Developing Algorithmic Thinking Using Crocheting Patterns as Educational Tool

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Abstract

In this paper we are emphasizing the importance of developing algorithmic thinking, for improving problem solving skills. Problem solving competence is important for one of the eight key competences defined at EU level - mathematical competence. We will find a relationship between Polya's problem-solving and algorithmic thinking, comparing basic Polya's principles to definition of algorithmic thinking. So, we are considering algorithmic thinking as an important role in high school education for developing mathematical competence. We are using crocheted geometrical shapes and finding a mathematical model to realize it through crochet. We developed algorithms for crocheted models, as a useful educational tool.

Key words: algorithmic thinking, problem solving, crochet, geometrical shapes
MSC: 97A80, 97M10, 97U99

1. Introduction

Eight key competences have been defined at EU level. Those competences represent a combination of knowledge, skills and attitudes that are necessary for "personal fulfillment and development; active citizenship; social inclusion; and employment" [1]:

- communication in mother tongue
- communication in foreign languages
- mathematical competence and basic competences in science and technology
- digital competence
- learning to learn
- social and civic competences
- sense of initiative and entrepreneurship
- cultural awareness and expression.

“Mathematical competence is ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations, with the emphasis being placed on process, activity and knowledge.” [2]

According to a given mathematical competence we are analyzing Georg Polya's principles of solving problems. In everyday life there are problems that have to be solved. Solving a problem means, to find an adequate procedure for its solution. Thinking about daily procedures is helping each person to develop basic algorithmic skills.

2. “How to solve it” - George Polya

The father of problem-solving, George Polya, gave specific advices to teachers, how they should teach mathematics. According to his principles, the main role of teaching mathematics is to develop thinking, clarity and persistence. The most important part of thinking, developed in mathematics, is solving problems. [3]

In his book “How to solve it”, published in 1945, Polya identifies four basic principles of solving problems [4]:

1. Understand the Problem
2. Devise a plan
3. Carry out the Plan
4. Look back.

These principles indicate that problem-solving process consist of following steps: [5]

- Identifying the problem
- Understanding the problem
- Representing the problem
- Solving the problem
- Communicating the results

3. Algorithmic Thinking and Algorithm

Gerald Futschek gave a definition of algorithmic thinking as a special problem solving competence, which consists of several abilities [6]:

- Analyze given problem
- Specify problems precisely
- Find the basic action that are adequate to given problems
- Construct correct algorithms to given problems using basic actions
- Think about all possible special and normal cases of a problem
- Evaluate algorithms
- Improve the efficiency of algorithms.

Beside these abilities, algorithmic thinking is influenced by many other human cognitive factors as abstract and logical thinking, thinking in structures, creativity and problem solving competence.

Let us compare Futschek's definition of algorithmic thinking with Polya's problem-solving process:

- Analyze given problems <=> Identifying the problem
- Specify problem precisely <=> Understanding the problem and Representing the problem

- Construct correct algorithms to given problems using basic actions and Think about all possible special and normal cases of a problem <=> Solving the problem.

This comparison represents real connection between algorithmic thinking and problem-solving process in mathematics.

Algorithmic thinking contributes to the understanding of problem solving and because of that has pedagogical value, as Donald Knuth said: "…a person that is taught how to deal with algorithms: how to construct them, manipulate them, understand them, analyzed them has a general-purpose mental tool which will be a definite aid to his understanding of other subjects. The reason for this may be understood in the following way: It has often been said that a person does not really understand something until he teaches it to someone else. Actually a person does not really understand something until he can teach it to computer, i.e. express it as an algorithm. " [7]

Algorithm is a precisely-defined sequence of rules telling how to produce specified output information from given input information in a finite number of steps, so "...the attempt to formalize things as algorithms leads to a much deeper understanding than if we simply try to comprehend things in the traditional way" [7]

4. Problem-solving, Algorithmic Thinking and Crocheting

Polya's Second Principle of problem solving, Devise a plan, says that there are many ways to solve a problem. It is important to find a strategy for a solution. It can be done by solving many problems. Choosing a strategy is very easy, Polya says. And he gave a partial list of strategies that could be used for solving problems [8]:

- guess and check
- make and orderly list
- eliminate possibilities
- use symmetry
- consider special cases
- use direct reasoning
- solve an equation
- look for a pattern
- draw a picture
- solve a simpler problem
- use a model
- work backward
- use a formula
- be ingenious.

Each crocheted geometrical shape has a specific rule that is used to build it.

Doing crocheting we could start from patterns and build a model (look for a pattern, solve a simpler problem) or we can try to decompose into the pattern a model that is already made (use model, work backward). In both cases we are drawing picture, using symmetry, considering all possible cases, using direct reasoning and finding a formula that describes the process of crocheting a pattern or a model.

