

## A QUANTITATIVE MODEL FOR EVALUATING THE IMPACT OF A POLLUTING STACK

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**Abstract.** This paper presents a mathematical model for a quantitative evaluation of the polluting impact of single stack polluting facilities. Parameters are introduced quantifying both the maximum probable impact and the long-term time averaged impact. Realistic pollutant dispersion and stack performance models are employed. Local meteorological statistics are taken into account for the prediction of the spatial distribution of long-term pollution loads. The local population distribution is used for the evaluation of the overall polluting impact of the unit. The parameters introduced are suitable for quantitative evaluations of alternative stack designs and locations and may be used in models optimizing the location and stack design of polluting facilities. The methodology is applied to the case of an existing unit in the region of Thessaloniki — Greece.

*Key words and phrases:* obnoxious facilities, location, stack height, quantitative model

### 1. INTRODUCTION

The location of obnoxious facilities, and especially of those causing atmospheric pollution, is one of the fields of operational research the importance of which has become eminent in recent years. During the last decade, the optimal location of polluting units has been the subject of intense research activity aimed towards the elaboration of models expressing effectively the spatial data of the problem. In these models the installation of the unit is permitted over a predetermined set of areas. Other areas are characterized as protected (e.g. towns, agricultural lands, protected natural habitats etc.) with varying protection requirements depending on the nature of the area. The aim of optimization is to define a unit location minimizing the polluting impact of the unit, expressed either by the total pollution load of the whole set of protected areas, or by the load of the most affected area. In several recent publications the emphasis was on increasing the realism of modelling

the geometrical data of the problem concerning the shape of protected or permitted areas, their relative position and the different protection requirements of each protected area. For the interested reader we refer to the works of Dasarathy and White [1], Drezner and Wesolowsky [2], Melachrinoudis and Cullinane [3] and [4], Karkazis and Karagiorgis [5] and [6] where such efforts are presented. A common element of all these works is that the polluting impact of the unit on a point within the affected area is exclusively quantified by the Euclidean distance between the unit and the point. In reality several other factors equally affect this impact as for example the meteorological conditions and the diluting performance of the unit's stack. The distance of a point from the stack affects non-linearly the pollution load when the point is located downwind of the stack but is irrelevant when the point is located upwind. The purpose of this paper is to suggest parameters quantifying in a more realistic way the polluting impact of a unit on the affected area. Such parameters should take into account both the meteorological conditions of the area in question and the physics of dispersion.

The parameters suggested quantify the impact of a unit discharging obnoxious effluent gases by means of a single stack. The first parameter is related to the maximum pollutant concentration caused by the operation of the stack under any probable meteorological conditions. The second quantifies the overall pollution load caused by the operation of the stack. Its derivation is based on the calculation of a time averaged pollutant concentration field taking into account a statistical analysis of all probable meteorological conditions in the region of interest. An appropriate average of these concentrations weighted by the local population density quantifies the overall impact of the unit.

These parameters can be used in models optimizing the location of the unit in the following way: The first would allow for the identification of permitted areas, where the location of the unit would result in maximum concentrations, over the protected areas, not exceeding (under any probable meteorological conditions) a limit defined by environmental considerations. The second parameter quantifies the overall polluting impact of the unit and the optimal unit location should minimize its value.

In the following section the mathematical dispersion model is presented and the stack performance parameters are introduced. In the third section the methodology is applied to the case of an existing unit in the region of Thessaloniki, Greece and the parameters suggested are used for the evaluation of candidate locations of a stack in the same region.

## 2. THE MATHEMATICAL MODEL

### 2.1. THE PLUME DISPERSION MODEL

The present application deals with pollutants which may be considered as chemically inert during dispersion, e.g. sulfur dioxide ( $\text{SO}_2$ ). The pollutant is discharged by means of a high stack. The dispersion process is shown schematically

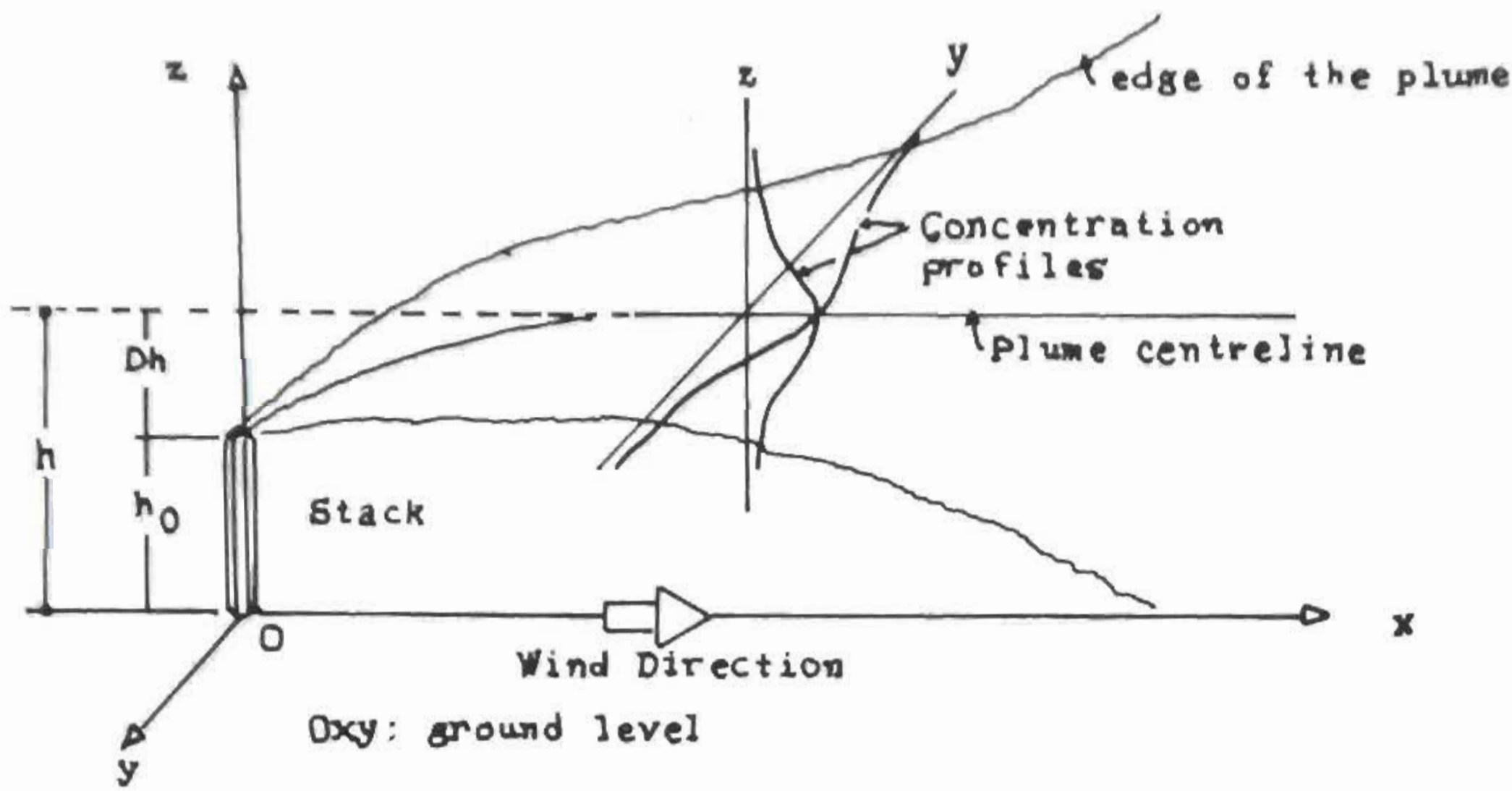


FIGURE 1: Coordinate system. Sketch of plume elevation and dilution.

