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CORRELATION ANALYSIS OF STUDENTS' SUCCESS IN SOLVING ANALYTIC GEOMETRY AND MULTIPLE INTEGRAL PROBLEMS

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Abstract. In this paper, we aim to contribute to the planning and implementation of education in higher mathematics education for students from non-mathematics study programs, specifically focusing on multivariable calculus, i.e., multiple integrals. Indeed, the outcomes of various empirical studies indicate that students from non-mathematical faculties struggle to grasp and comprehend multiple integrals and multivariable functions in general. The research presented in this paper aims to ascertain whether there is a significant correlation between students' achievements in multiple integrals and their achievements in applying knowledge and skills from analytical geometry (to define sets of points in the plane and space, determined by lines, curves, planes and surfaces). Additionally, the study investigates whether this correlation potentially varies based on the various instructional teaching approaches. The presented empirical research was conducted at the Faculty of Engineering, University of Kragujevac, with 72 second-year students, divided into two groups. The results indicate that the given linear correlation is statistically significant and positive. Moreover, the differences in correlation coefficients calculated for two groups of students who acquired knowledge in multiple integrals through different instructional approaches are not statistically significant. These findings underscore the need to devote substantial attention to the teaching of multiple integrals, especially in devising methods that enable students to visualize specific mathematical concepts in both plane and space. Additionally, a precise definition of integration domains, and an accurate specification of variable bounds, should be emphasized in the multiple integrals teaching and learning process.

MathEduc Subject Classification: D65

AMS Subject Classification: 97D60

Key words and phrases: Multiple integrals; visualization; calculus; multiple representations; analytic geometry.

Introduction

The main goal of this research is to establish the influence of the proper students' knowledge and skills about defining sets of points in plane and in space, which students gain during their mathematical education regarding analytical geometry in plane and in space onto their achievements in solving multiple integral problems. In many educational studies, the researchers are trying to identify and compare different methodologies regarding teaching and learning process of various mathematical contents, and so it is for calculus contents as well. On the other hand, for the most mathematics contents it is very important for students that they have already acquired mathematical knowledge regarding the teaching content that they should learn and understand in the present moment. As is already stated in some research regarding teaching and learning of multiple integrals [11, 14, 16, 19] when students encounter double or triple integral tasks, two kinds of problems may appear. One of them is regarding the process of calculating multiple integral that is directly connected with students' knowledge and skills regarding calculation of the definite integral. In that manner, students must master solving definite integrals and have procedural knowledge about decomposition, about substitution, and about integration by parts. The other problem regards the lack of students' success in defining the integration area and setting boundaries for the variables. This is basically the first part of solving multiple integrals process, because in the most cases, the task is posed in manner that the integral function is given and students are informed about which mathematical objects, with their intersections, determine the integration area for the multiple integrals. So, for students to start calculating multiple integrals, they must use graphical representation of objects, and use the algebraic representations of those objects and to solve some equations and inequalities to determine the boundaries for the multiple integral. This part of the task shouldn't be too hard for the students of mathematics study programs, because before the course of Mathematical Analysis in which they learn about multiple integrals, they have entire Analytical Geometry course in which they acquire this type of knowledge and skills. On the other hand, for students from technical, mechanical, and other non-mathematics academic study programs, understanding properties of different surfaces in space and understanding their interrelations is not so easy because in their mathematical curriculum, less attention and less time is paid for these contents. It is therefore crucial to provide students with appropriate visualization of educational content, coupled with an analytical approach to learning, for them to achieve successful learning outcomes.

