Adventures of one triangle

Biljana Stojičić, physics teacher
Zemunska gimnazija, Zemun, Gradski park 1
Belgrade, Serbia
E-mail: bbs100@open.telekom.rs

Zorica Marinković, math teacher
Zemunska gimnazija, Zemun, Gradski park 1
Belgrade, Serbia
E-mail: zorica.f.m@gmail.com

Abstract

Our goal was to join isometric transformations with real life and playful activities. Properties of shape pattern schemas are formative idea for representation of isometric transformations. We found very interesting to use a kaleidoscope in that purpose because of flat mirror properties. Using kaleidoscope and kaleidoscope made images students will be able to demonstrate an understanding of key concepts in that field of school mathematics and discuss it on different levels.

Key words: isometric transformations, kaleidoscope, learning

MSC: 97D40

Methodological basis

An axial symmetry maps each point A in another point A_1 in the same plane, so that the axis of symmetry is a perpendicular bisector of the segment AA_1. The axial symmetry is indirect isometric transformation - it preserves the distances between points (distances between the points of the original figure are equal to the distances between the points of the symmetrical figure) but it changes the orientation. A commonly used example of axial symmetry is described as the mirror image of an object. One way to explain what is axial symmetry and what are its characteristics is drawing in the coordinate system. In this way, the attention is shifted away from the use of ruler and compass in geometric constructions to the characteristics of the transformation. With the help of a mirror placed on a coordinate axis, which is also the axis of symmetry, perpendicular to the plane of the coordinate system, students can verify the solution. Introducing two or more mirrors we can show the composition of isometric transformations in an interesting and very easy way.

Activities for students

Axial symmetry and one mirror

Drawing the image of the triangle ABC under axial symmetry of the coordinate axis *y* we get triangle $A_1B_1C_1$.

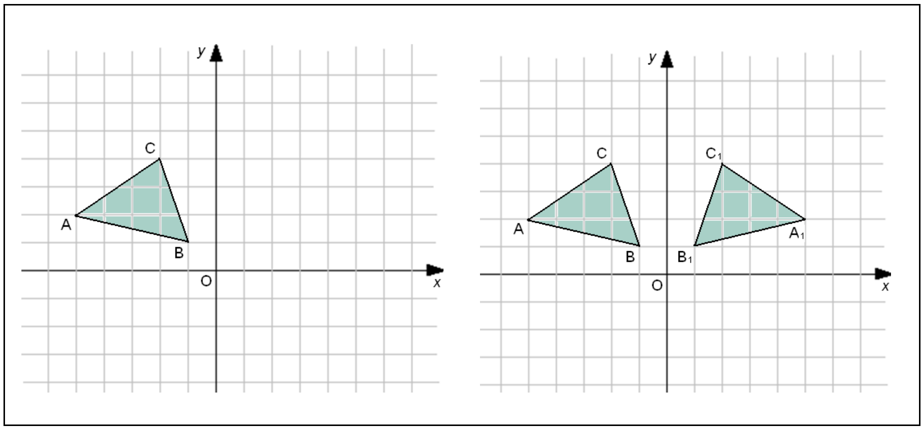

Figure 1: *Axial symmetry*

How we can use the mirror to verify the solution?

On the photo below is the expected result where it is clearly visible axial symmetry as a mirror reflection.

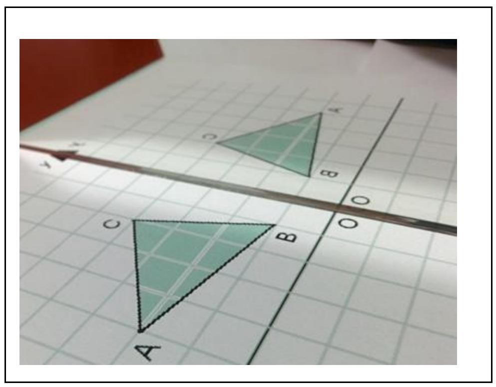

Figure 2: *Mirror reflection*

Axial symmetry and two mirrors

What will happen if we put two mirrors at an angle of 90° along the coordinate axes, orthogonal on coordinate plane, with meeting point of mirror lines in origin?

