Axial reflection and plane mirror reflection in analytic geometry

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Abstract

The idea for this work came from experience acquired through International Summer Schools on Visual Mathematics (in Eger 2013. and in Belgrade 2014.) in the frame of the TEMPUS IV. Our main idea was to join thematics in three school subjects: mathematics, physics and art, and to find corresponding mathematical models. In the field of isometric transformations and analytic geometry, math is connected with physics and art through plane mirrors, mirror curves and lot of other interesting phenomena and examples in everyday situations. We presented this work on a school lesson through practical work of our students.

Key words: axial reflection, plane mirrors, mirror curves

MSC: 97M10

1. Methodological basis

We compared the curricula of three school subjects (mathematics, physics and art) as well as corresponding standards for ours high schools and we have found many possibilities to work interdisciplinary. We have created the lesson "Axial reflection and plane mirror reflection in analytic geometry". Through this lesson students can develop mathematical and science competencies (PISA) and interdisciplinary approach using examples from art and everyday life. Math competencies are mathematical thinking skill, mathematical argumentation skill (connections and integration for problem solving), modelling skill (translating "reality" into math model), representation skill, symbolic, formal and technical skill, interdisciplinarity (translating from natural language to symbolic/formal language), communication skill (this includes expressing oneself, in a variety of ways, on matters with a mathematical content, in oral as well as in written form, and understanding others' written or oral statements about such matters), aids and tools skill (knowing about, being able to make use of aids and tools that may assist mathematical activity).
Science competencies are explaining phenomena scientifically, evaluating and designing scientific enquiry and interpreting data and evidence scientifically.
In this lesson, we achieved all competencies.
We planed group work to. Every group got an activity to solve. At the end, every group presented their solution.

2. Activities for students

Plane mirror in the Cartesian coordinate system

Working instructions: When a ray of light is reflected on a flat mirror, the reflection obey the following laws of reflection. The incident ray, the normal line and the reflected ray belong to the same plan. The angle between the incident ray and the normal, called the angle of incidence, equals the angle formed by the reflected ray and the normal.

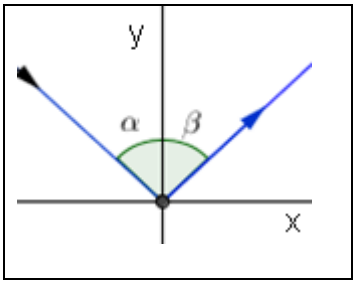

Figure 1: *The law of reflection*

Group 1:

Task 1 Plane mirror is on the position $y=b$. How to set a laser beam so that, after mirror reflection, hits a given point A? Calculate the angle φ. ($b > 0$, $0 < m < b$, $n > 0$)

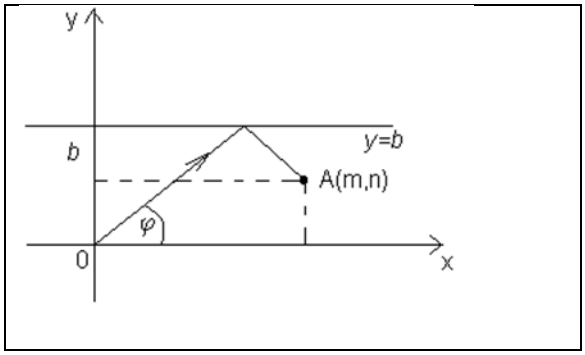

Figure2: *Task 1*

Answer: The angle of reflection is equal to the angle of incidence. Based on the properties of axial symmetry, one can obtain that $tg\ \varphi = (2b-n)/m$.

Task 2 The laser beam parallel to the y-axis hits the mirror at point A. Calculate the angle x at which is needed to tilt the mirror so that the laser beam hits the origin after reflection. Find the equation of straight line that represents a mirror in that position.

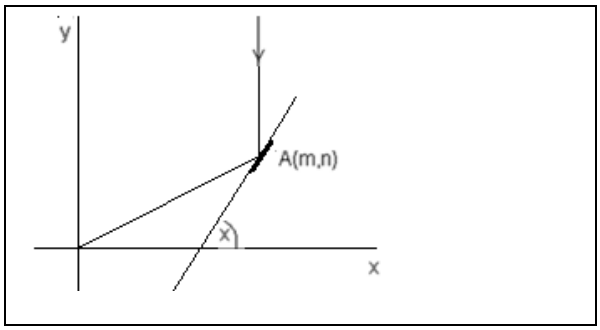

Figure 2: *Task 2*

Answer: Triangle ΔAOV is isosceles (OA=OB). Based on the Pythagorean theorem :
$OB = OA = \sqrt{m^2 + n^2}$ and $AC = \sqrt{m^2 + n^2}$ +n.

