Extreme Values of Function in GeoGebra Style

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Abstract

The process of student transitioning from elementary to advanced mathematical thinking in learning of calculus is followed by many difficulties. The role of the teacher is very important in planning, designing and applying adequate methodic solutions by which smooth transitions from lower to higher levels of abstraction, and mathematical thinking, are secured. Methodic solution wich can be efficient in teaching and learning calculus, is visualization, and animation of the basic terms, and processes in combination with symbolic records and definitions. By applying according computer programs, the teacher can realize the teaching process in visual environment. Educational software can link visual and symbolic representations of mathematical objects in dynamic, and interactive environment. This paper presents the possibility of didactic shaping of calculus material using GeoGebra. In dynamic worksheets algebraic and geometric interpretations of concepts of local extreme values of the functions, are connected.

Key words: extreme values of function, GeoGebra, cognitive-visual approach.

MSC: 97U30, 97D40, 97U50.

1. Introduction

Until the sixties, mathematical education was based on a formal definition of concepts and predominantly deductive methods. The cognitive psychologists findings on the biological basis of cognition, about the work of the brain hemispheres, and the study of how people perceive, learn, remember and think, had an impact on mathematical education. Since then, efforts have been made to apply the cognitive theory in the classroom so that mathematical education can be improved.

Tall and Vinner stated that one of the key reasons for failure in mathematical education is the difference *"between the mathematical concepts as formally defined and the cognitive processes by which they are conceived"* [1]. In their work two ways of adopting mathematical concepts are stated: the concept definition and the concept image. The concept definition is a form of words and symbols used to specify that concept. The term concept image is defined as "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes". During the formation of the concept, the relationship between the concept definition and the concept image should be reciprocal [1].

Educational research suggests that in teaching practice we should find the optimal relationship between the concept definition and concept image, and should strive to the balanced relationship between logical-analytical and visual-creative thinking. Visualizing concepts that are being introduced or processed in combination with symbolic entries and definitions contribute to the efficiency of the educational process.

Realization of the cognitive-visual approach means that we apply graphics, schemes, tables, concept maps, presentations, interactive learning materials, applets, animations etc. in teaching process. Many computer applications can provide an environment that allows teachers to display mathematical concepts and processes, and students to explore, analyze and draw conclusions.

2. Student difficulties in learning calculus

Functions first derivative is one of the basic topics of calculus with which students of high school first encounter in fourth grade. Students have difficulties accepting the geometrical interpretation of the first derivative itself. Even bigger problems occur when the basic theorems, which link the functions first derivative with monotonicity and the extreme values of the functions, are to be applied. Difficulties occur because students, besides the great knowledge of basic math, must form a new way of thought, which is called *"Advanced mathematical thinking"* in literature (thinking which is based on formal definitions, axioms and theorems, with which, logical deduction is applied).

The role of the teacher in the process of transitioning students from elementary to advanced thinking is very important. The results of pedagogic researches give certain guidelines to teachers, in order to make that transition as simple as possible for students. When adopting new terms, it's better to go from specific and simpler to more abstract and complex. The existing representations which students have about certain terms need to be connected and on that basis new terms should be built. Students easier, and better adopt new terms of calculus when in teaching, concepts of image and definition, are combined. This approach allows students to make connections between symbolic and visual representations, and thus increase their cognitive abilities.

3. Cognitive-visual approach to teaching calculus by using GeoGebra

GeoGebra is a dynamic mathematical software where mathematical objects are shown in two ways: algebraic and graphic (geometric). Users can simultaneously use a computer algebra system and an interactive geometric system. This tool extends the concepts of dynamic geometry to the fields of algebra and mathematical analysis.

Function first derivative, definite integral and other concepts of calculus are simply presented in GeoGebra, both algebraically and graphically. It is a powerful tool for visualization and animation, so that students can be shown various positions and relationships of given objects. Therefore, GeoGebra can display, in addition to examples of the concepts, examples of mathematical processes through dynamic action. GeoGebra is used in teaching and learning of calculus as a cognitive tool for explaining, exploring and modeling of mathematical concepts and processes. GeoGebra environment encourages students` projects in mathematics, experimental and guided discovery learning.

4. Interactive Examples in Teaching and Learning Calculus with GeoGebra

In this paper we are presenting examples of how Geogebra is used in classrooms to explain and explore concept of local extreme values of functions. Dynamic worksheets which we are presenting, the author has prepared and used in classroom with students of the fourth grade of high school. The basic idea while preparing GeoGebra dynamic worksheets is connecting algebraic and geometrical interpretation of the concept of local extreme values of functions.