in Figure 1. Initially the effluent gases, under the influence of their vertical momentum and buoyancy, are elevated to an additional height  $Dh$  above the stack exit. The vertical component of the plume motion fades gradually with the distance from the stack. From a certain point downwind of the stack the plume moves horizontally with the mean wind velocity. Atmospheric turbulence continuously dilutes the effluent gases and the plume cross-section is increased with the distance from the stack, whereas ground-level concentrations are decreased. A Gaussian plume model is adopted for the estimation of atmospheric dispersion (Turner, [7]). Concentrations are assumed to follow Gaussian distributions. The ground-level concentrations, of interest here, are given by equation:

$$c(x, y, z = 0) = \frac{\dot{Q}}{\pi \sigma_y \sigma_x \bar{u}_h} \exp\left(-0.5 \cdot \left(\frac{y^2}{\sigma_y^2} + \frac{h^2}{\sigma_z^2}\right)\right) \quad (1)$$

where  $\dot{Q}$  is the pollutant emission rate,  $h$  the height of the plume centreline,  $\bar{u}_h$  is the mean wind velocity at height  $h$  and  $\sigma_y, \sigma_z$  are the normal distribution standard deviations, usually referred to as diffusion parameters. The diffusion parameters are functions of the distance from the stack and they depend on the degree of atmospheric stability. The degree of stability quantifies the intensity of atmospheric turbulence and depends, mainly, on the mean temperature gradient of the lower atmosphere. Pasquill and Gifford [8], based on experimental observations, provided curves of  $\sigma_y$  and  $\sigma_z$  as functions of the distance from the stack for six characteristic stability conditions. These characteristic stability conditions cover satisfactorily the whole range of cases occurring in reality (from very stable to strongly unstable atmosphere). In this application the curves of Pasquill and Gifford were approxi-

mated by polynomial expressions of the form:

$$\sigma_y = ax^b, \quad \sigma_z = a_0 + a_1x^{n_1} + a_2x^{n_2} + a_3x^{n_3}. \quad (2)$$

The coefficients and exponents in equations (2) were defined using an appropriate number of original data (see Appendix). Equations (2) were found to reproduce the data satisfactorily, the largest deviations being less than 1%.

In order to calculate the concentration field (equation 1), the total plume elevation ( $h = h_0 + Dh$ ) and the wind velocity at this height ( $\bar{u}_h$ ) have to be specified. The wind velocity at any height  $h$  is estimated from the semi-empirical boundary layer law (Strom, [9]):

$$\bar{u}_h = \bar{u}_0 \cdot (h/z_0)^m \quad (3)$$

where  $\bar{u}_0$  is the wind velocity at a height  $z_0$  where wind velocity data are available (usually  $z_0 = 10$  m). The exponent  $m$  depends on the stability conditions, and the values suggested by Strom [9] were adopted; i.e.  $m = 1/7$  for neutral conditions,  $m = 1/10$  for stable and  $m = 1/5$  for unstable conditions.

The method of Briggs [10] is applied for the estimation of the plume elevation. The plume elevation depends on the wind velocity at the stack exit and on the momentum and buoyancy of the effluent gases. The momentum and buoyancy content of the effluent gases are quantified by the parameter  $F_b$ :

$$F_b = gV_0(D_0/2)^2(1 - T/T_0) \quad (4)$$

where  $g$  is the acceleration of gravity,  $T$  is the absolute temperature ( $^{\circ}\text{K}$ ) of the atmosphere at the stack exit,  $T_0$  is the absolute temperature of the effluent gases,  $V_0$  is the velocity of the gases at the stack exit and  $D_0$  is the diameter of the stack exit. The elevation of the plume is estimated from the equation:

$$Dh = c_x F_b^{1/3} x_0^{2/3} / \bar{u}_s, \quad h = h_0 + Dh \quad (5)$$

where  $\bar{u}_s$  is the wind velocity at the stack exit ( $= \bar{u}_0(h_0/z_0)^m$ ),  $F_b$  was previously defined (eq. 4) and the quantity  $x_0$  is defined as a function of  $F_b$  as follows:

$$\begin{aligned} \text{if } F_b > 51.6 \text{ then } x_0 &= 119F_b^{0.4} \\ \text{if } F_b < 51.6 \text{ then } x_0 &= 49F_b^{0.625} \end{aligned}$$

( $x_0$  is in meters the distance from the stack where the plume practically reaches its final elevation  $Dh$ ). Finally  $c_x$  is an experimentally derived coefficient with a value  $c_x = 1.6$ .

The presentation of the mathematical plume dispersion model concludes with a description of the procedure employed for the estimation of the concentration field for a given application. The data for each application are the stack parameters (i.e. the stack height  $h_0$  and diameter  $D_0$ , the pollutant emission rate  $Q$ , the effluent gases temperature and velocity) and the meteorological data (i.e. the wind mean velocity vector and the category of atmospheric stability), the set of  $\sigma_y$ ,  $\sigma_z$

equations, and the value of the exponent  $m$  of the wind velocity law (eq. 3) corresponding to the category of atmospheric stability, are selected (see Appendix). The stack parameter  $F_b$  and the wind velocity at the stack exit  $\bar{u}_s$ , are calculated from equations (4) and (3), respectively. The plume elevation is subsequently defined by equation (5). Thus all parameters appearing in equation 1 are known and the concentration field may be calculated.

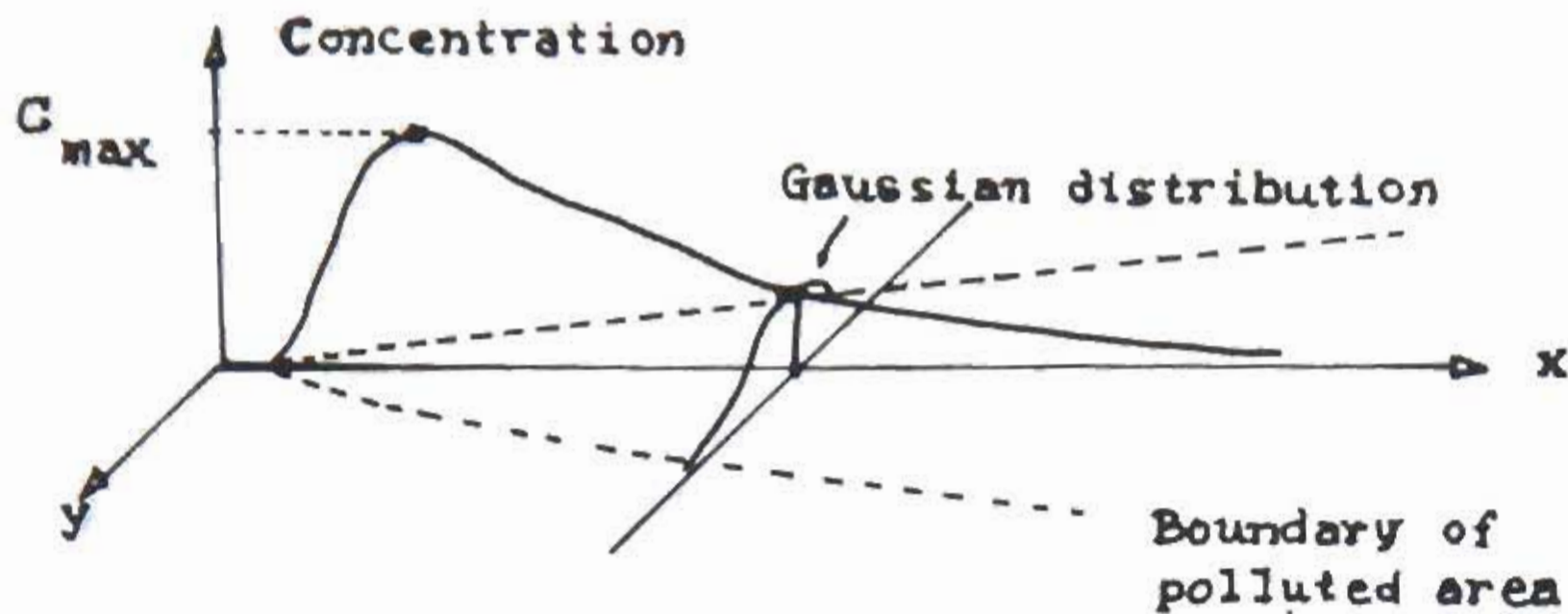


FIGURE 2: Sketch of ground-level concentrations distribution.