Theoretical background

To solve concrete problems from analytic geometry and from multivariable calculus students need to visualize the adequate mathematical concepts in plane and/or in space. According to [20], to visualize means to construct, create, or make connections between an external mathematical object or its representation (a diagram, a table, or a picture) and a mental (internal) construct or image and use analytical approach to develop and advance understanding. Interaction with the mental image can be through physical models, manipulatives, sketches, computer-based static outputs, or animations such as simulations [20]. The author of [2] has stated about visualization: "It's the ability, the process and the product of creation, interpretation, use of, and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of thinking about and developing previously unknown ideas and advancing understandings". According to [20], the process of visualization involves the interplay and interrelation between the mental cognitive framework and the tangible or digital mathematical object or its representation, whether that representation is internal or external. Sheikh explains that this connection can manifest in two possible directions. Visualization can entail mentally constructing objects that are derived from externally observed entities, or it can involve creating objects or scenarios on an external medium, like paper, a chalkboard, or a computer screen, which correspond to concepts or processes held in an individual's mind. Tall [21] argued that dismissing visualization equates to disregarding the origins of numerous fundamental mathematical concepts. He pointed out that during the initial phases of formulating theories by students like functions, limits, and continuity, visualization played a pivotal role in generating foundational ideas. Gutierrez [6] asserted that visual depictions, encompassing images, graphs, or illustrations, whether conceived mentally, sketched on paper, or generated through dedicated software applications, can enhance conceptual comprehension, and play a notable role in the process of discovery.

To manage analytic geometry in space and multiple integrals, students must have some kind of spatial ability. Spatial ability refers to cognitive functions and competence that is crucial in solving problems that involve manipulating and processing visuo-spatial information [7, 13]. Studies identified indications of robust spatial-mathematical correlations and the application of spatial interventions leading to enhanced mathematical comprehension [25]. McGee [15] defines spatial ability as a collection of four concepts:

(1) imagines the rotation of an object, e.g., xy projection onto 3D object;

(2) fold a net and unfold an object, e.g., sector of a circle in 2D into a cone in 3D;

(3) imagine movements such as translations, rotations, enlargements of 3D objects;

(4) transform or manipulate spatial patterns into other arrangements, e.g., object in rectangular to spherical or cylindrical coordinate systems.

Zimmermann [27] discussed the role of visualization in calculus: "the role of visual thinking is so fundamental to the understanding of calculus that it is difficult to imagine a successful calculus course which does not emphasize the visual elements of the subject". In the study [8], the author reached the conclusion that fostering the ability for visualization, which impacts the correlation between graphical representations and other forms of representations, enhances the performance in resolving definite integral problems. Delice et al. [3] concentrated on evaluating the sketches created by university students while tackling volume problems involving integrals. Their research demonstrated that students possessing both algebraic and spatial skills achieve success in the problem-solving process. In a separate study [10], the authors endeavored to establish whether spatial visualization skills exert influence on grades in calculus courses. Their findings indicated that spatial visualization skills have the potential to serve as a predictor for success in a calculus course. Tall [22] has stated, "of all the areas in mathematics, calculus has received the most interest and investment in the use of technology".

Multiple research studies propose that dynamic visualizations, in contrast to static visualizations, offer greater advantages to students, including those in engineering fields [9]. The level of interactivity within visualizations plays a pivotal role, with a higher degree of interaction improving comprehension, fostering deeper learning, and enhancing interactive visualizations. This approach has found extensive application in the realm of science and engineering education [18]. Software tools are frequently employed to visualize calculus concepts [16, 24]. In their study [23], the authors demonstrated that their students, utilizing *GeoGebra*, invested more time in analyzing correlations between formulas and their graphical representations. Furthermore, *GeoGebra* proved beneficial in helping students with limited prerequisite knowledge, necessary for problem-solving, to enhance their understanding. The challenge of imagining and sketching in three dimensions posed significant difficulties for students tackling multivariable calculus [11]. Given the positive impact observed in numerous studies regarding this approach's effect on students' comprehension, the integration of technology into calculus education needs serious consideration [26].

Concerning the role of calculus from the perspective of mathematics educators, they generally view calculus as an essential component of the mathematics curriculum and regard it with a higher level of significance and complexity compared to other mathematical subjects such as algebra and geometry [1]. One of the possible reasons is precisely that dealing with calculus requires certain knowledge of algebra and geometry.

