500 Years of Melancholia

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Abstract

In 1514 Albrecht Dürer made his now worldly famous engraving 'Melencolia I'. It enjoyed a general admiration through times, but recently some 'experts' pretended an ellipse on this drawing was constructed erroneously. They based their argumentation on a drawing Dürermade in his book 'Underweysung der Messung' and on the fact Dürer used the word 'egg line' instead of 'ellipse'. The present contribution however invalidates these arguments.

Key words: ellipse, conic section, Dürer

MSC: 01A40, 51N05, 53-03

1. Introduction

'Melencolia I' is the title of a 500 years old etch(see figure 1) by Albrecht Dürer (1471- 1528). This is, for once, a sure fact about an old artwork: not only did the artist sign and date the work in the bottom right corner, the title is clearly shown by a mythical animal flying in the skies, while the date figures on the bottom row of a magic square. Today Dürer is sometimes called the German Leonardo da Vinci (1452 – 1519), an honor he would have enjoyed as he went at length trying to meet the Italian master. His nationality however may be more a matter of discussion, as his father was the goldsmith Ajtósi, who had traveled about a thousand kilometers to emigrate to Nuremberg from Ajtós, a Hungarian village near Gyula in Southeastern Hungary. His name refers to the word 'door' or 'Tür', and 'Thürer' in fact means 'door maker' or 'ajtós' in Hungarian.

The 'I' after Melencholia would not refer to the number 1, but to the expression 'Melencholia Imaginativa'. The melancholic state of the depicted intellectual absorbed in thought is illustrated through several symbols, such as the unused scientific equipment, the magic square, the human skull on the geometric solid, the hourglass showing time running out, the empty scale, the rainbow in the sky, the enigmatically positioned ladder and so on. However, the interpretations of the different features are not the topic here; they have been the subject of numerous publications, the one more speculative than the other. Rather, we are interested in the ellipses drawn in the etching, since some recent publications, moreover spread on Facebook, accuse Dürer of having drawn erroneous curves. Daniel Silver gave his publication in 'American Scientist' the title 'Slicing a Cone for Art and Science'. Another author, Theodor Pavlopoulos, summarized it under the title "Albrecht Dürer's egg line". Chris Impens, an esteemed mathematics professor of the University of Ghent (now retired), shared it on his '@ Valvas' as 'Dürer fails his drawing assignment'. The critics were not afraid to state that "any high school student beats Dürer" or that "he

[Dürer] didn't see what any child can see". So, what about it, do these accusations towards one of the greatest graphical artists make sense?



Figure 1: This drawing would have an erroneously drawn ellipse; do you see it?

2. Dürer's ellipses

On the etching, there is a bell above the magic square, and there would be a problem with the elliptic shape of the mouth of the bell: following the above critics, its right end would wider than its left end. They pretend Dürer drew an *oval*, instead of an *ellipse*. An oval looks more like an egg: etymologically the word is related to the Latin word 'ovum' or 'egg'. It is only symmetric with respect to its long axis, while an ellipse is also symmetric with respect to its short axis. However, when drawing an ellipse with a horizontal great axis over the bell's mouth, the error seems very, very minor. And when the horizontal great axisis shifted over 2.5°, there even seems to be no error at all. Surely a bell can hang in a slightly skew position, why not (see figure 2). Moreover, the other ellipses such asthe contours of the scales and of aflask, are correct (see figure 3).

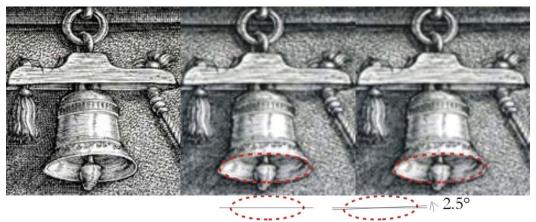


Figure 2: The bell on 'Melencolia I' (left), with an ellipse with horizontal great axis drawn over it (middle) and with a slightly rotated ellipse drawn over it (right).