So, $tgx = \frac{\sqrt{m^2+n^2}+n}{m}$.

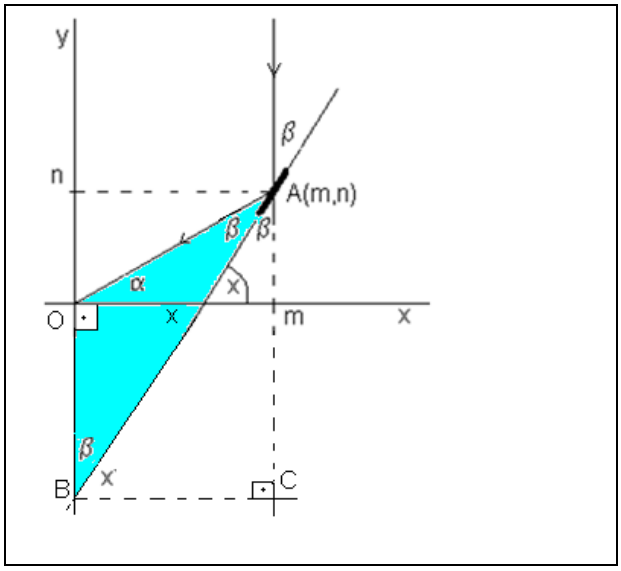

Figure 3 : *Illustration for task 2 solution*

It is necessary that the students use a mathematical model for the real life situation: laser beam and flat mirror are represented as a straight lines in coordinate system. Through solving this task student will develop modeling skills, interdisciplinarity, mathematical thinking skills and scientific enquiry.

Group 2:

Mirror curves in the Cartesian coordinate system in a plane (1)
Working instructions: A mirror curve is a closed polygonal line reflected at the sides of a rectangle and possibly at one or more double-sided mirrors placed horizontally or vertically, midway, between neighbouring grid points belonging to a rectangular grid. The polygonal line makes angles of 45° with the sides of the rectangle and internal mirror. It can be notice that an incident ray and a reflected ray belong to the same plane. Square grid with mirrors is given in a coordinate plane.

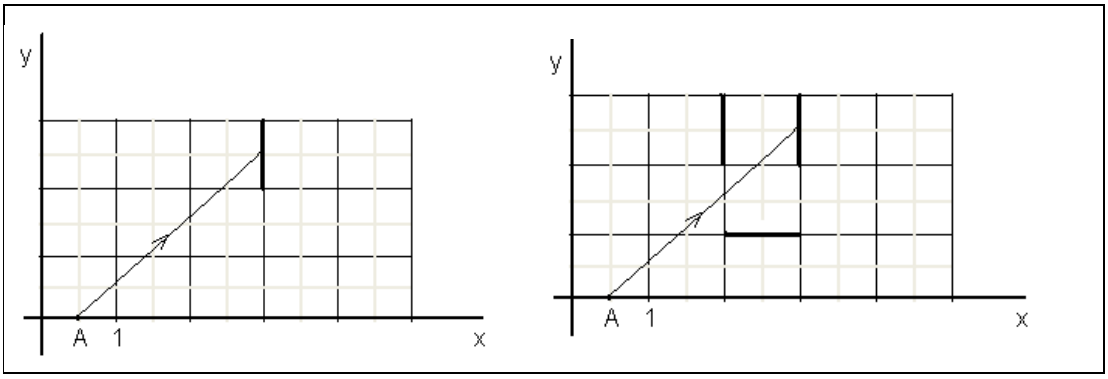

Figure 4: *Square grid*

Task 3 Continue drawing as it is started. Be aware that the angle of reflection is equal to the angle of incidence. In this case it is 45°. Wt is the result?
Answer: Monolinear curve.

Task 4 Calculate the sum of all gradients of straight lines whose parts form a mirror curve.
Answer: All the gradients are 1 or -1. Becouse of monolinearity sum is 0.

