Sierpinski triangle and pyramid

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Abstract

Sierpinski pyramid is one of the most beautiful structures that allow us to hide mathematics in game and art creation by letting students enjoy in exploring fractals, its patterns and properties. That way they become more interested in rich mathematical explorations and connections. Although the primary school mathematics curriculum is not anticipated with fractals as teaching lesson, it can be very usable in many different domain of elementary school mathematical teaching, such as fractions, area, similarity, and so on. In eight grade, it plays a big role in developing students knowledge of tetrahedron and pyramid properties.

Keywords: fractals, models, Sierpinski triangle, Sierpinski pyramid

MSC: 97U30 , 97A80

1. Introduction

Some of the main purposes of mathematics teaching are the formation of positive attitudes towards mathematics, development of critical thinking and spirit of inquiry, mastering the cognitive processes necessary for an understanding of mathematical concepts, acquisition of mathematical knowledge and skills necessary for modeling and solving real problems and development of cooperative learning and mathematical communication skills. Teachers should enable students to learn how to use mathematical skills in daily life and ensure that children will not be discouraged by mathematics. But in reality it's not that easy to achieve established aims. Overall achievement in mathematics is considered quite low and students have difficulties in comprehending mathematical concepts, and with the time, they become turned off from mathematics. Therefore, in order to entice students to embrace math and inspire them to learn and discover its beauty, teachers should use visualization, creativity and modeling as suitable tools for students' learning process.

Teachers of mathematics in elementary school, considering the years of their students, need to build a connection between mathematics and artistic projects, and to provide children with a chance to participate in such activities so they can accomplish much more in mathematics. Opportunity to be a participant of both International Summer Schools for Visual Mathematics and Education (Summer University and Experience Workshop) organized in the frame of the TEMPUS IV Project "Visuality & Mathematics: Experiential Education of Mathematics through Visual Arts, Sciences and Playful Activities" (2013. and 2014.), become a springboard of my searching for possibilities to integrate art, science or play-centered

learning into mathematics teaching programs, to investigate its pedagogical results, in order to develop or improve my students knowledge and make it more meaningful and accessible.

In 8th grade of primary education students are learning about pyramids, investigating it both geometrically and algebraically. Among computational problems facing later, at the very beginning of studying pyramids, many students happened to struggle with drawing and constructing representations of three-dimensional geometric objects in two-dimensional plane, and also with its net representation. Although in most cases teachers are using a variety of tools and showing models to class, still it seems to be not enough to build a strong and appropriate understanding.

2. Sierpinski gasket in Classroom

The conversation with students about Swedish packaging company "Tetra-Pak", the root of its name and packaging invented in 1952., the one made of tetrahedron-shaped cartons, raised their interest in dealing with geometry and mathematics. Seeing that students are very interested in something that they could talk about with their parents, also seen in an exhibition dedicated to the time of their parents youth, I realized that the best way to motivate them is to find a model they could work on.

Sierpinski pyramid and its tetrahedron structure was a good way to integrate art and play into studying mathematics. It is found that fractals generally shows complex geometrical character and shape, set up on simple mathematical foundation.

Fractals are not part of curriculum at the elementary level of math teaching. They are well known for their nice computer graphics pictures, but fractals are best known as spirals and examples from nature like fern leaf, broccoli, river networks, snowflakes, etc. Therefore, fractals can be easily connected with other school subjects. All this makes fractals very interesting for investigation. I organized my students in groups and gave them a task to research topic connected with fractals. First group of students had to discover what fractions are, where we can find them in a world around as and to show some interesting examples. The next two groups had to investigate Sierpinski triangle and Sierpinski pyramid. Fourth group had to find out about Waclaw Sierpinski, a Polish mathematician who created Sierpinski triangle, also known as Sierpinski gasket, in 1915. All groups presented their researches to the other students.

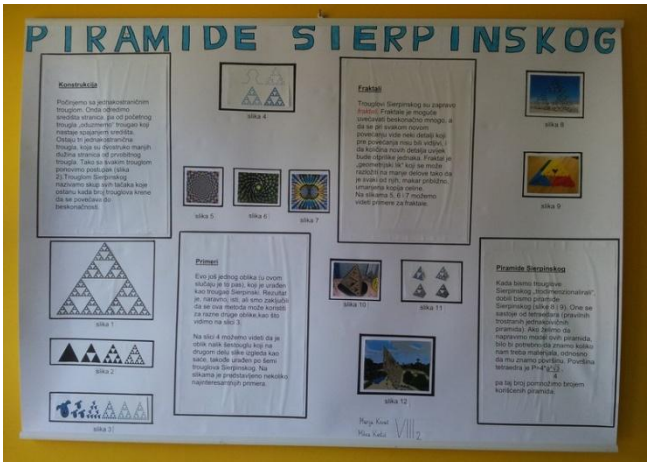


Figure 1: *Project preparation*

What the fractals are? There is not exact mathematical definition, but we could say that it is a geometric shape in which an identical pattern repeats itself. It means that mathematical fractals are iterative; they are formed by applying procedure over and over again. Its other geometric characterization is self-similarity:

the shape is made of smaller copies of itself. Model of Sierpinski triangle is prototypical representative of the fractals in two dimensions. It can be constructed from an equilateral triangle in two different techniques, in both cases extremely simple to make. The first one is to repeatedly subtract smaller and smaller triangles. That way the original triangle is divided into four equal triangles and the middle one is removed. Each of the remaining is divided the same way, and this process is carried on indefinitely. The other method also starts with an equilateral triangle but repeatedly adds new triangles to the construction. In each step three identical constructions are grouped together into a larger one. The second method is maybe more accurate to connect with making sculptures, but the first one is showing fractals beauty from both mathematical and visible point of view.
My students have discovered it by drawing iterations on white board, all the way till continuous patterns could be noticeable. At the end they had a task to count, noticing that a number of triangles used as patterns increase by a factor of 3 each step. That way they relate geometric ideas to numbers.

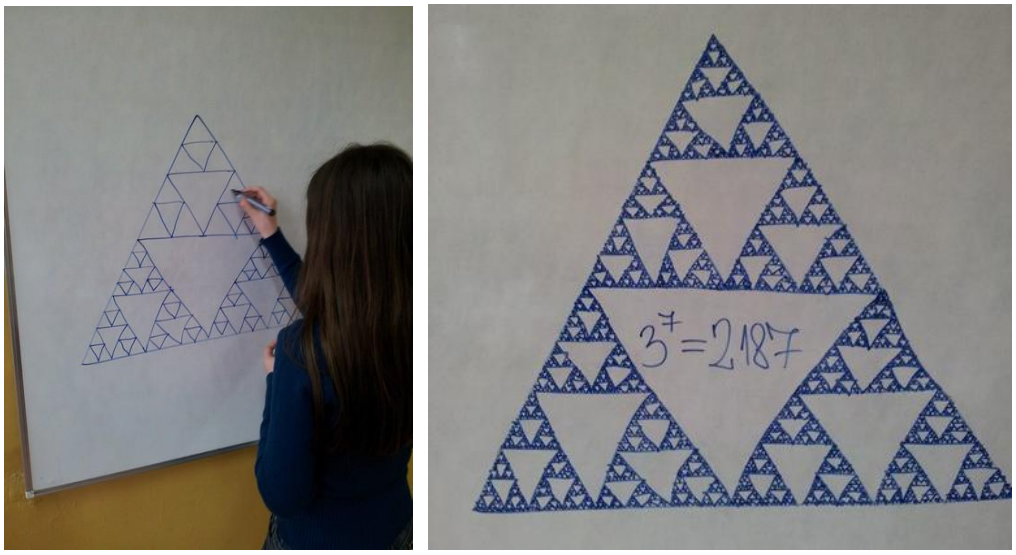


Figure 2: *Exploration of Sierpinski triangle on the board*

Sierpinski triangle is very intricate, and yet so simple to understand. Knowing how to create repeating and growing patterns, understand relations, and functions doesn't necessary mean that students won't make mistakes. Noticing and correcting them is also important part of learning process.

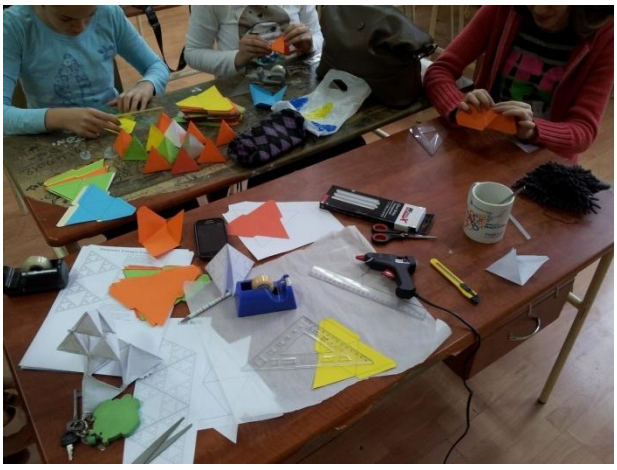
Figure 3: *Sierpinski triangle project*

Objectives

Depending on tasks we put in front of our students we can expect them to be able to visualize, build, and draw geometric objects, apply appropriate tools and formulas to determine measurements, develop precision in measuring lines and in finding or constructing midpoints, build, understand and use formulas for areas (any shaded triangle at the n-th stage) and for that need construct, analyze and interpret tables, perform or describe line symmetry, develop mathematical arguments about all geometric relationships, solve problems involving fractions, ratios and proportions, discover fraction that represent part of triangle and calculate the fraction of area which is (not) cut out, for any step, etc.