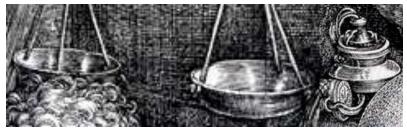


Figure 3: More ellipses on Melencolia.

It is also said when examining other masterpieces of Dürer'elliptic shapes sometimes stand out awkwardly", and the example is given of the central panel of the Dresden altarpiece (about 1498). Here it is stated that "the shape of the crown held obliquely by putti over Virgin's head, which ought to be elliptic, is rather an arbitrary, elongated closed curve". However, not only is it the 'awkwardness' a matter of opinion (and in the case of Dürer's art it is itself a rather awkward expression), but the confusion of the ellipse with the egg shape certainly is not illustrated in this curve (see figure 4). Moreover, when checking out some other ellipses, such as on the engravings Saint Jerome in His Study' (1514) and 'The Last Supper'(1523), the egg shape Dürer would have considered is not confirmed, not at all (see figure 5). And yet Dürer's critics state that "he had the wrong idea of what an ellipse looked like".



Figure 4: The Dresden altarpiece (1498)with an 'awkward' (?) elliptic crown.

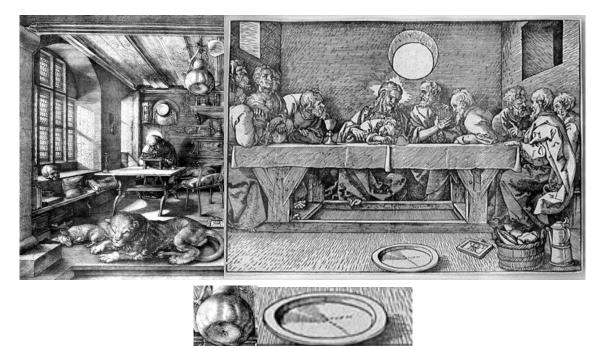


Figure 5: 'Saint Jerome in His Study' (1514) and 'The Last Supper' (1523); note the nicely drawn ellipses on these engravings.

3. The critics' arguments

An important 'argument' in the discussion is a linguistic one about the vocabulary used by Dürer. He translated the word 'ellipse' into German as 'eyer linie' or 'egg line', since he preferred to use the German language rather than Latin. This was not so uncommon in those days: for instance, the Tuscan Leonardo da Vinci wrote in his regional version of Italian, the Flemish mathematician and engineer Simon Stevin in Dutch and Martin Luther translated the complete Bible from Latin. Just as Leonardo and Stevin (and Luther?) often had to invent words, Dürer did so for his language. Perhaps we should keep in mind

mathematical expressions in Latin were creations as well with words borrowed from every day's Latin language, at least for Latin speaking people; or else they were compositions using existing words, in Latin or in Greek. For example, Dürer's word for 'spiral' was a 'Schneckenlinie' or 'snail-line', but 'spiral' itself was a borrowed word from everyday life, because 'spira' meant 'turn' or 'twist'. Soopting for a word like 'egg line' for 'spiral' is as understandable as choosing the word 'spiral' for that mathematical notion.

A funnier example is the word used by Simon Stevin for a sphere, in Dutch. His word was 'kloot', which means, in today's Dutch, nothing else but 'testicle', but in Stevin's days it simply was a synonym for sphere. However, no science critic would ever think about accusing Stevin for confusing the earth's shape with that of a testicle. These language issues and 'confusions' (a very debatable topic) are very well known to mathematicians from Germany, The Netherlands or Flanders, but perhaps uncommon to specialists not familiar with this European context.

Another, and in fact the sole valid argument of the critics, is Dürer's drawing on page 30 of his book 'Underweysung der Messung' or 'Teaching Measurements' (Nürnberg, 1525). It shows a drawing for the plane intersection of a cone (see figure 6). It would become a classic in all courses on Monge's projection method. In this drawing the ellipse is indeed slightly wider where the cone is wider. The critics state that Dürer made 'an elaborate techniqueto accurately draw the ellipse', but anyone who followed descriptive geometry in high school would laugh away theadjective 'elaborate'. The logic of the method is simple, but the execution is more problematic. In fact, anyone who ever made this drawing with compass, ruler and pencil experiencedit is not so easy get a smooth ellipse. One easily gets an egg shape or else an ellipse with sharp vertices on the longer axis.