Group 3:

Mirror curves in the Cartesian coordinate system in a plane (2)
Working instructions: A mirror curve is a closed polygonal line reflected at the sides of a rectangle and possibly at one or more double-sided mirrors placed horizontally or vertically, midway, between neighbouring grid points belonging to a rectangular grid. The polygonal line makes angles of 45° with the sides of the rectangle and internal mirror. It can be notice that an incident ray and a reflected ray belong to the same plane. Square grid with mirrors is given in a coordinate plane.

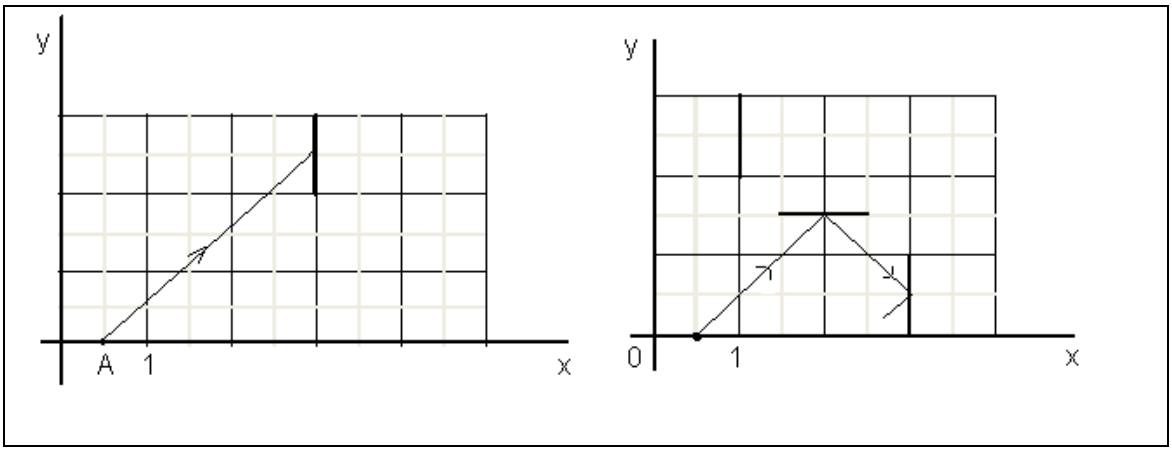

Figure 5: *Square grid*

Group 4:

Task 5 Continue drawing as started. Be aware that the angle of reflection is equal to the angle of incidence. In this case it is 45°.
Answer: Monolinear curve.

Task.6 Calculate the length of the resulting mirror curve.
Answer: Formula for distance between two points $A(x_A,y_A)$ and $B(x_B,y_B)$ is
$d = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

Students have opportunity to work on mathematical problem using interdisciplinary approach. Knowledge from the sphere of arts can enrich the mathematics and vice versa.

Group 5:

Number of images
Working instructions: Look at the picture below. Two plane mirrors are set at acute angle. To see the dependence between measure of the angle and the number of images you can construct some cases using a compass and ruler. Recall the characteristics of axial symmetry.

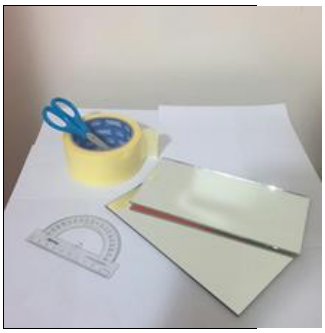

Figure 6: *The materials*

Task 6 Connect two mirrors using adhesive tape. Does the number of images depends on the angle between the mirrors? Explain.
Answer: Number of images: $n=[360^0/\alpha]-1$

ANGLE (α)	Number of mirrors observed	Number of images observed
10^0		
20^0		
60^0		
90^0		
120^0		
150^0		
180^0		

Figure 7: *Table for obtained results*

Task 7 Two mirrors (*a* and *b*) are placed at angle of 45°. Point A is located between two mirrors. How many images have point A? (*y*-axis is the image of *x*-axis in respect to the mirror *a*, point D is image of point C in respect to the mirror *a* , and so on.) Explain the connection between mirror reflection and axial symmetry? Calculate the coordinates of all the images.

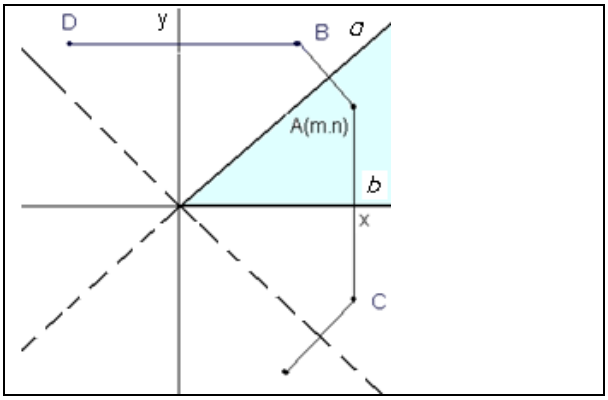

Figure 8: *Illustration for task 7*

Answer: Look at the figure 9.