3. Sierpinski pyramid

As part of lecture preparation, a teacher gives each group or pair of student material for work and explains what their assignments are. By using models students cut tetrahedron nets out of paper. Then, they have to fold them so they could make pyramids - tetrahedrons. Students have an assignment (already knowing what we are going to make) to count how many tetrahedrons they need to make a structure, for each level, so that become 4 iteration Sierpinski pyramid.
They counted that it would be needed 256 basic tetrahedrons. After putting 3 of them at a bottom and fourth above, so that its base vertices touch others 3 tops, students have made 64 new structures. Using founded pattern and gluing smaller pieces together, students got 16, later 4 and at the end one beautiful structure - work of art and also 3D math model. All the time, teacher is supposed to lead and monitor the progress of building.



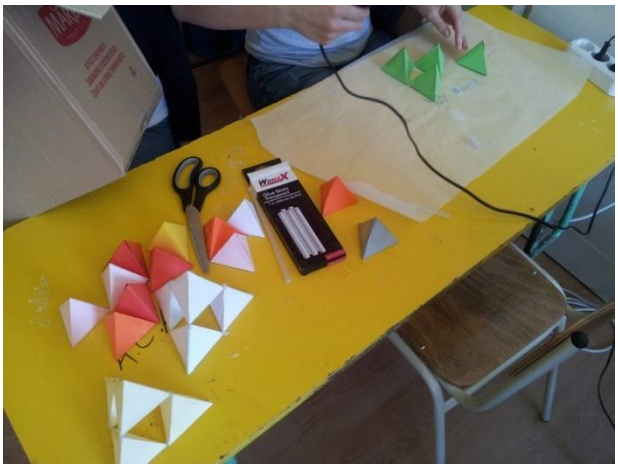


Figure 4: *Sierpinski pyramid project*

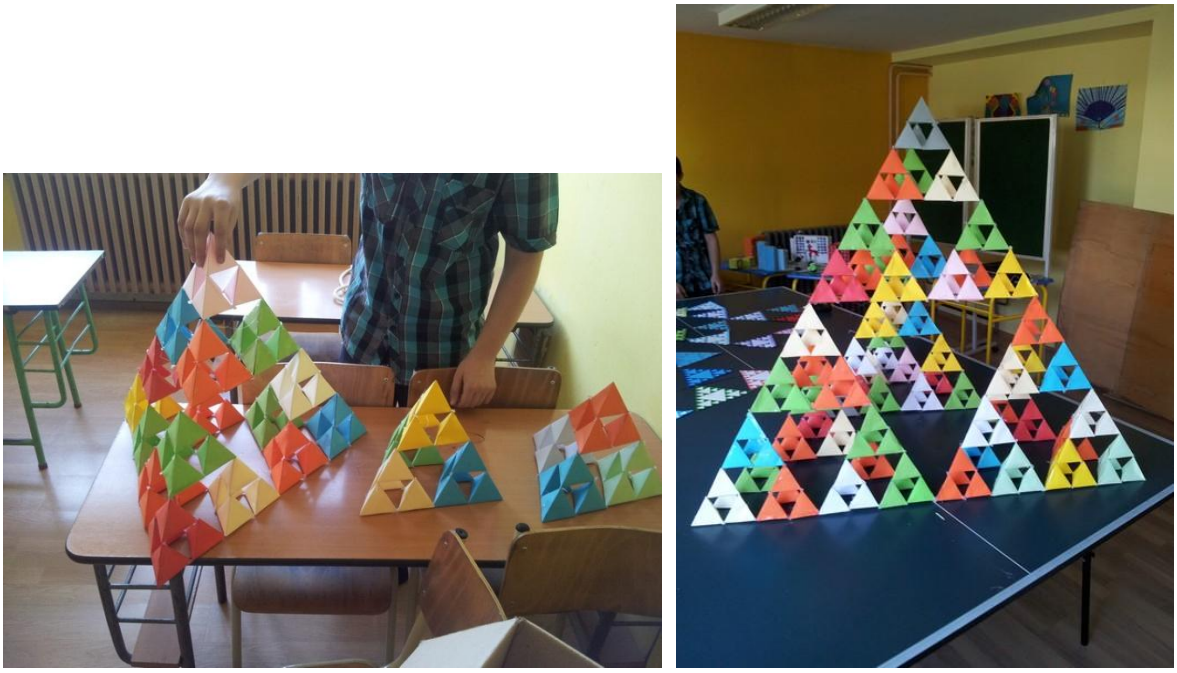


Figure 5: *Sierpinski pyramid project*

Objectives

In order to explore the main fractal concepts students could do the investigation on their own. By building this structure students become an active participants in their learning, motivated for discovering properties of tetrahedron and pyramid in general.
One of the main goals is teaching students to use mathematical models to represent and understand quantitative relationships. It helps them to identify and describe the attributes of pyramid, and specially tetrahedron, based on various attributes (e.g. faces, edges, and corners), to identify and analyze pyramid nets, find it and calculate by formula for height, area and volume of pyramids at any given step - different iterations. Also they can count the area of paper used to make structure, and area of inside “holes”.