The form of the ground-level concentration field predicted by the Gaussian plume model is shown in Figure 2. For a given distance ( $x$ ) from the stack the concentrations have a normal distribution with a local maximum at the projection of the plume centreline ( $Ox$ ). The concentration field is symmetric in respect to the plume centreline projection; the concentrations rise steeply close to the stack, reach a maximum value and then continuously decrease further downwind.

## 2.2. STACK PERFORMANCE PARAMETERS

### 2.2.1 The Largest Ground-Level Concentration $C_m$

This parameter quantifies the physical diluting performance of the stack. For a given stack geometry ( $h_0, D_0$ ) and constant operation characteristics ( $\dot{Q}, F_b$ ) the maximum pollutant concentration  $C_{\max}$  depends on the meteorological conditions during the operation of the stack, namely on the wind velocity modulus ( $\bar{u}_0$ ) and on the category of atmospheric stability ( $S$ ):

$$C_{\max} = f(\bar{u}_0, S). \quad (6)$$

The *Largest Ground-Level Concentration*  $C_m$  is defined as the maximum of  $C_{\max,i}$  values under all probable meteorological conditions:

$$C_m = \max(C_{\max,i}) \quad (7)$$

where the index  $i$  denotes the whole range of conditions reported in the meteorological statistics of the region in question. The practical importance of this criterion is

rather obvious: it gives the maximum value of the pollutant concentration resulting from the operation of the unit under any probable meteorological conditions.

### 2.2.2. The Overall Pollution Load Parameter OPLP

The second parameter quantifies the overall polluting impact of the stack on the population of the areas surrounding the stack. Let  $(\eta, \xi)$  be a fixed Cartesian Coordinate system on the level of the area surrounding the stack, with its centre on the location of the stack. For each point  $M(\eta, \xi)$  within the area affected by the stack, a cumulative pollutant concentration is defined as the frequency weighted average of all the concentration values occurring during a given period of time at this point:

$$\bar{c}(\eta, \xi) = \sum_i f_i c_i(\eta, \xi). \quad (8)$$

The summation refers to the given period of time during which a finite set of meteorological conditions (i) occur with frequencies  $f_i$ .

Equation (8) allows for the estimation of cumulative pollutant concentrations all over the area affected by the operation of the stack for a set period of time. Equal cumulative concentration contours may be subsequently derived to give a clear picture of the spatial distribution of the long-term pollution load caused by the operation of the stack. More important, from an environmental point of view, are the cumulative concentrations occurring in densely populated areas around the stack. To quantify the importance of this fact, an *Overall Pollution Load Parameter (OPLP)* is introduced as the average of all cumulative concentrations weighted by the local density population:

$$\text{OPLP} = \frac{\sum_{\eta, \xi} P(\eta_i, \xi_i) \bar{c}(\eta_i, \xi_i)}{\sum_{\eta, \xi} P(\eta_i, \xi_i)}. \quad (9)$$

$P(\eta_i, \xi_i)$  is the population of the element of the area surrounding the point  $M(\eta_i, \xi_i)$ . It is obvious that the OPLP's value is not only a function of the stack design parameters but also of the location of the stack in relation to a set of affected areas.

In the case study presented in the next section of this paper the following assumption, concerning the physics of dispersion, was employed. It was assumed that the stability conditions of the atmosphere were neutral (category *D* in the classification of Pasquill and Gifford). This assumption will be justified later; it suffices here to say that neutral conditions are the most frequently occurring in Mediterranean coastal areas and may be considered as fairly representative of the stability conditions prevailing in such areas. With this assumption, the meteorological conditions refer exclusively to the wind velocity vector. This situation drastically simplifies the calculation of both the  $C_m$  and OPLP values for a given stack.

The Largest Ground-Level Concentration, defined in equation 7, has now the simpler form:

$$C_m = \max(C_{\max}(\bar{u}_{0,i})) \quad (10)$$

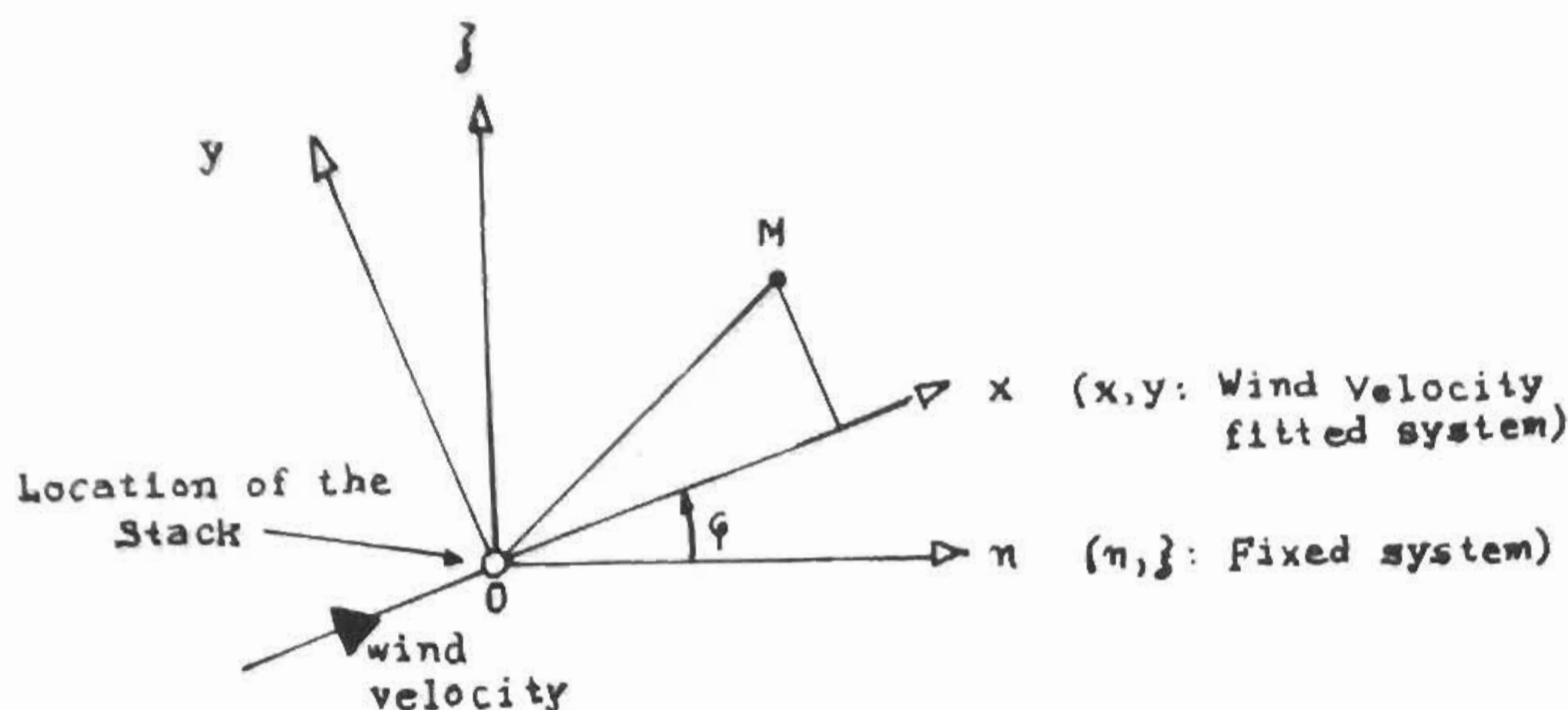


FIGURE 3: Calculation of cumulative concentrations. Coordinate systems.

where the wind velocity modulus  $\bar{u}_{0,i}$  assumes all the values reported in the meteorological statistics ranging from very small values (practically no wind) to a maximum value  $\bar{u}_{0,max}$ . The  $C_{max}$  values were calculated for a large number of  $\bar{u}_0$  values in the range ( $\bar{u}_0 \rightarrow 0$ ) to  $\bar{u}_{max}$ , continuous curves  $C_{max} = f(\bar{u}_{0,i})$  were drawn and  $C_m$  was defined as the largest  $C_{max}$  value.