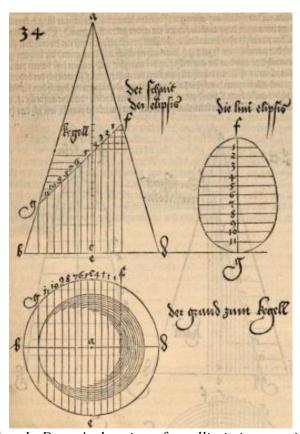


Figure 6: Albrecht Durer's drawing of an elliptic intersection of a cone.

I taught this course for years and remember I sometimes turned an imprecise ellipse on a student's copy into a whale when it resembled too much to an egg or into a goldfish when the vertices were too sharp. This happened very often and seemed unavoidable to many students when making this drawing for the first time. But in Dürer's case I would not have added any comments: knowing the techniques and materials at his disposal, his drawing surely was good enough for any teacher of Monge geometry. It looks slightly like an egg, sure, but it is not exaggeratedly wrong.

Perhaps we are too spoiled today by computer drawings to understand the technical difficulties when making such a drawing by hand. Note a similar remark about a drawing can be made for Dürer's hyperbolic intersection of a cone: the obtained hyperbole does not seem to have asymptotes. The hyperbole curves slightly inwards. Also, the top of the hyperbole is too sharp and the curve is tooplump (see figure 7). This again will be recognized immediately by any teacher who ever taught descriptive geometry, since it was a very common inaccuracy in the pre-computer era. One can wonder why the art critics did not seize this opportunity to comment Dürer's geometric insights once more, while they were so eager to see deformedellipses.

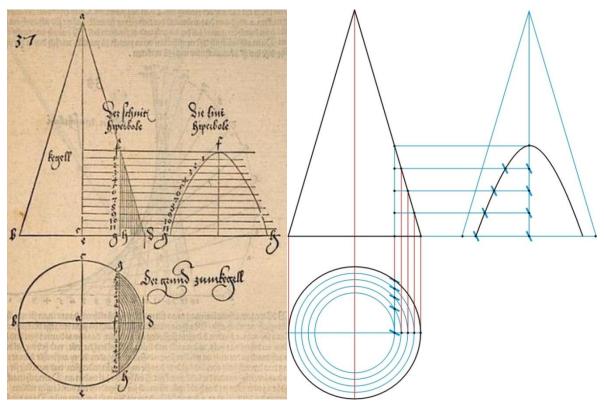


Figure 7: Durer's drawing of a hyperbolic intersection of a cone, and a modern version.

4. The critics' own errors

Thus the critics' arguments are doubtable, but their own contributions are not much better. For instance, they added a 'modern drawing' in which they redrew Dürer's drawing as to explain it, adding a lengthy explanation of the method, instead of simply referring to any high school text book on descriptive geometry. The drawing makes their argumentation worse: two intersection points have an important error of precision (see question marks in figure 8). Their drawing, though clearly made on computer, will turn out into a much less precise ellipse than Dürer's case. The Dürer critics also addsome mathematical facts

about the ellipse, and one can wonder why they do so too, unless it is were for demagogical purposes. The additional mathematical information does not impress anyone with a high school degree.

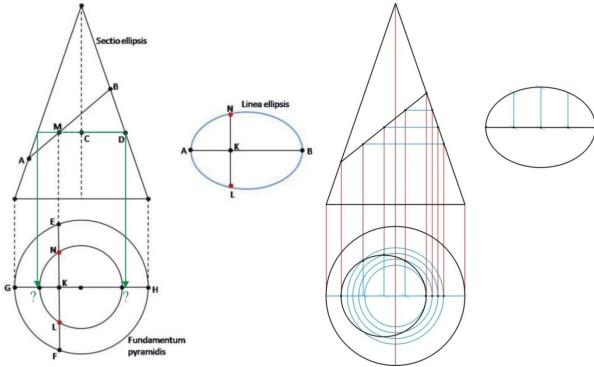


Figure 8: The critics' modern erroneous 'explanation' (the question marks are additions), and a correct modern version.