The problem which is given to the students is as it follows: link each of four given functions with its derivative function. Functions are given with their graphs, and their derivative functions are given analytically, namely with formula (figure 1).
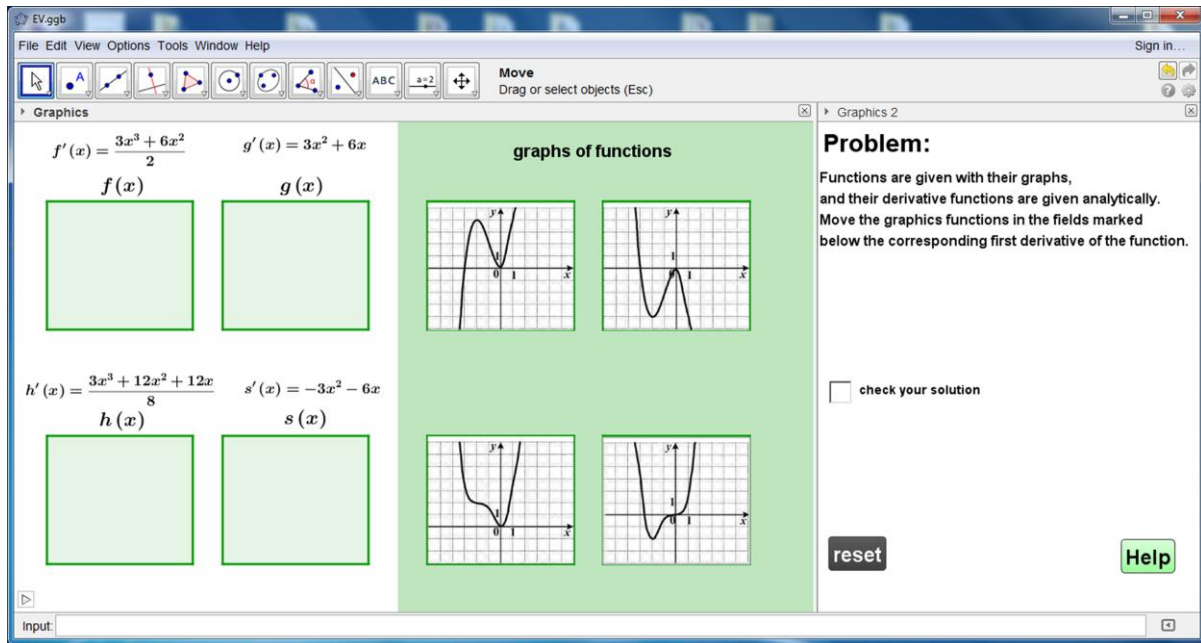

Figure 1: *Problem and home menu.*

Before solving the problem, the following is to be repeated with students: the notion of the critical number (critical point) of the function, the notion of the local maximum and minimum of the function, necessary and sufficient conditions for local extreme points and procedure for determining local extremes of the function. Students on the basis of the graph of each function, get essential data of critical numbers and local extremes of the function. As the derivative functions are given by formula, by conducting the corresponding algebraic procedure, students first determine the critical numbers, and then the points of local extremes. For the students to link the function with its derivative function, they first need to connect the facts that they have learned by graphical and algebraic solving of the problem.

Students can check if they solved the problem correctly (check your solution), as well solving the problem again ("reset" button). In a case that students haven't solved the assignment correctly, or if during solving they have certain difficulties and concerns, they can use additional explanations by clicking the "help" button.

The assistance that students can receive is organized in multiple levels, and is shown by checking the corresponding boxes or clicking on the button (Figure 2).
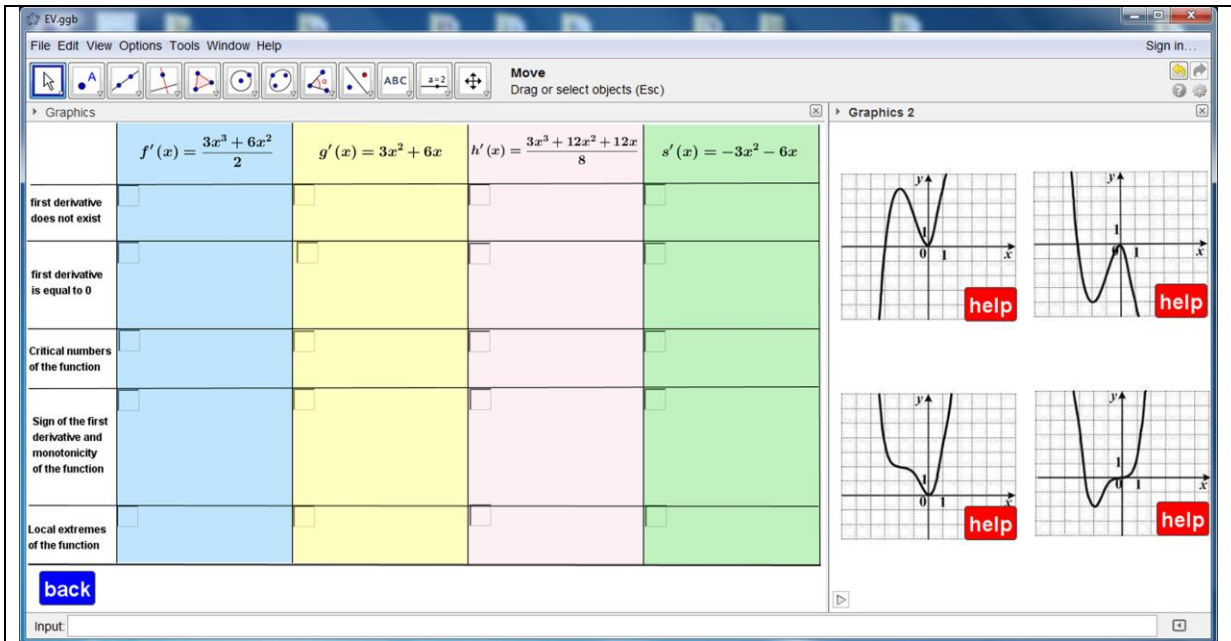
Figure 2: *Assistance in solving problems.*