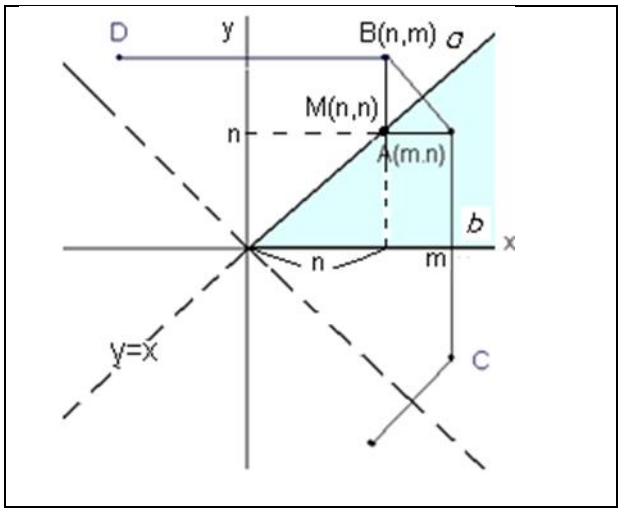

Figure 9: *Illustration for answer 7*

Similar situations in every day life can be found in the amusing parks, movies, fascinating toys like kaleidoscope and instruments like periscope to the dressing rooms. Working on this problem students will develop their tools skills (knowing about, being able to make use of aids and tools that may assist mathematical activity) and science competencies (explaining phenomena scientifically, evaluating and designing scientific enquiry and interpreting data and evidence scientifically).

Figure 10: *Illustration for answer 6*

Group 6:

How to set the plane mirror

Working instructions: What is the middle line of the triangle?

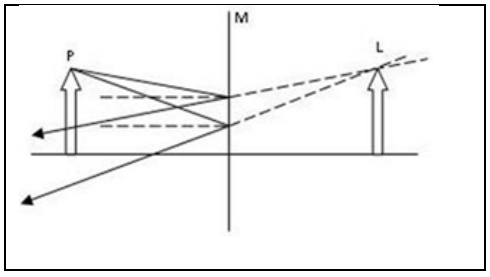

Figure 11: *Creating image in a plane mirror*

Task 8 The flat mirror should be put on the wall. What must be the length of the mirror and how high it should be set, so that man, 192 cm high, can see himself in full?

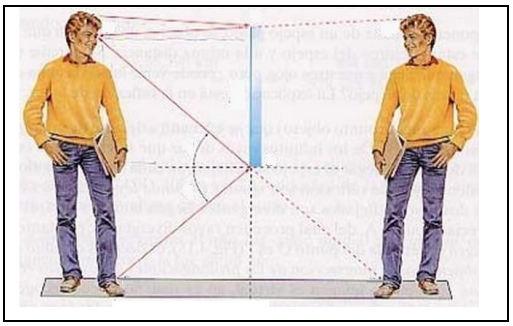

Figure 12: *Illustration for task 8*

Answer: Using the properties of treangles ΔDD_1B_1 and ΔDA_1D_1 and their midle lines lenght of mirror must be at least 192/2 cm . Point M should be aligned with the line of the eyes (FM).

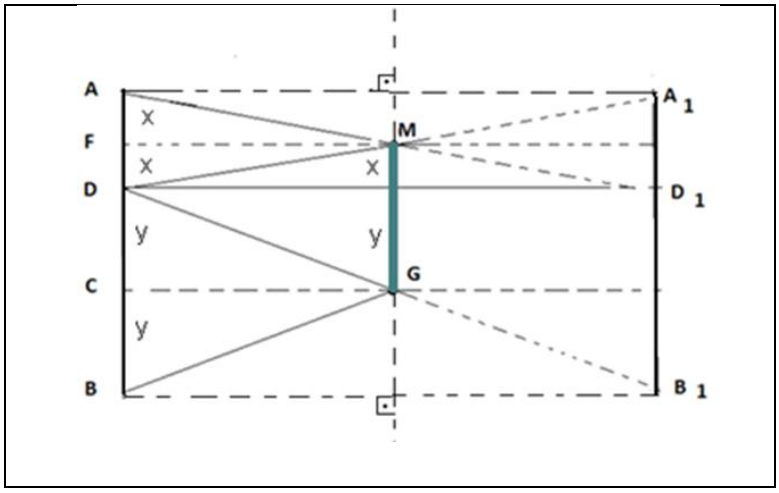

Figure 13: *Illustration for answer 8*

Group 7:

Task 9 If F1 is the reflected image of the object F from the plane mirror, where the mirror should be placed? Draw and determine the coordinates of the endpoints of the shortest such mirror.

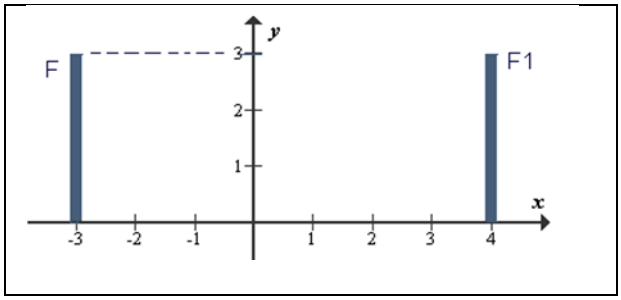

Figure 14: *Illustration for Task 9*

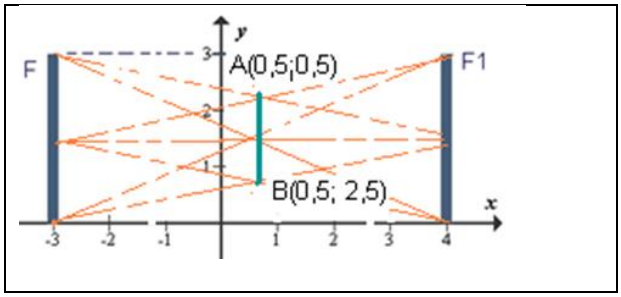

Figure 15: *Illustration for answer 9*

This is an example that can be solved in many different ways. Selecting the method of analytic geometry task is appropriate for our students and their curriculum. The same problem can be used in work with the students of different ages, from elementary school like playful activity and in different situations in everyday life.

Homework: Homework was the same for all the students. They got instructions how to make a kaleidoscope and periscope. It was suggested to make some photos of their own products and experiments that could be used for school exhibition. On the Figure 16 is photo made by Stefan Stojicic, student at Zemunska gimnazija high school.

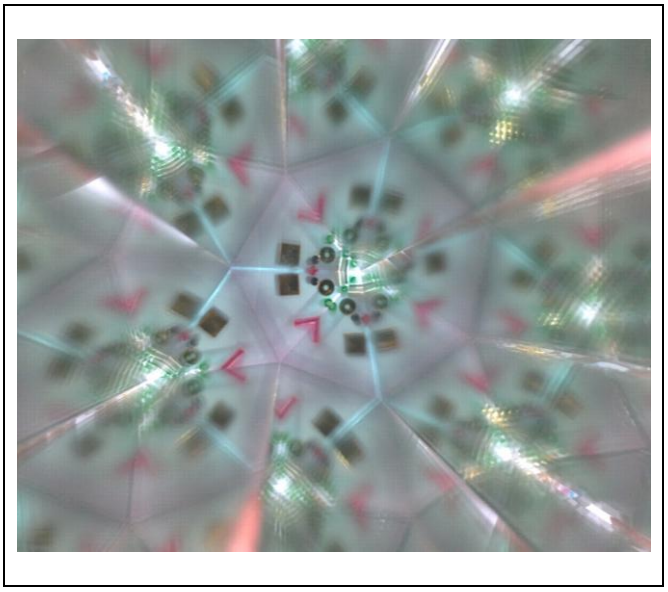

Figure 16: K*aleidoscope made image*

3. Summary

Reasoning and sense making is the core of all mathematical learning and understanding. Reasoning is the process of conclusions based on evidence or stated assumptions—extending the knowledge that one has at a given moment. Sense making develops understanding of a situation, concept, or context by connecting it

with existing knowledge. A high school mathematics and science learning based on reasoning and sense making provides more qualitative knowledge to the students.

We think that this could be achieved by coordinated activities which connect different school subjects, incorporating examples from everyday life and focusing activities on outcomes, as it is presented in our lesson plan.

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