In the study [5], the authors claim that mechanical engineering students have difficulty understanding multiple integrals. One of the errors that students made during multiple integration was that some students could not sketch the regions of integration correctly, which later influenced their success in solving tasks. Coordination and conversions between the graphical and analytical representations of the integration domain are often found to be highly beneficial, if not essential, as emphasized by [4]. Several misconceptions have been identified in research [12] regarding double integral misconceptions by engineering students. Two of them include challenges related to the graphical representation of surfaces that form the integration region, and the transition from Cartesian to polar coordinate systems. Other difficulties refer to altering the order of integration, challenges in establishing the integral and algebraic complexities. Furthermore, in another study [17] that explored students' comprehension of the limits of integration in double integrals using the APOS theory framework, it was observed that some students possessed a basic ability to formulate integrals but appeared to lack a solid geometric understanding of the significance of these limits which points to a deficiency in their foundational knowledge.

Purpose of the study

Having in mind, as we stated earlier, that there are efforts to a certain extent in identifying methodological approaches for teaching and learning multivariable calculus with introducing technology [10, 16, 20], we wanted to investigate are there differences in strength of influence (correlation coefficient) of students' knowledge and skills about determining and defining sets of points with given mathematical objects (in which visualization of mathematical objects have great impact) on the students' success in solving multiple integral tasks, for different teaching and learning approaches (approach, which is conducted with the use of dynamical software, and an approach in which technology is not implemented, but both with an accent on the visualization of mathematical objects). For those purposes, we conducted the research in which we carry out teaching of the multiple integrals with 72 second-year students from the Faculty of Engineering in Kragujevac, where the accent is put on the visualization and analysis of the objects that determine the integration area and on the setting boundaries for the given variables. To determine the presence or absence of the significant difference in the linear correlation between students' knowledge about the defining set of points in plane or in space and students' achievements in solving multiple integral tasks, whether technology is meaningfully used in teaching, or it is not used, students were divided into two groups. Practical classes in which students learned about multiple integrals were realized in the traditional way, with writing and drawing on the blackboard by the teacher and students and with continuous discussions in one group, while in the second group students used the applications *Graphing Calculator* and *3D Calculator* during their learning process.

Research questions

In our research, we had two research questions.

Q1. Is there a statistically significant linear correlation between students' success in solving tasks in which they must define the appropriate set of points in plane and/or space and set boundaries for the variables and their success in solving multiple integral tasks (which will implicate that it is necessary for students to master contents from analytic geometry to solve multiple integral tasks)?

Q2. Is there statistically significant difference between the correlation coefficients for solving analytical geometry tasks and multiple integral tasks for two different groups of students—the students who used appropriate mobile applications for the visualization for the parts of plane or/and space and for the students who didn't use them, but made appropriate images using paper and pen (chalk and blackboard)?

Material and methods

Theoretical lectures from the Mathematics 3 course were conducted in an identical way in both groups of students, and in those classes the teacher introduced the students to theoretical concepts related to multiple integrals, making an analogy with a definite integral. When it comes to practical classes, classes with one group took place in a traditional way and we will call that group the control group. In control group the teacher used the blackboard to visualize objects that determine the integration area, and students made sketches in their papers. The teacher also combined the graphical and algebraic representations of those mathematical concepts during practice. Lessons with another group of students were realized with the use of *GeoGebra* applications *Graphing Calculator* and *3D Calculator*. We will call the latter group the experimental group, since the teaching practice in that group of students has been changed to a greater extent. What should be emphasized is that the teacher, in the work with both groups of students (in addition to the complete implementation of the procedure for calculating multiple integrals when the limits of the variables were clearly specified), emphasized the detailed analysis of the area by which the integration is performed. Namely, the students of these study programs, in their curriculum, didn't have a great extent of classes for the realization of teaching content related to analytical geometry in space, while when it comes to content related to analytical geometry in plane, they previously showed some gaps in their (prior) knowledge.

Having in mind that for the appropriate determination of the domain over which the integration is performed, as well as for the correct determination of the limits of the variables, it is necessary for students that they have proper knowledge about the elementary functions of one variable, solving equations and inequalities, knowledge about basic surfaces, lines and planes in space, so the teacher paid special attention for the analysis of given mathematical objects and their interrelations. In working with the students of the control group, the teacher associated both an algebraic representation and a graphic representation on the blackboard of each mathematical object that determines the domain on which integration is performed. On that occasion, he pointed out to the students that changes in the algebraic notation of mathematical concepts also affect the geometric properties of these objects. After determining the domain on which the integration is performed, and after determining the limits for the variables, he moved on to the calculation process of multiple integrals.