Meteorological statistics allow for the derivation of diagrams such as those shown in Figure 4 giving for each direction  $\varphi$  the average wind velocity and the frequency of occurrence of winds within the angle  $(\varphi \pm \Delta\varphi/2)$ :  $f(\varphi) \Delta\varphi$ . In this context the frequency  $f_i$  appearing in the definition of cumulative concentrations (equation (8)) may be considered as the frequency of winds within the angle  $(\varphi \pm \Delta\varphi/2)$  i.e.  $f_i = f(\varphi) \Delta\varphi$  and the summation in equation (8) should refer to all possible  $\varphi$  values:  $0 \leq \varphi \leq 2\pi$  (Figure 3). With these simplifications the cumulative concentration equation may be re-written as follows:

$$\bar{c}(\eta, \xi) = \sum_{\varphi=0}^{\varphi=2\pi} f(\varphi) \Delta\varphi c_{\varphi}(\eta, \xi). \tag{11}$$

The concentration  $c_{\varphi}(\eta, \xi)$  at the point  $M(\eta, \xi)$  due to winds blowing in the direction  $\varphi$ , given by equation (1), has now the form:

$$c_{\varphi}(\eta, \xi) = \frac{\dot{Q}}{\Pi \sigma_y \sigma_z \bar{u}_h(\varphi)} \exp\left(-0.5 \cdot \left(\frac{y^2}{\sigma_y^2} + \frac{h(\varphi)^2}{\sigma_z^2}\right)\right) \tag{12}$$

$(x, y)$  are the coordinates of the point  $M(\eta, \xi)$  in relation to the Cartesian system  $(x, y)$  where direction  $x$  is the direction of the winds considered;  $\bar{u}(\varphi)$  is the harmonic mean velocity of winds blowing during the set period of time in the direction  $\varphi$ . (Harmonic instead of linear averaging gives more accurate results in this case, since in equation (12) the velocities appear in the denominator of the concentration

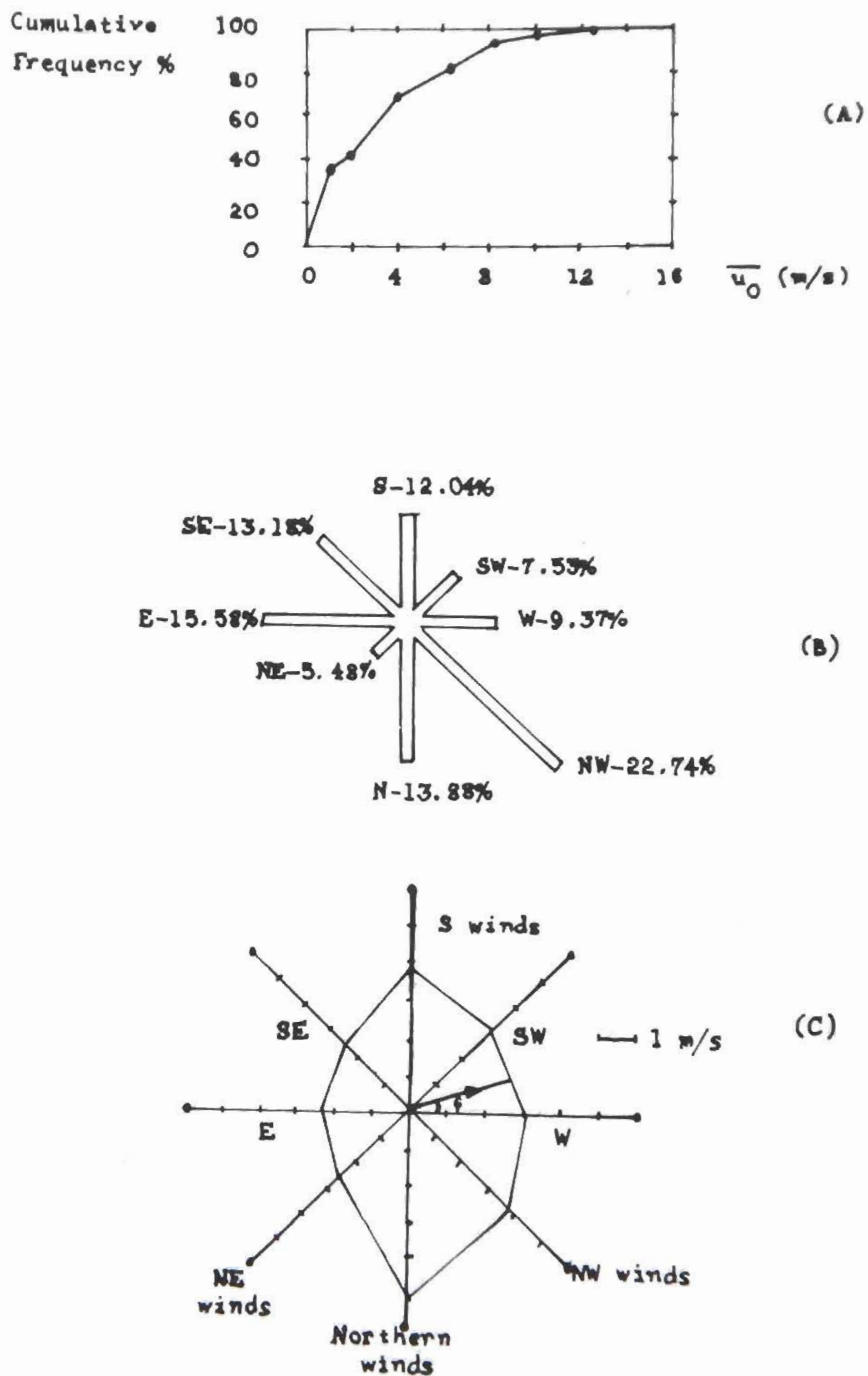


FIGURE 4: Meteorological statistics for Thessaloniki. (Annual basis). (A): Wind velocity — cumulative frequency curve, all directions. (B): Frequencies of occurrence of winds in the 8 main directions. (C): Average wind velocities.



function.) The diffusion parameters  $\sigma_y$  and  $\sigma_z$  are now functions of  $x$ . The dispersion height  $h(\varphi)$  is also a function of  $\varphi$  since it depends on  $\bar{u}(\varphi)$  through equations (3) and (5). A combination of equations (11) and (12) gives the final expression for the cumulative concentrations:

$$\bar{c}(\eta, \xi) = \sum_0^{2\pi} \frac{\dot{Q}f(\varphi)\Delta\varphi}{\Pi\sigma_y\sigma_z\bar{u}_h(\varphi)} \exp\left(-0.5 \cdot \left(\frac{y^2}{\sigma_y^2} + \frac{h(\varphi)^2}{\sigma_z^2}\right)\right). \quad (13)$$

### 3. RESULTS AND DISCUSSION

The stack performance analysis presented so far will be applied to the case of an existing large chemical plant located in the region of Thessaloniki — Greece (unit of “Northern Greece Chemical Industries” in Thessaloniki). The location of the unit and the set of towns affected by the operation of the stack are shown in Figure 5. The data of the existing stack are: stack height  $h_0 = 60$  m, stack diameter  $D_0 = 1.5$  m,  $\text{SO}_2$  emission rate  $Q = 285$  g/s, velocity of effluent gases  $V_0 = 6$  m/s, temperature of effluent gases  $T_0 = 200^\circ\text{C}$ .



