Celtic Knots and Greatest Common Divisor

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Abstract

According to the curriculum for the fifth grade, divisibility of integers is a topic in the first term. As a preparation for working with rational numbers, pupils learn notions of greatest common divisor and least common multiple of given numbers. An opportunity to give a lecture which stands apart from this demanding material arises after the introduction of the greatest common divisor. The lecture we are talking about was invented by the author and was inspired by mirror curves and Celtic knots.

Key words: divisibility, great common divisor, mirror curves

MSC: 97U30, 97D80, 97D40

In many ways it is a huge jump for primary school children from the lower level classes to upper level classes. Pupils are given a lot of new material, with little time to absorb it and almost no time to review it through a series of exercises. This is accompanied by an increased formalism in writing Mathematics. The "weaker" students are lagging behind and the best ones are losing interest in lectures. It is a good idea to wake them up from time to time by giving new and unusual content and/or form in the lectures. The best idea is to make an unexpected correlation with other subjects.

According to the curriculum for the fifth grade, divisibility of integers is a topic in the first term and takes up 18 lectures. After getting acquainted with the basic notions and properties of divisibility, pupils learn criteria of divisibility, prime and composite numbers and learn to factor numbers into primes. After that, as a preparation for working with rational numbers, they learn notions of greatest common divisor and least common multiple of given numbers.

An ideal opportunity to give a lecture which stands apart from this demanding material arises after the introduction of the greatest common divisor. The lecture we are talking about was given in primary school "Lazar Savatic" in Zemun, 24.4.2014. by the author and was inspired by a lecture by Ljiljana Radovic, Ph.D. given at the European Summer School for Visual Mathematics and Education in Eger, Hungary in July 2013. This summer school was a part of a Tempus project "Visuality and Mathematics".

In the introductory part of the lecture, accompanied by music of a contemporary Serbian band "Orthodox Celts", students were given simple questions, in an informal manner, about the topics covered up to that point in the area of prime divisors, common divisors with the aim of reviewing the covered material. Then students were asked question related to the history of Celts, especially those relating to the cultural heritage of Celts in Serbia and to Celtic settlements in the Balkans. An excitement and enthusiasm of the pupils were marked.

The basic idea was to give, after the initial warming up a more detailed story on the Celtic tribes in Europe, expanding on the material covered in History classes and relating it to educational broadcasts

on the Celts. The second part begins with a gentle return to Mathematics, involving prime and composite numbers, twin prime numbers, relatively prime numbers etc. The main part of the lecture is the fusion of these two threads: we talk about Celtic knots and great common divisor (GCD) and pupils use nonstandard tools, at least for a Mathematics lecture, as crayons and drawings pads.

All the pupils were quite enthusiastic during the lecture. They stayed focused and interested, there were no problems regarding discipline. The slower pupils easily adopted the pattern of Celtic knots and enjoyed drawing them as much as the best ones. The main goal of the lecture was fully realized: the pupils got a review of the covered material mixed with a fascinating story of an ancient culture of Celts and were able to use the basic notion of GCD in a concrete situation. They were given a chance to experiment on their own.

We give bellow a detailed preparation for this lecture, given with a lecture timetable.

Lecture Dynamics

Introductory part

Listening to the Orthodox Celts band music

Part I

The Celts are Indo European people which during Stone Age inhabited region spanning from present day France to Czech Republic. They lived in the Balkans as well. They moved, under pressure by Germans, to the west and to the South of Italy, Spain and Brittany. Celtic culture is present today in Ireland, Scotland and Wales. Today they are considered to be "the key nation of Europe".
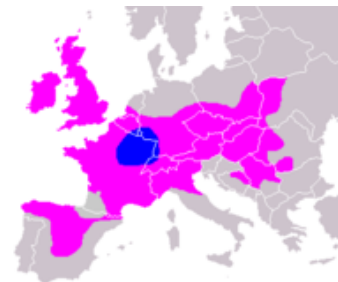
The Celts were very warlike and brave people; however, they were not well organized. It was difficult for them to follow regulations and they fought rather individually. They lived in village, kinship communities. They often engaged in warfare among themselves and so they could not unite. There was no national feeling of being Celt.