When it comes to working with the students in the experimental group, the process of calculating the integral proceeded as usual (as well as in the control group of students), while the procedure for solving the task that preceded the implementation of the calculation process was changed and modernized. In order to better visualize the corresponding mathematical concepts, the teacher introduced the students to two very simple and receptive to them, *GeoGebra* applications—*Graphing Calculator* and *3D Calculator*. The teacher introduced the students to the fact that for the purposes of drawing graphs of functions of one variable, for solving equations and systems of equations using the graphical method, as well as for graphically representing mathematical objects in a plane such as circles, ellipse, parabolas, hyperbolas etc. the *Graphing Calculator* application can be used, while the *3D Calculator* application can be used for graphing of two-variable functions, for sketching graphs of lines, planes, surfaces (such as paraboloids, cones, cylinders etc.) in space, as well as for representing different geometric objects in space.

The students then installed the given two applications on their mobile devices, which they were asked to bring to the practical classes (with the instruction that it might be more beneficial to use laptop devices due to the screen size, in order to see the given objects as well as possible, but of course they could also use their mobile phones). When solving specific tasks, the teacher let students to independently enter algebraic representations of mathematical objects that define the domain of integration, in the appropriate application, then to analyze the obtained images and try to write down the integration domain using mathematical notation, as well as to determine the limits for the given two or three variables. Then the students would cross out the given pictures in their own notebooks, in order to better master and to better understand the given procedure. Of course, in order for the students to better understand the determination of the limits of variables when solving triple integral problems in the 3D, the students drew the corresponding objects using *3D Calculator*, and then the given projections of the bodies on the plane, using the *Graphing Calculator*. The students entered some mathematical objects in an explicit form, but to understand the introduction of variable change using polar coordinates (in the plane), that is variable changes using cylindrical or spherical coordinates (in space), they entered some objects in a parametric form. In order to better understand and acquire proper knowledge about the given changes of variables, students defined sliders and included leaving a trace option, so they could visually experience the impact of changing one of the variables to the change in the position of a point in the plane or in space. On Figures 1, 2 and 3 there are graphical and algebraic representations of mathematical objects for some concrete tasks.

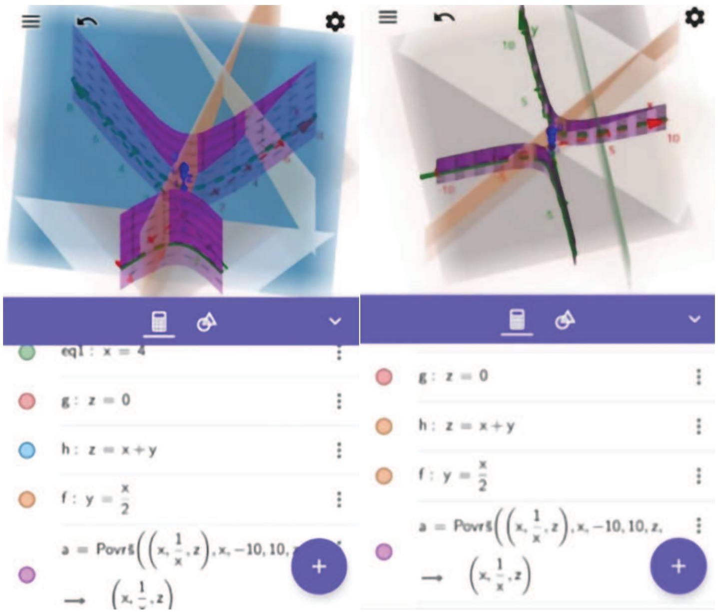

Fig. 1. Graphs corresponding to the task in which the integration region is defined with planes $z = 0$, $x = 4$, $z = x + y$, $y = \dfrac{x}{2}$, and with cylinder $y = \dfrac{1}{x}$