The main goal is to construct an algorithm based on a mathematical model of crocheting a specific pattern or a specific model.

5. Mathematical Foundations of Crocheting

The material used for doing crocheting is a yarn, a three dimensional line. With this line, the crochet is building meshes, which are the basic elements of crocheting.

One can build with a crochet needle the following types of meshes:

1. the chain stitch **O**
2. a single crochet **X**
3. a double crochet **∓**

After the double crochet you can also build a triple crochet, a quadruple crochet and so on, but those can be seen as modifications of a double crochet.
These symbols are internationally used to draw crocheting patterns.

A main property of crocheting is also notable in the term of the chain stitch. Logically, by using a yarn, we are obliged to build one mesh after the other. We are building chains of meshes or rows. After the first row, we can do a flip stitch to turn the work around and to build another row above the row before. If we do that several times, we will get several rows.

One single mesh is as high as wide.
So actually it is not far away to see it as a Cartesian coordinate system, dimension N x N, where each mesh is a crossing point of both sets. If we would be extremely correct, we should see it, because of the three dimensionality of the crocheted object, as a coordinate system dimension N x N x 1 - because the surface is as thick as one mesh.

Our measurement in crocheting does not necessarily have to be in centimetres or millimetres. Because of that we are able to define a new measure - 1 mesh.

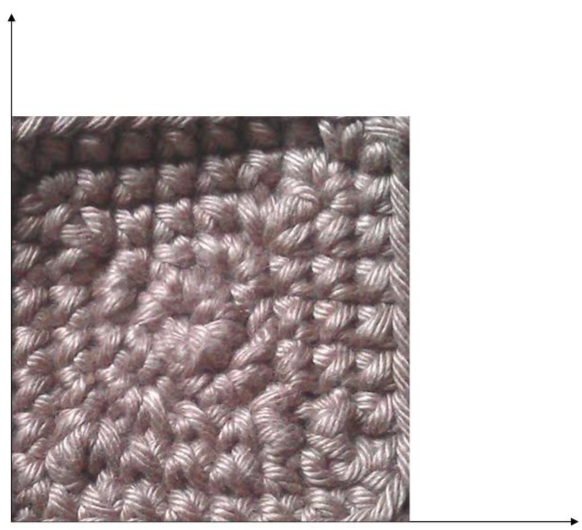

Figure 1: *Cartesian coordinate system, N* x *N*

6. Examples of Algorithms

We were curious: is it possible to crochet a perfect circle, triangle, square? And, if it is: is there an algorithm that will describe building regular 2D geometrical shape, or one can rely on intuitive approach?

Crocheting a Circle

Let us consider a perimeter of circle given with formula P=2·*Pi*·*r*, where r is radius of a circle. Since we can not have a half of a mesh or a third of a mesh, there is a need to take approximation of number Pi as Pi ≈ 6.28 ≈6 .

Assume that *r* = 1 mesh, then P=6*1 meshes = 6 meshes. And, more further, let it be r=2 meshes, then we have P=6*2=12 meshes. Then, if r=3 meshes, P=6*3=18 meshes.

So, we have now following results:

r = 1 => P=6 meshes

r = 2 => P=12 meshes

r = 3 => P=18 meshes.

The picture below shows the idea of crocheting circle.

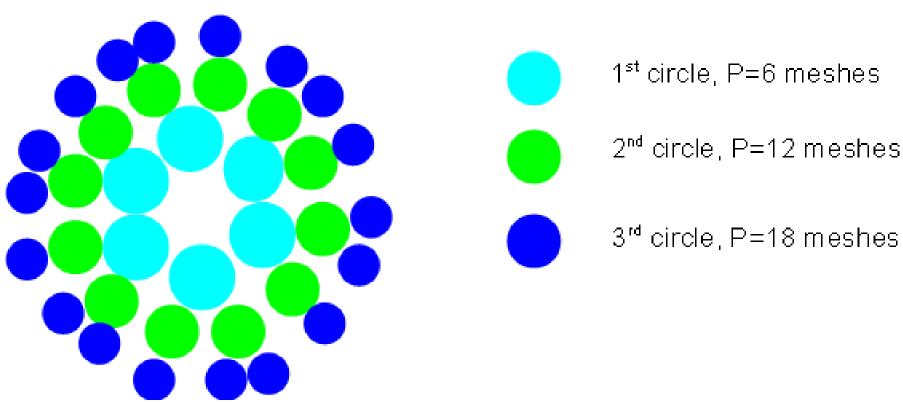

Figure 2: *Crocheting a Circle*

Observing the results of perimeter we can conclude that those numbers are elements of arithmetic progression with a common difference d=6 and initial term a_1=6.