Art and culture were highly developed. They learned, from the Greeks and Romans, to make ceramic and bronze pots. They ornamented them by abstract and geometric motifs. Also, they made fine arms, built elaborate fortresses, coined copper coins…

Celtic knots are one the most famous legacies of the Celtic civilizations. They are ubiquitous today.
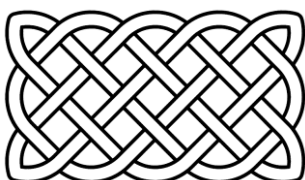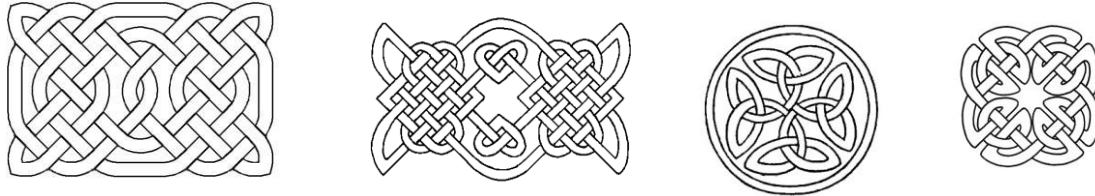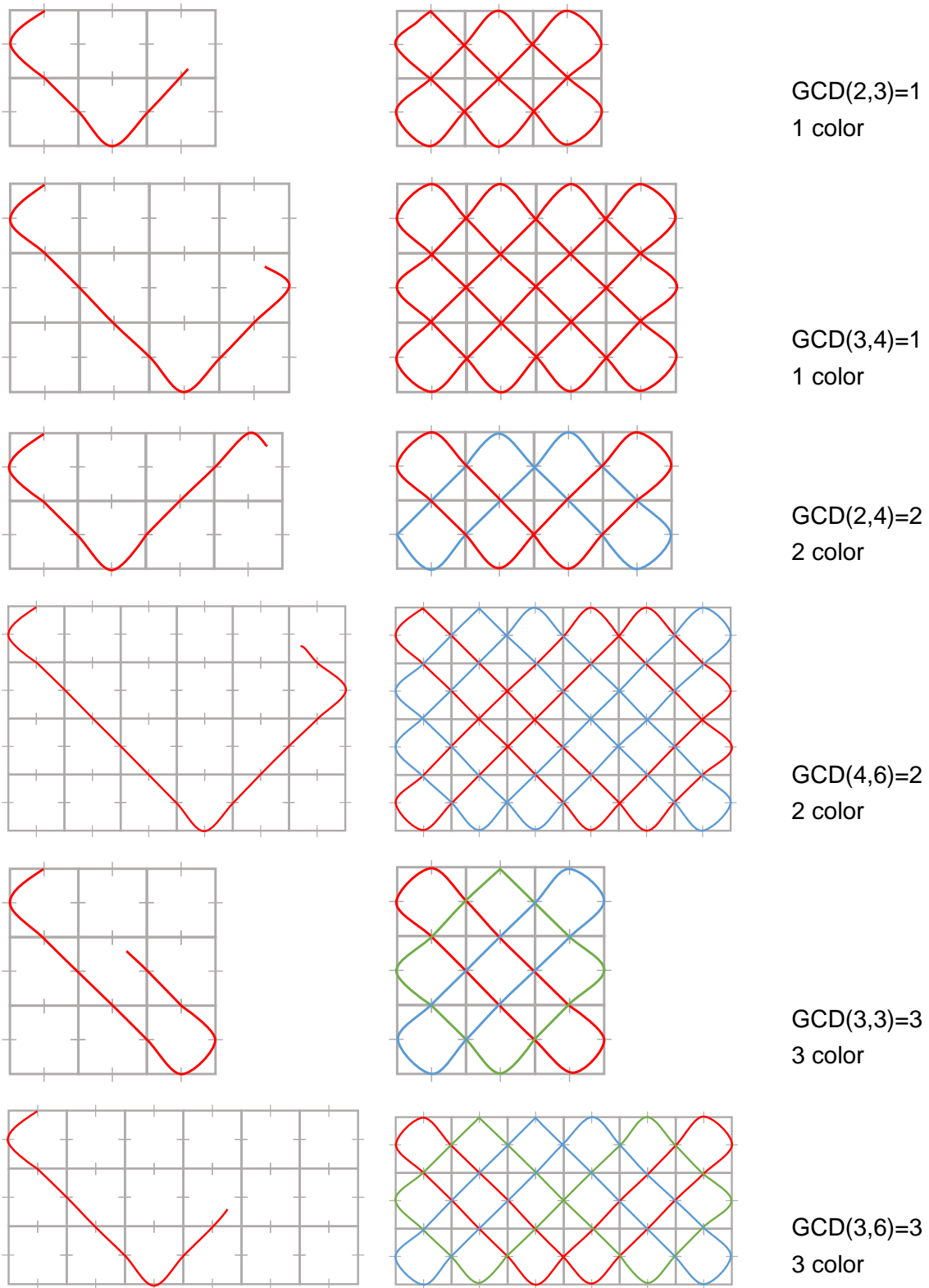
Figure 1: Examples of Celtic knots

Part II

We start with numbers, natural numbers, and quotients asking different types of quotients:

- Even and odd numbers;

- Prime numbers and composite numbers, twin primes, relatively prime numbers, common divisors, the greatest common divisor.