So, admittedly, Dürer's drawings of the elliptic and hyperbolic intersections of a cone have inaccuracies, but there is no proof at all Dürer thought an elliptic intersection gets broader where the cone gets broader. He never said so; it is a mere very subjective exaggeration. Moreover, even if there would have been an egg shape in bell Dürer drew because he would have thought an ellipse gets wider at some end, the egg shape would have been in the wrong position. The egg should not be wider to the right, but to the bottom. The ellipse corresponds to the intersection of a cone formed by light beams reaching the eye and the plane of the bottom of the bell (see figure 9).

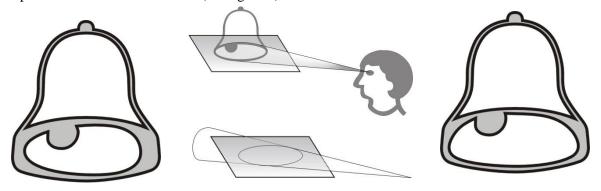


Figure 9: The egg shape of the bell on the 'Melencholia' engraving following the critics' alleged egg theory (left), and as it should have been following their hypothesis (right).

A final consideration is that Dürer's 'Underweysung der Messung' with the egg shape cone intersection appeared in 1525, while the allegedother erroneous ellipses are on drawings or paintings from before the appearance of that book. Thus, Dürer could hardly have based his bell ellipse on 'Melancholia I' (1514) or round crown on the Dresden altarpiece (1498) on the technical construction in the 'Underweysung der Messung'. Of course Dürer could have had in his mind that erroneous egg concept of the ellipse during his entire live, the critics will object, but it surely contradicts their also uttered hypothesis Dürer made his artwork based upon technical geometric drawings he first prepared. And this they 'support' by the well-known literature about the exchange, in one direction or in both, between art and science during the Renaissance. No, the geometric representation of the ellipse came later.

When I first read the text of the American Scientist, I thought it would be an interesting topic for the Dutch popular science magazine EOS for which I revealed many similar errors of other famous artists or artworks (some of them reached Scientific American or international blogs). In the past, I did not hesitate to reveal Rinus Roelofs' discovery of a mistake in a geometric drawing by Leonardo da Vinci, or to disclose an arithmetic error in Norbert Attard's Fibonacci numbers in his 100,000 euro artwork in De Panne (Belgium), or a ridiculous hexagon in the Belgian Atomium landmark and an equally hilarious 'bridge scale model' in the Australian million dollars traveling exhibition enterprise 'Leonardo the Genius'.But accusing Dürer based on the very meager language and inaccuracy arguments given by his critics does not make sense. On the contrary, their negative 'Hineininterpretierung', to use a word in Dürer's language,can only make a reader melancholic.

References

- 1. Daniel Silver, Slicing a Cone for Art and Science, American Scientist, Sep 1, 2012.
- 2. Theodor Pavlopoulos, *Albrecht Dürer's egg line*, blog: "The Peacock's Tail", http://pavlopoulos.wordpress.com/2009/07/27/albrecht-duerers-eyer-lini/, July 27, 2009.
- 3. Dirk Huylebrouck, *A Mistake by Leonardo da Vinci*, EOS magazine 04 2011; reproduced in a shortened version as *Lost in Triangulation: Leonardo da Vinci's Mathematical Slip-Up*in the web edition of Scientific American, 04 2011, http://www.scientificamerican.com/article/davinci-mathematical-slip-up/.
- 4. Chris Impens, *Dürer fails his drawing assignment*, @ Valvas, http://ci47.blogspot.be/2012/09/durer-fails-his-drawing-assignment.html, 08 September 2012.