When students start playing with pyramid they will be able to discover its different shadows - plane figures (triangle, square, rectangle) and recall some of the math concepts they learned before, such as

writing plane formulas and counting by it. Using model makes easier the process of discovering pyramid symmetries, scale factor, similarities, fractions, ratios and proportion. Also, they will be able to use it for coding, to represent and understand quantitative relationships, to calculate number of tetrahedrons, used in each iteration of the pyramid (they could figure out the algorithm 4^a, for a-th step).

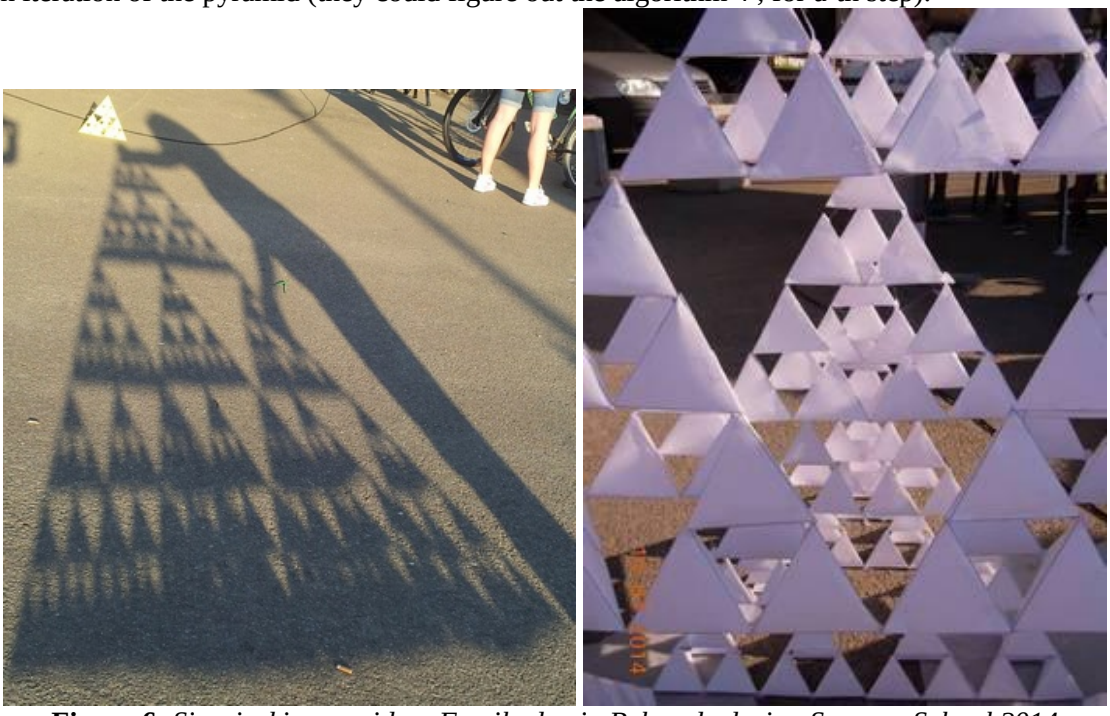


Figure 6: *Sierpinski pyramid on Family day in Belgrade during Summer School 2014*

In many cases students who have trouble with math don't want to try to learn it, mostly because they just don't understand it or have no motivation to even try. A lot of students take an attitude that math is too hard, they don't need it in life, or they don't even care. But in reality, students actually don't know how to learn something. If not being able to visualize some representation or rule, they loose interest in dealing with it. This challenge gives students an opportunity to beat those problems. Even more, they can enjoy in beauty of their mathematical, playful and artistic deed, that is, as they say, „even more beautiful in real life then on a photo or video presentation".

Figure 7: *Sierpinski pyramid-result of the project*

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