FIGURE 5: Greater area of Thessaloniki. set of affected towns and location of the existing stack.

As already mentioned, neutral atmospheric stability conditions were accepted in this analysis. Pasquill and Gifford [8] suggest six categories of atmospheric

stability ranging from strongly unstable to stable conditions. A given set of meteorological conditions is classified in one of these categories on the basis of the wind velocity and temperature gradient of the lower atmosphere. In the absence of temperature gradient data, cloudiness and solar radiation data may be used. Meteorological statistics for the region of Thessaloniki contain data concerning the wind velocity vector, the cloudiness and solar radiation. It should be pointed out however that these statistics do not provide data for the *combination* of these parameters and, therefore, cannot be directly used for a stability classification. A scenario had to be devised to allow for such a combination. As a first approximation, large samples of the parameters involved (conforming with the statistics) were generated and combined at random by a computer. Application of the classification criteria on the combined data gave the following frequencies of stability conditions:

- 1) Unstable atmosphere (categories A and B): 16%
- 2) Weakly unstable, neutral or weakly stable (categories C,D,E): 56%
- 3) Stable atmosphere (category F): 28%.

Pasquill and Gifford [8] pointed out that neutral or close to neutral conditions are the most probable at coastal sites (in moderate climates) and may occur up to 80% of the total time. Cagnetti et al. [11], based on wind velocity and temperature gradient data, found that neutral conditions occur with a frequency of 70%–80% over most coastal Italian towns. It is probable that the crude stability category classification employed in this work led to an underestimation of the frequency of neutral conditions in our case. Given that neutral conditions are undoubtedly the most probable, and that a reliable deterministic stability classification was not possible with the data available, neutral stability conditions were accepted as fairly representative of the conditions occurring in reality.

With this assumption, the meteorological conditions refer exclusively to the wind velocity vector. The wind velocity data used in this work are shown in Figure 4. Figure 4a shows an annual cumulative frequency diagram for all direction wind velocities. Figure 4b shows the annual frequencies of winds in the 8 main directions and Figure 4c shows the harmonic mean wind velocity in each of these directions. The calculation of the cumulative concentration field requires a continuous wind velocity function  $u(\varphi)$  giving the average wind velocity and frequency density for any direction  $\varphi$ . Accordingly the wind velocity was assumed to vary linearly between the main wind directions as shown in figure 4c, whereas the frequency of each main wind direction was spread evenly to an angle ( $\pm 22.5^\circ$ ) surrounding each main direction.

### 3.1. PERFORMANCE OF THE EXISTING STACK

The relationship between the Maximum Ground-Level Concentration and the wind velocity modulus  $C_{\max} = f(\bar{u}_0)$ , is shown in Figure 9 ( $h_0 = 60$  m). The Largest Ground-Level Concentration  $C_m$  has a value of  $0.98 \text{ mg/m}^3$  occurring at wind velocity of 1.75 m/s. Wind velocities of this order are quite frequent as may be seen in Figure 4a. In Figure 6 the concentration field corresponding to this wind velocity is

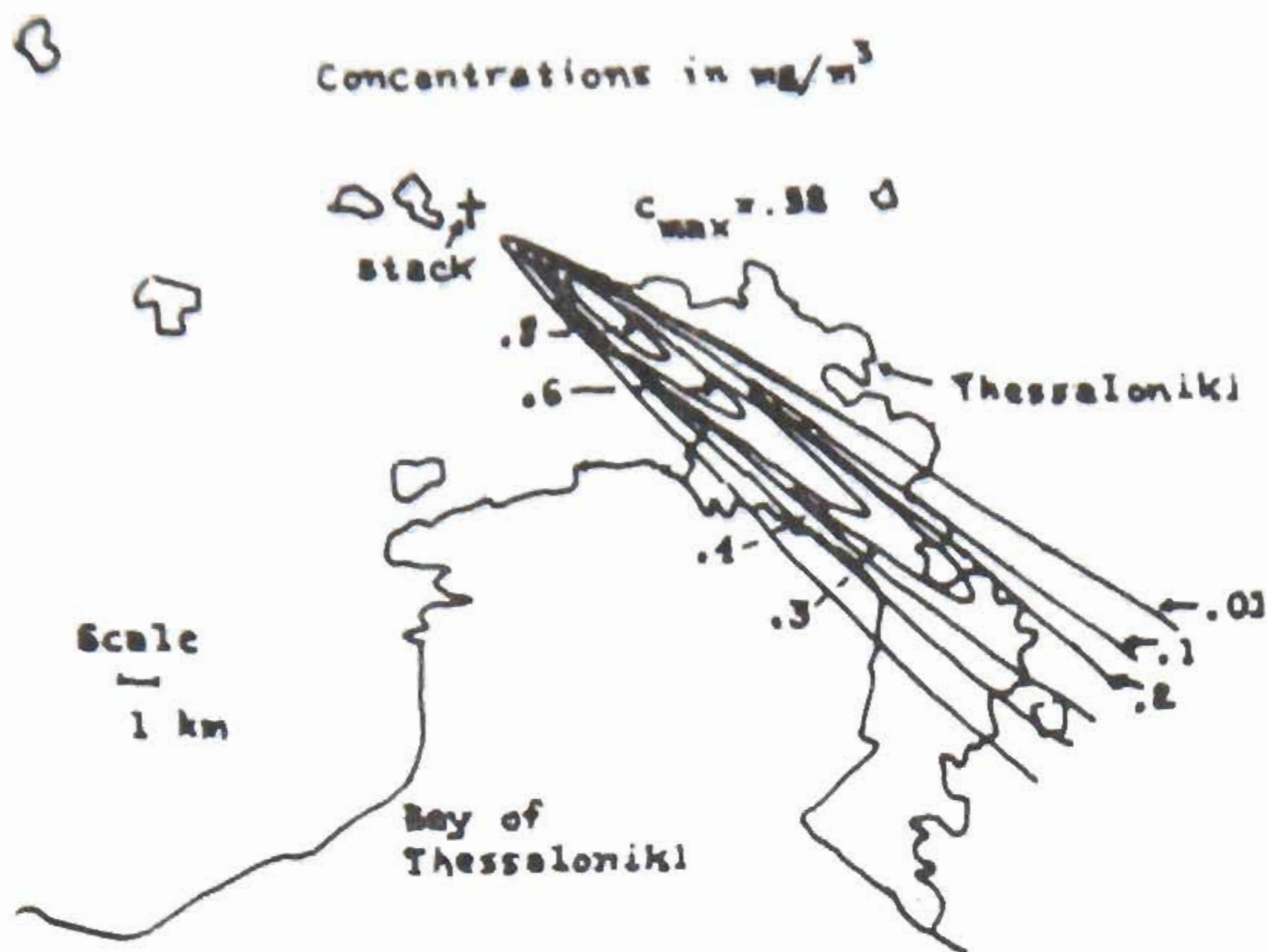


FIGURE 6: Concentration field. North-western wind  $u_0 = 1.75$  m/s. Existing stack  $h_0 = 60$  m.

shown when North-Western winds blow. (NW winds are the more frequently occurring as shown in Figure 4b). It is clear from the equal concentration contours given in Figure 6 that high  $\text{SO}_2$  concentrations result from the operation of the stack over the city of Thessaloniki and that the maximum value of  $0.98 \text{ mg/m}^3$  occurs at the north part of the city. "Acceptable" values of  $\text{SO}_2$  concentrations are  $0.9 \text{ mg/m}^3$  whereas a "satisfactory" value would be  $0.45 \text{ mg/m}^3$  (Canadian regulations, Greek regulations). The situation would look even worse if the impact of other  $\text{SO}_2$  polluting plants located close to the unit were examined here (e.g. the EKO refinery) and if other  $\text{SO}_2$  pollution loads (traffic, domestic heating) were taken into account. The results seem to contradict the study of Zerefos et al. [12] based on the numerical solution of an  $\text{SO}_2$  transport equation, where values lower than  $0.5 \text{ mg/m}^3$  were predicted over the region of interest for wind velocities close to the one examined here. However it should be taken into account that the numerical grid used in this work had a cell of  $(3 \times 3 \text{ km})$  whereas the present predictions show that the width of the concentration field ( $c > 0.1 \text{ mg/m}^3$ ) varies between 0 and 2 km. It is therefore clear that the coarse grid used for the numerical solution of Zerefos et al. could not allow for the resolution of a detailed concentration distribution.