Based on previous we have following algorithm:

Step 0: build a chain of 3 or 4 meshes
Step 1: build a circle using the chain from Step 0

Step 2: make a circle with 6 meshes (initial term of arithmetic progression)
Step 3: build a circle with 6+6=6+1*6=12 meshes, by stitching two meshes in every 6/6=1st mesh of previous circle
Step 4: build a circle with 6+12=6+2*6=18 meshes by stitching two times in every 12/6=2nd mesh of a previous circle (and one stitch in all other meshes)
Step 5: 6+18=6+3*6=24 meshes, by stitching two times in every 18/6=3rd mesh of every previous circle (and one time in all other meshes)
Step N: 6+(n-1)*6 meshes by stitching two times in every [(n-1)*6]/6=(n-1)th mesh (and one time in all other meshes).

Since we have to think about all special cases there is a question: what if we want to make initial circle with random number of meshes? In this case we have to modify the given algorithm. The modification consists of modification Step 2 and Step 3.

Step 2: build a circle with K meshes
Step 3: IF K mod 6 = = 0 THEN DO Step4
ELSE
(find the smallest number M that is greater than K) AND (M mod 6 == 0)
AND (make a circle with M meshes) // M is initial term of arithmetic progression

Figure 3. *Crocheted Circle*

Crocheting a Square

Let us think of a perimeter of a square. Since mash can approximate a unit square, we are counting unit squares in the perimeter. Let a be a length of a side of a square.

a = 1 there is 1 mash in a “perimeter”
a = 2 there are 4 mashes in a “perimeter”
a = 3 there are 8 mashes in a “perimeter”
a = 4 there are 12 mashes in a “perimeter”

...
$a = n$ there are 4x(n-1) mashes in "perimeter"

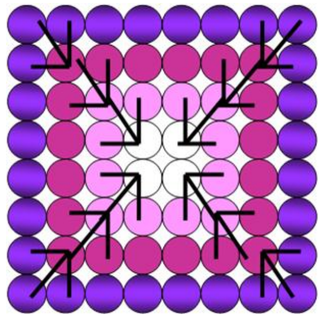

Figure 4: *Crocheting a Square*

In each new square, we need to have 8 new meshes. According to this we are able to construct an algorithm for crocheting a perfect square based on arithmetic progression (initial term of arithmetic progression a_i=4, common difference d=4):

Step 0: build a chain of 2 or 3 mashes
Step 1: build a circle using the chain from Step 0
Step 2: make a square with a = 2, with 4 mashes (initial term of arithmetic progression)
Step 3: make a square with a = 4, with 4 + 8 = 12 meshes, by making 3 mashes in every 4/4=1st mash of a previous circle
Step 4: make a square with a = 6, with 12 + 8 = 20 mashes, by making 3 mashes in every 12/4=3nd mash of a previous circle
Step N: make a square with $a = n$, with 4x(n – 1) mashes, by making 3 mashes in every 4x(n-3)/4=(n-3)th mash of a previous circle

Figure 5: *Crocheted Square*

Conclusion

To develop mathematical competence, one of eight key competences that have been defined at EU level, we compared Polya's principles of problem-solving to definition algorithmic thinking.

Developing algorithmic thinking each person could develop basic mathematical competence.

The main goal is to construct an algorithm based on mathematical model of crocheting a specific pattern or a specific model.

In our own experience, students should work on

- making a clear idea of a model
- visualisation of a model
- calculation of some function (in our example perimeter for a sequence of circles/squares)
- ability to make a drawing of a model
- skill of crocheting, so she/he could make experiments
- think and rethink, going back, if experiment fail
- make an algorithm, i.e. put a solution in a reusable form, from beginning to the end

This is an example how students could be encouraged to develop their mental processes which will lead them to succeed in problem solving. Combination of thinking, visualizing, drawing and handcraft make this sort of teaching usable for students with different cognitive learning styles - visual, auditory, and kinaesthetic/ tactile.

That is why we are considering crocheting patterns as a good educational tool for developing algorithmic thinking and using it in a teaching mathematics.

References

[1] Developing Key Competences for School in Europe, http://eacea.ec.europa.eu/education/eurydice/documents/thematic_reports/145EN.pdf

[2]http://europa.eu/legislation_summaries/education_training_youth/lifelong_learning/c11090_en.htm

[3] http://cmc-math.org/members/infinity/polya.html

[4] http://www.math.wichita.edu/history/men/polya.html

[5] L. Zsakó, P. Szlávi, *ICT Competences: Algorithmic Thinking*, Acta Didactica Napocensia, Volume 5, November 2, pp. 50. 2012.

[6] G. Futschek, *Algorithmic Thinking: The Key for Understanding Computer Science*. In Lecture Notes in Computer Science 4226, Springer, pp. 159 - 168. 2006.

[7] D. E. Knuth ,*Computer Science and Relation to Mathematics*, American Mathematical Monthly, 81, pp. 323-343. 1974.

[8] http://www.math.wichita.edu/history/men/polya.html