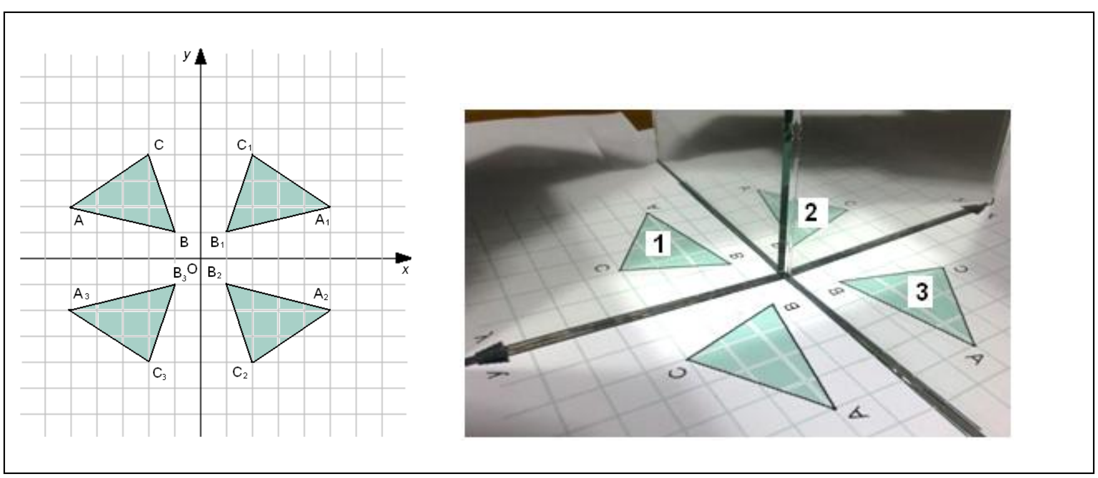

Figure 3: *Mirror reflection with two orthogonal mirrors*

In this situation, we can comment on axial symmetry but on central symmetry and rotation as well. The composition of two axial symmetries with the perpendicular axes is a central symmetry with regard to the point where two axes of symmetry meet. For example, given triangle ABC is symmetric under the central symmetry with the triangle 2. This is a good opportunity to note that the composition of two indirect transformations is direct transformation.

If the angle between axes is less than 90°, the composition of corresponding axial symmetries is a rotation with an angle of rotation equal to doubled angle between the axes. The smaller angle between mirrors means more images. On the figure 4 are shown two almost parallel mirrors and almost endless set of triangles. With two parallel mirrors we can show translation.

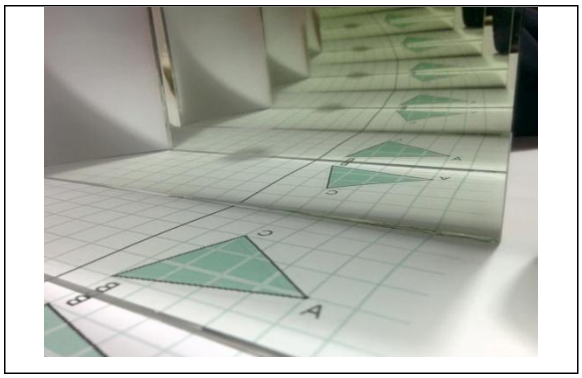

Figure 4: *Mirror reflection with two mirrors*

Axial symmetry and kaleidoscope

Even it is considered as a child’s toy, kaleidoscope is also a simple optical device. A kaleidoscope operates on the principle of multiple reflections, where several mirrors are placed at an angle to one another. Usually there are three rectangular mirrors set at 60° to each other so that they form an equilateral triangle. The objects in the box can be fragments of rock or minerals, gemstones, beads, glass or any other small things. We made a simple kaleidoscope with three mirrors bonded together by adhesive tape. It should be a real adventure for our triangle ABC and playful activity for students.