The cumulative concentration field will be considered next. Equal cumulative concentration contours are shown in Figure 7. It is clear from this figure that the higher exposures (or cumulative concentrations) occur over the city of Thessaloniki due to the high frequency of NW winds (22.74%) and over the towns of Diavata,



FIGURE 7: Cumulative concentration field. Existing stack  $h_0 = 60$  m.

Nea Magnissian and Sindos due to the high frequencies of Eastern winds (15.58%). Examining Figure 7, one may understand that cumulative concentration maps as the one shown in this Figure are useful for a rational selection of the unit location in the case where the unit affects several inhabited areas. Such maps can also be used for the distribution of anti-pollution public funds between affected local authorities.

### 3.2. EFFECTS OF STACK HEIGHT ON $C_m$ AND OPLP

It is clear from equation (1) that the higher the dispersion takes place, the lower the ground level pollutant concentrations. The dependence of concentrations on  $h$  is strong through both the exponential term:  $\exp(-0.5(h/\sigma_z)^2)$  and through the wind velocity at the height of dispersion  $\bar{u}_h$ , which again depends on  $h$  through equation (3). The dispersion height is the sum of the stack height  $h_0$  and the plume elevation  $Dh$  which is related with all the main stack parameters ( $D_0, V_0, T_0$ ) and the wind velocity and temperature gradient. Thus, higher dispersion heights may be achieved either by increasing the stack height or by an appropriate choice of the remaining stack parameters. Nonetheless, it should be pointed out that these parameters are linked through the equations governing the operation of the stack, and that their choice affects both the stack construction and operation costs. Fur-

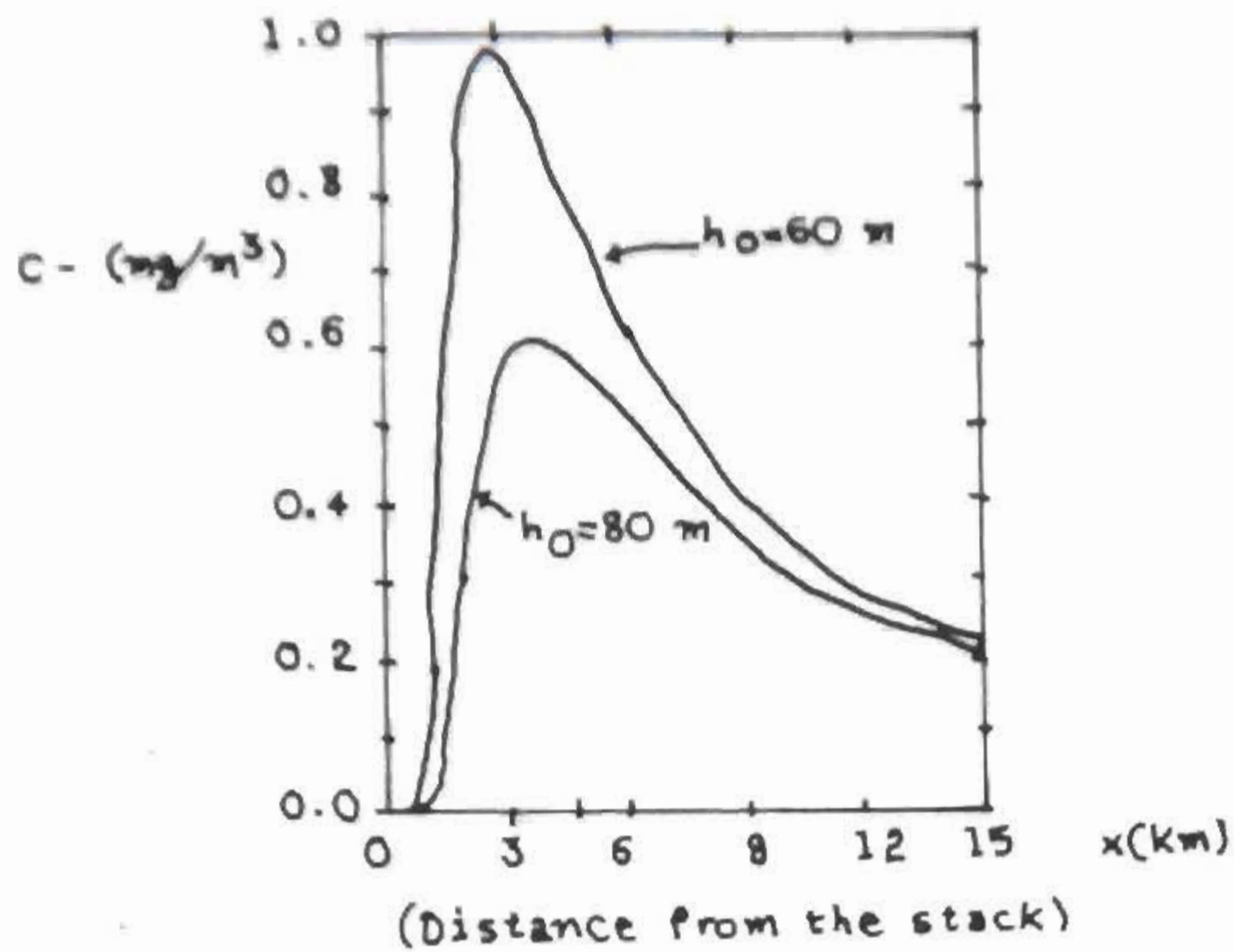


FIGURE 8: Concentrations ( $c$ ) at the projection of the plume centreline for two stack heights. Wind velocity 1.75 m/s.

thermore, stacks which result in larger dispersion heights, though more expensive to construct and operate, reduce the overall polluting impact of the unit and the related pollution costs. It is clear from this outline that the choice of the stack

