIME : 85.31 No. OF PREC. RESTR. : 137

station number	WORK ELEMENTS																slack time
1	01	02	08	22	26	34	41	00	00	00	00	00	00	00	00	00	0.01
2	09	12	13	20	21	23	25	30	32	00	00	00	00	00	00	00	0.01
3	03	04	05	06	07	27	31	33	36	37	00	00	00	00	00	00	0.01
4	10	11	14	16	17	18	19	35	38	42	43	49	00	00	00	00	0.26
5	15	24	28	29	39	40	44	47	48	50	00	00	00	00	00	00	0.01
6	45	46	51	52	53	58	59	63	86	00	00	00	00	00	00	00	0.01
7	56	65	66	67	69	70	80	98	00	00	00	00	00	00	00	00	0.01
8	60	62	64	71	88	95	00	00	00	00	00	00	00	00	00	00	0.01
9	55	73	74	76	77	82	87	92	00	00	00	00	00	00	00	00	0.00
10	61	75	78	79	81	85	89	96	00	00	00	00	00	00	00	00	0.01
11	54	57	68	83	93	94	00	00	00	00	00	00	00	00	00	00	0.08
12	72	84	90	91	97	99	00	00	00	00	00	00	00	00	00	00	1.01

EFFICIENCY=99.86% SMOOTH INDEX=1.05 TOTAL SLACK TIME=1.43

Figure 6. Final computer program results with smoothed station assignments