The levels of help for derivative functions are (figure 3):
1) Real numbers for which the first derivative of the function is not defined.
2) Zeroes of the first derivative.
3) Critical numbers of the function.
4) Sign of the first derivative and monotonicity of the function.
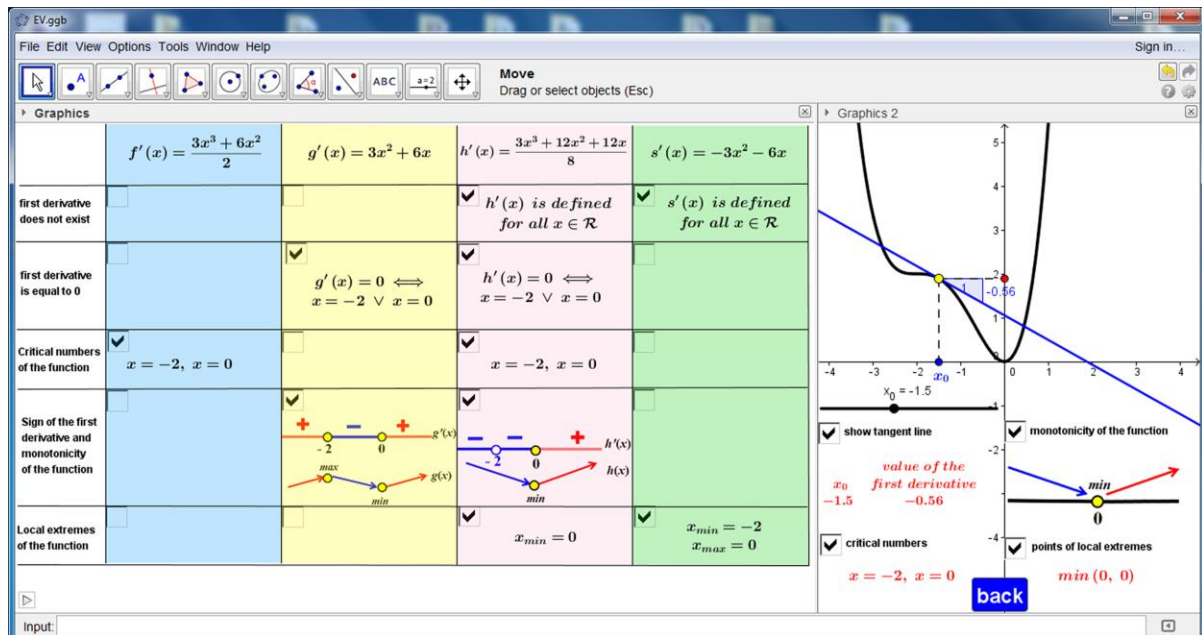5) Local extremes of the function.

Figure 3: *Levels of help for derivative functions and graphs of the functions.*

Assistance with graphs of the functions is given in following levels (figure 3):
1) Tangents of the graph of the function and value of the first derivative - Students can move the slider and observe the movement of the point on the function curve and the tangent line which is constructed at that point. The slope of the tangent line is geometrically represented by the corresponding right-angled triangle. Also moving the slider, students can follow the changes in the value of the functions first derivative.
2) Critical numbers of the function.
3) Monotonicity of the function.
4) Points of local extremes.

According to their needs, students can choose any type of help, and with independent research they can come to certain conclusions and knowledge. After using selected levels of assistance, by clicking the "back" button, students have the possibility to return to solving the problem, and checking the solution.

Impressions and findings of students in a class where they used GeoGebra worksheets, point to their satisfaction because they had the option to solve the problem independently, and to create the way by which they get to the final solution, by themselves. Solving the problem students experienced as a game with images, and they were very motivated to solve this GeoGebra puzzle.

5. Conclusions

One way to overcome difficulties students faces in learning calculus is for the teacher to shape the learning environment which connects symbolic and visual representations of terms, concepts and processes of calculus. Because of the dual representation of the mathematical objects, algebraic and geometric, GeoGebra is a powerful cognitive-visual tool for teaching and learning calculus.

Visual and dynamic interaction between the user and GeoGebra environment helps students to form their visual understanding and thinking. Also, students can easier apply and link their visual understandings with formal-symbolic language of calculus. In that way their understanding of the basic concepts and connections between them, is fast and proper. GeoGebra allows teacher to encourage and guide students to independently research, discover, and through their own experience obtain new knowledge. Didactically designed contents in GeoGebra allows students to independently forge their own way by which they obtain new knowledge.

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