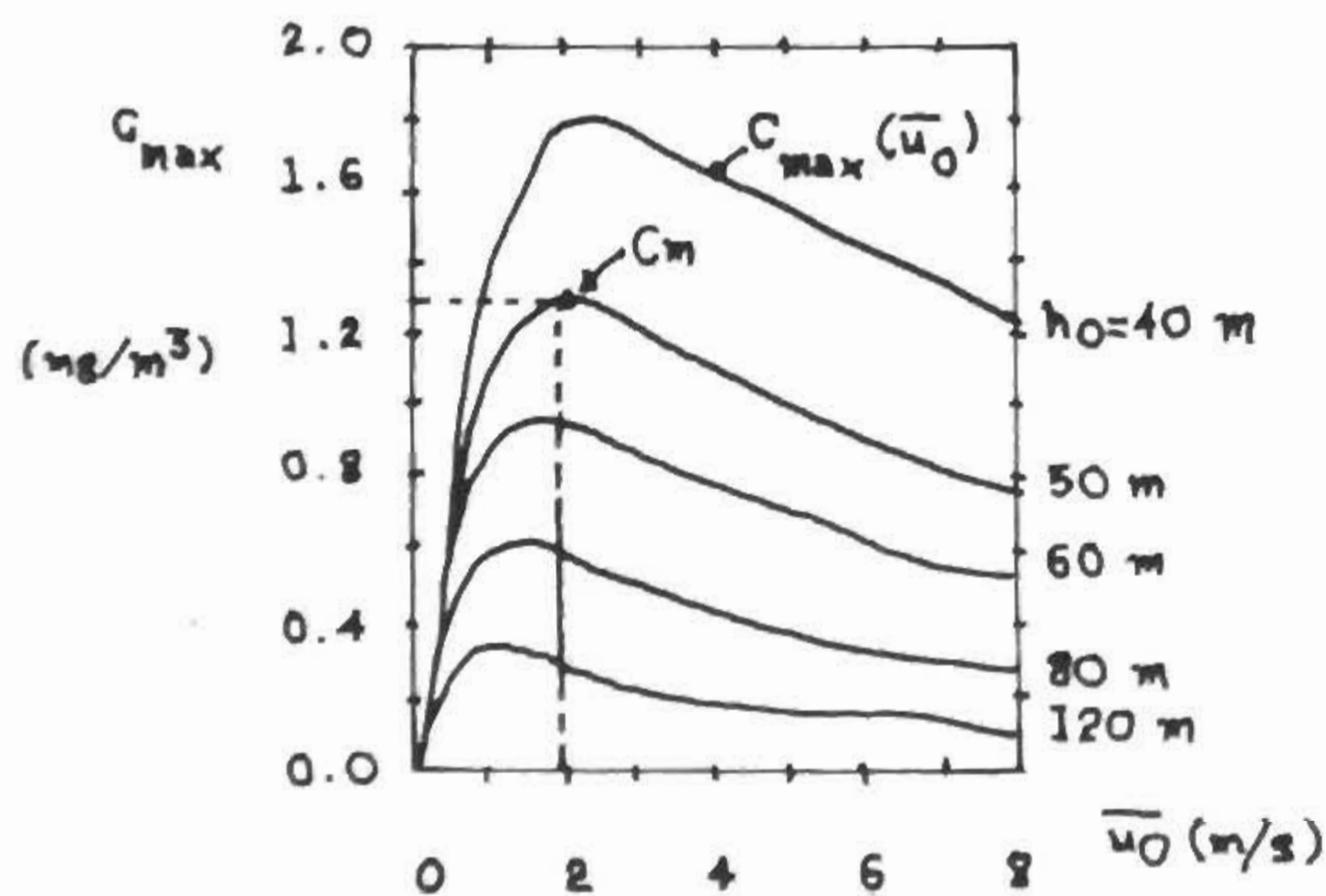


FIGURE 9: Relation between maximum concentration  $C_{\max}$  and wind velocity for five stack heights  $h_0$ .

design parameters may be a subject of optimization aiming to minimize the sum of construction, operation and pollution costs. Such an analysis is beyond the scope of this paper and could be suggested as a future research topic. In this work, only the

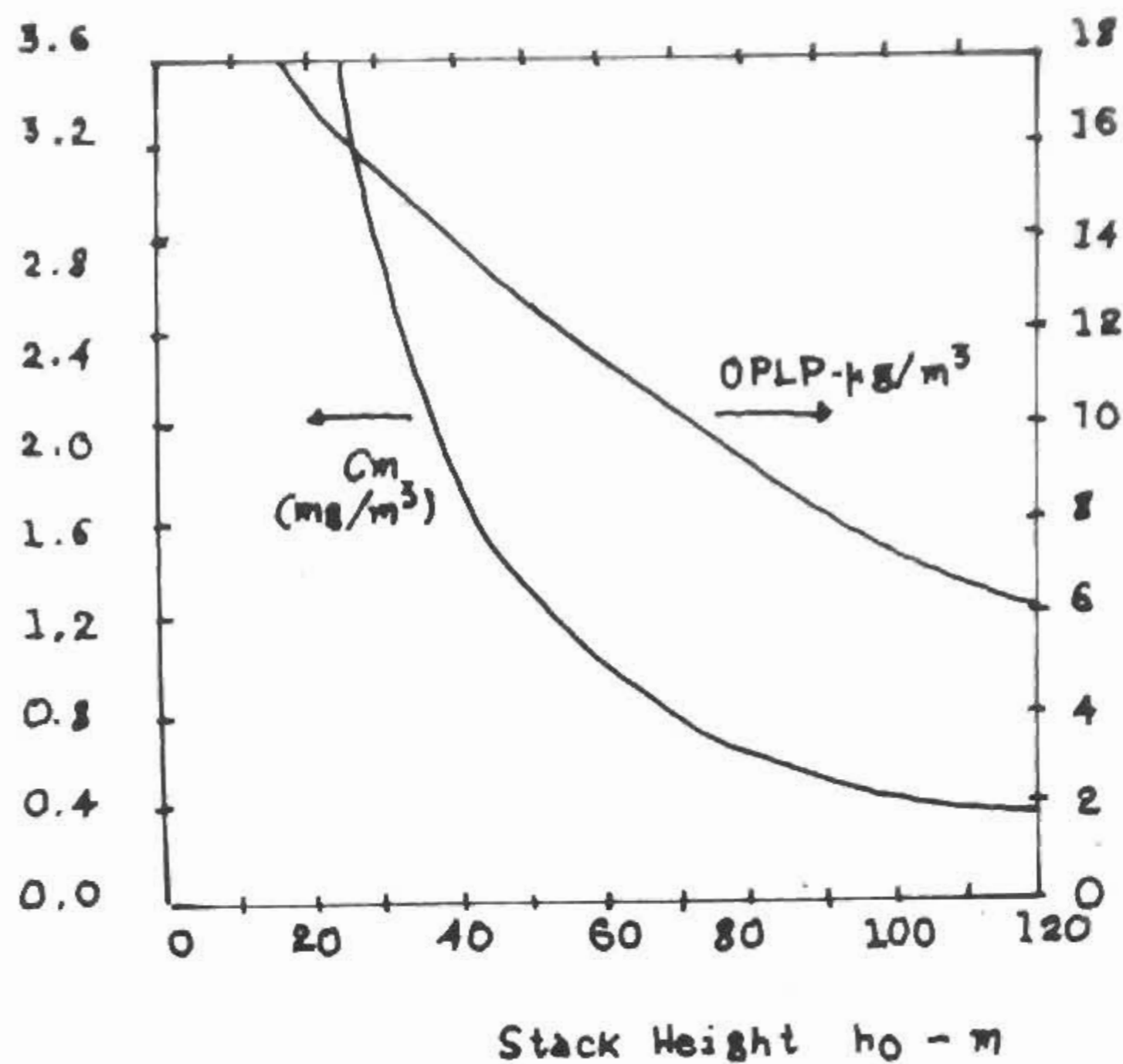


FIGURE 10: Overall Pollution Load Parameter OPLP and Largest Ground-Level Concentration  $C_m$  as functions of the stack-height.

influence of the stack height on  $C_m$  and OPLP will be examined and the remaining parameters will be assumed constant. It should be kept in mind, however, that the same effects could probably be achieved by appropriately changing the remaining stack parameters.

The relationship between the maximum ground-level concentration  $C_{\max}$  and the wind velocity ( $\bar{u}_h$ ) is shown in Figure 9 for several stack heights. In all cases the wind velocities corresponding to the Largest Ground-Level Concentration  $C_m$  occur quite frequently, as may be seen in Figure 4a. The relationship between  $C_m$  and the stack height is shown in Figure 10. A very steep reduction of  $C_m$  occurs as the stack height is increased (particularly in the region of lower stack heights). The practical significance of the introduction of the parameter  $C_m$  is clear from Figure 10. If an upper limit of the pollutant concentration caused by the unit (under any probable meteorological conditions) is defined by broader environmental considerations, a lower limit for the stack height may be directly derived from the function  $C_m = f(h_0)$ .