During the exercises, students solved problems related to double integrals, for calculating the integral with a given integral function, as well as problems in which the double integral had to be applied to calculate the area of a part of the plane. Similarly, students solved tasks related to triple integrals, for calculating triple integrals with a given integral function, as well as tasks in which the triple

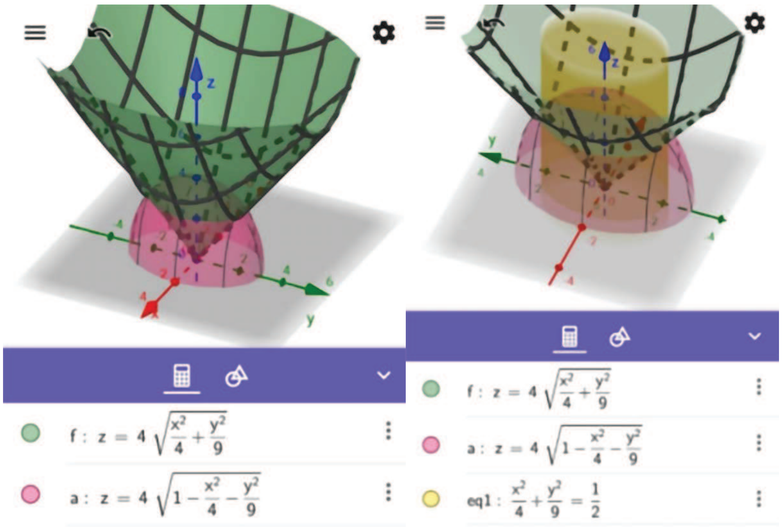

Fig. 2. Graphs corresponding to the task in which the integration region is defined with ellipsoid $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{16}=1$ and with conus $\frac{x^2}{4}+\frac{y^2}{9}=\frac{z^2}{16}$

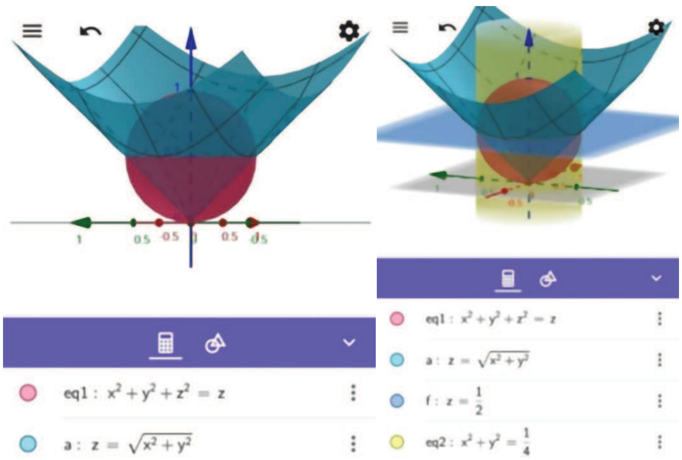

Fig. 3. Graphs corresponding to the task in which the integration region is defined with sphere $x^2+y^2+z^2=z$ and with conus $z=\sqrt{x^2+y^2}$

integral had to be applied to calculate the volume of a part of space. In the work with students of both groups (control and experimental), identical tasks were solved. For this purpose, nine practical classes were realized, both with control and experimental group of students.

Results and discussion

Statistical analysis of results of the pretests

After forming the experimental and control group, both groups of students solved the pre-test. The students had 30 minutes to complete the test which contained four tasks. For solving the tasks, students had to display theoretical and practical knowledge regarding solving definite integrals (with knowledge about definite integral properties, method of substitution, integration by parts method). Students could not use any help in solving tasks on the pre-test (computers or telephones). The maximum number of points on the pre-test was 20.

In the pre-test there were no statistically significant differences between the groups tested, the experimental and control group at the level of significance of 0.05. The results of the statistical analysis are given in Table 1.

Table 1. Statistical results of the pre-test

Group	Number of students	Mean	Std. deviation	Student's t-test		
				df	t	p (2-tailed)
Experimental	35	11.09	3.89	70	−0.6	0.55
Control	37	10.51	4.16			

Statistical results of the test

The maximum number of points that students could score on the test was 50. The number of points that students could achieve on the test for each task is given in Table 2. There were no negative points on the test.