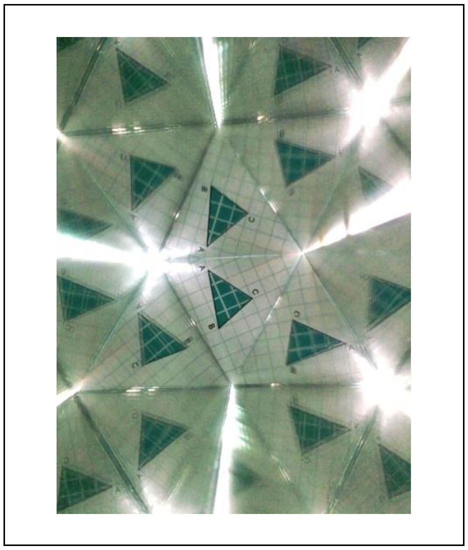

Figure 5: *Triangle in kaleidoscope*

Useful and fun exercise for isometric transformation may be colouring pictures such as it is started on the Figure 6. It is needed to imagine that the sides of the triangle ABC are actually mirror lines. On this image we can recognize all kinds of isometric transformations in the plane.

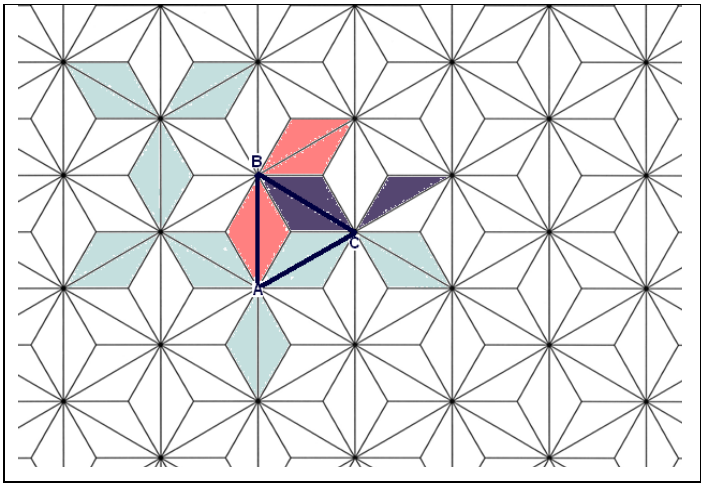

Figure 6: *Pattern for colouring* 1

On the figure 6 the smallest possible equilateral triangle is shown and coloured with three colours. Depending on the task and imagination of the author, it is possible to set the mirror (reflection) axes on different positions, as it is on the figure 7.

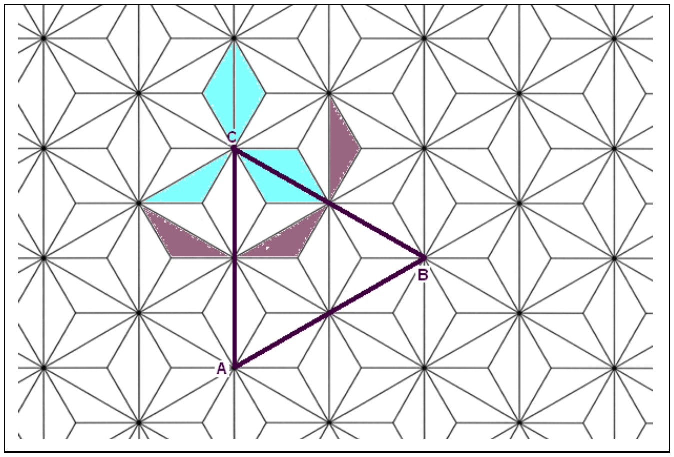

Figure 7: *Pattern for colouring* 2

Summary

In this lesson we were focused on how to integrate properties of isometric transformations, especially axis symmetry, and play-centred learning into mathematics teaching in first grade high school. Similar activity can be used as an idea how to integrate experience-centred mathematics education into art teaching programs. Triangle is the simplest geometrical figure where we can see what is going on with orientation, which geometrical transformation is direct and which is indirect and what happens after their composition. Kaleidoscope is excellent educational tool for developing a learner's perception, aesthetic sensibility, mathematical, logical, combinatorial, and spatial abilities and thinking skills through interdisciplinary approach.

References

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