- Which kinds of numbers are 7, 27, 41, and 91? If they are composite, explain why. If they are prime, explain why. At what point we terminate the process of checking divisibility?

- Consider the number 2475. Factor this number into prime factors. How many divisors are there? Write down some of them. Is there a number whose product of digits is equal to 2475? Why?

- What do we can say about numbers 4 and 9? They are not prime, but they are coprime. How do we write down this fact?

- And what about 52 and 117? What do we do if we are not sure that GCD (52, 117) = 1? Is it going to be simpler if we factor each of the given numbers into prime factors? 52=4·4·13, 117=3·3·13. What is the conclusion?

The main body of the lecture

We present a method of drawing Celtic knots using Mirror curves. Mirror curves are trajectories of (imaginary) rays of light emitted from edge mid-points in the regular square grid RG [a, b]. , with one-sided mirrors on their external sides and eventually, two-sided mirrors placed between cells, coinciding with internal edges or perpendicular to them at their mid-points. After a series of reflections, a ray of light creates a closed path − a component of the mirror curve. Here we are working with RG [a, b] with no internal mirrors. The ray of light reflects and is emitted from the mid-point of an edge, reflects in the mid-points of the external edges of bordering squares, and travels through the grid. If the numbers a and b are relatively prime, i.e., if GCD(a,b)=1 we obtain a single curve which uniformly covers RG[a,b]. Otherwise, a multi-component mirror curve is obtained, where its number of components is c = GCD(a,b) .For each component we can use different color.

GCD(2,3)=1
1 color

GCD(3,4)=1
1 color

GCD(2,4)=2
2 color

GCD(4,6)=2
2 color

GCD(3,3)=3
3 color

GCD(3,6)=3
3 color

Figure 2: Diferent rectangular grids with Celtic knots designed as Mirror curves

Optional homework

- Write an essay, up to one page long including illustrations, on Merlin the wizard and king Arthur, characters of one the most famous Celtic legends.

- Make a Celtic knot from wire, ribbons, shoestrings…

Suggested readings:

- Book "The World of the Celts", Arnulf Krause, (publisher Laguna in Serbian, see [3]) – history and myth on a legendary people;

- Book/movie "La solitudine dei numeri primi", Paolo Giordano (publisher Dereta in Serbian, see [2]).

Homework problem:

- Visit the site on making Celtic knots *gwydir.demon.co.uk/jo/knots* **Making Celtic Knots,** with background music **Orthodox Celts**, a Belgrade rock band that performs traditional Irish music combined with elements of rock and roll.

Notes and comments:

Pupils should be given printed homework assignments.

The model lecture was given in the class 5_3 on 24. April 2014.

Idea for realization of this lecture originated from the Summer School of Mathematics in Eger, July 2013, organized as a part of Tempus project: "Visualization of Mathematics".
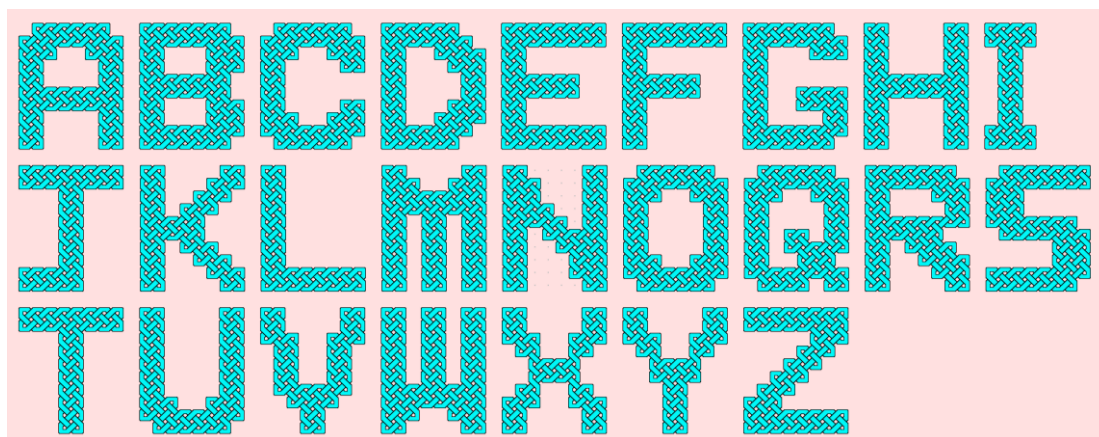
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