The relationship between the Overall Pollution Load Parameter OPLP and the height of the stack is shown in Figure 10. Though the OPLP values are reduced as the stack height is increased, the reduction is significantly less steep than the Largest Concentration  $C_m$  reduction. This difference could be expected since the OPLP is a spatially averaged concentration value and average concentrations are less sensitive to changes of the stack height. This may be clearly seen in Figure 8

showing the concentrations at the plume centreline projection for two stack heights ( $h_0 = 60$  and  $80$  m) for a length of  $15$  km downwind of the stack and a wind velocity of  $\bar{u}_0 = 1.75$  m/s. The maximum concentration values differ by  $38\%$  but the concentration differences are gradually reduced to  $10\%$  at a distance of  $15$  km, whereas the average values over this length differ by  $27\%$ .

The data shown in Figure 10 support the following conclusion: Even though increases of the stack height may alleviate local pollution problems through a significant reduction of maximum concentrations, the overall impact of the unit (as quantified by OPLP) is significantly less sensitive to stack height changes.

### 3.3. THE OPL PARAMETER AS A TOOL FOR THE EVALUATION OF CANDIDATE STACK LOCATION SITES

The OPL Parameter (quantifying the overall polluting impact of a single stack on a population) may be used for an evaluation of candidate sites for the location of a single stack affecting a set of inhabited areas. In order to demonstrate such a procedure, the relationship between the location of the stack and its Overall Polluting impact was studied for the case examined so far. Several locations in the vicinity of the existing stack were considered, lying in four directions: South-North, East-West, SW-NE, NW-SE. The meteorological conditions are the same for all stack locations and therefore the cumulative concentration field remains unchanged centered at the new location of the stack. The only change that occurs is the way the cumulative concentration field combines with the population density field. The resulting OPLP values (normalized by the OPLP value at the location of the existing stack) are given in Table 1. It is clear from this table that the OPLP is quite sensitive to changes of the stack location. Removal of the unit by  $2.5$ – $2$  km to the South-West leads to a  $36\%$  reduction of OPLP. This strong pollution load reduction can be explained if the equal cumulative concentration contours shown in Figure 7 are examined; when the stack is removed to the SW the higher cumulative concentrations occur out of the city of Thessaloniki, over sparsely inhabited areas or over the bay of Thessaloniki. It is interesting to note that along the South-Eastern relocation path, the OPLP first increases (it reaches a maximum of  $122\%$  when the unit is removed by  $1.5$ – $2$  km) and then decreases. Again the form of the cumulative concentration field explains this behaviour. The initial increase is expected since as the unit is removed to the SE, the locus of higher concentrations (say  $c/c_{\max} \geq 0.8$ , Figure 7) occurs exactly over the city of Thessaloniki. If, however, the unit is further removed to the SE, the locus of low concentrations ( $c/c_{\max} \leq 0.2$ ) surrounding the stack (Figure 7) occurs over the city of Thessaloniki and this leads to the OPLP reduction mentioned before.

In the case considered so far several towns are affected by the operation of the stack and the city of Thessaloniki has by far the larger population among them. It should be expected that locations of the unit which have a smaller impact on the city of Thessaloniki would give lower overall pollution loads. (Such locations are obviously those lying to the SW of the existing unit — Figure 7.) If, however, several towns with comparable populations were affected by the stack, then the

		$\Delta x$ in km											
		← WEST					●	EAST →					
		-2.5	-2.0	-1.5	-1.0	-0.5	0.0	+0.5	+1.0	+1.5	+2.0	+2.5	
$\Delta y$ in km	NORTH →	+2.5	73				84					84	
	+2.0		79				89				93		
	+1.5			84			94			97			
	+1.0				90		96			97			
	+0.5					94	100	106					
	0.0	71	77	81	87	94	100	109	117	123	124	123	
	← SOUTH	-0.5					93	102	107				
		-1.0				81	101		115				
		-1.5			73		93			122			
		-2.0		68			89				121		
		-2.5	64				89					115	

TABLE 1. Values of the Overall Pollution Load Parameter OPLP at several locations of the stack. The OPLP values were normalized by the OPLP value at the location of the existing stack.

$h_0$ (m)	20	30	40	50	60	70	80	90	100	110	120
OPLP %	148	134	121	112	100	92	82	75	67	60	54

TABLE 2. Values of OPLP for several stack heights at the location of the existing stack. The OPLP values were normalized by the OPLP value of the existing stack ( $h_0 = 60$  m).

more favorable stack locations would not be so obvious and the OPLP analysis would be necessary to disclose them among a set of candidate sites.

Tables 1 and 2 summarize the results of this study, in a way convenient for further economic considerations, since they directly quantify alternative ways for an OPLP reduction either by changing the stack location or by increasing the height of the stack.

It is clear from the results presented in this section that the proposed stack performance model could be used for an evaluation of candidate locations of a stack affecting a given set of inhabited areas. Site selection involves an evaluation of pollution impacts using the model on a finite number of candidate sites (i.e. a trial and error procedure). This form of the model is sufficient for the great majority of practical applications where a limited number of candidate sites need to be considered, and a few applications are necessary to disclose the more favorable site. Nonetheless, it should be emphasized that the present form of the model cannot be used for optimization. An interesting extension of the work presented here would be the development of an algorithm optimizing the location of the stack by minimizing the Overall Pollution impact of the stack as this is quantified by the model discussed here.



## 4. EPILOGUE

Effort was made to demonstrate the practical significance of the suggested parameters in preliminary stack design and location studies. It was demonstrated that the stack parameters may be used for a quantitative evaluation of alternative stack designs and locations. The replacement of Euclidian distances from the stack — commonly used in locational models as a sole indication of the stack's impact — by the cumulative concentrations introduced in this paper would greatly increase the validity of the site selection.

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## APPENDIX

Coefficients and exponents of the diffusion parameters ( $\sigma_y, \sigma_z$ ) polynomial approximations

A: *Strongly unstable conditions*

Valid for  $100 < x < 2000$  m

$$\sigma_y = 0.393x^{0.893}$$

$$\sigma_z = -13.008 + 7.296 \cdot 10^{-7} \cdot x^3 - 5.352 \cdot 10^{-4} \cdot x^2 + 0.32842x$$

**B: Unstable conditions**Valid for  $100 < x < 12000$  m

$$\sigma_y = 0.3041x^{0.893}$$

$$\sigma_z = 7.2117 + 3.9309 \cdot 10^{-5} \cdot x^2 + 0.103544x - 0.79592x^{1/2}$$

**C: Weakly unstable conditions**Valid for  $100 < x < 10000$  m

$$\sigma_y = 0.2257x^{0.893}$$

$$\sigma_z = 52.3701 + 0.00222x + 10.9144x^{1/2} - 33.1432x^{1/3}$$

**D: Neutral Conditions**Valid for  $100 < x < 100000$  m

$$\sigma_y = 0.1533x^{0.893}$$

$$\sigma_z = 7.5364 - 0.00232x + 3.08615x^{1/2} - 7.145424x^{1/3}$$

**E: Moderately stable conditions**Valid for  $100 < x < 100000$  m

$$\sigma_y = 0.1047x^{0.893}$$

$$\sigma_z = 93.2747 - 4.1321x^{1/2} + 81.865x^{1/3} - 135.642x^{1/4}$$

**F: Stable conditions**Valid for  $100 < x < 100000$  m

$$\sigma_y = 0.0804x^{0.893}$$

$$\sigma_z = 8.391311 - 1.06948x^{1/2} + 19.4459x^{1/3} - 27.3357x^{1/4}$$