Table 2. Maximum number of points of the per task

Task	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Maximum number of points	3	3	3 + 3	3 + 3	3	6	6	6	6	5

In the first five tasks, the request in the task was not to solve the multiple integral tasks, but to define the proper set of points in plane or in space that is bounded with given mathematical objects. In those five tasks, students had to define ordered sets of points after introducing change of variable (one task for polar, one for cylindrical, and one for spherical variables), and in one task students should redefine the ordered pair of points and to present the independent variable as dependent one and vice versa.

In the second five tasks, students solved multiple integrals (two double integrals and three triple integrals). While solving those five tasks, students should solve one double integral after introducing polar coordinates, one triple integral after introducing cylindrical coordinates, one triple integral after introducing spherical coordinates (see Appendix).

To illustrate the connection between the results achieved by the students while solving the first five tasks and the students' achievements in solving other five tasks, the linear correlation of the two variables was examined. Pearson's correlation coefficient of variables, which represents the number of points students achieved by solving the first five tasks on the test and the number of points students achieved by solving the second five tasks on the test, is equal to 0.809 (Table 3 and Figure 4) which is considered as a strong and positive, statistically significant correlation ($p < 0.0005$). This result indicates that students who achieved better results in solving the first five tasks had better achievements in solving the second five tasks on the test.

Table 3. Correlation statistics between the number of points that students achieved by solving the first five tasks and the second five tasks on the test

		Total number of points that students achieved for solving second five tasks
Total number of points that students achieved for solving first five tasks	Pearson Correlation Coefficient	0.809
	p (2-tailed)	< 0.0005
	N	72

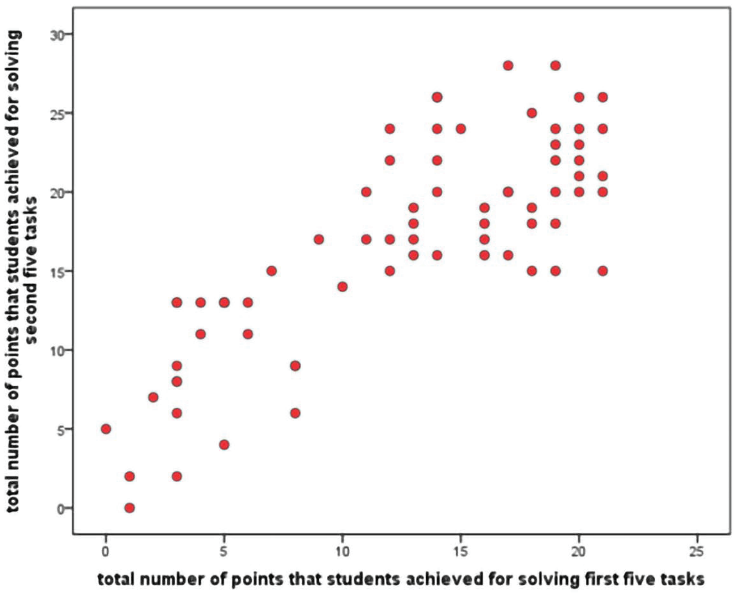

Fig. 4. Correlation between the number of points that students achieved by solving the first five tasks and the number of points achieved by solving the other five tasks on the test

We also wanted to examine the influence of the students' knowledge about the change of variables (with polar, cylindrical, or spherical variables) needed for solving some types of double and triple integrals on the students' success in solving the multiple integrals in which these changes should be introduced. In Figure 5 and Table 4, we can see that there is also significant ($p < 0.0005$) strong and positive ($r = 0.703$) correlation between these two variables, which implies that for the better students' achievement in defining the area in plane or space after introducing change of variables, students achieve better results in solving multiple integral tasks where the mentioned change is necessary.

Table 4. Correlation statistics between the number of points that students achieved by solving the tasks in which they needed to introduce the switch of variables

	Total number of points that students achieved for solving multiple integrals tasks in which they should transform coordinate system after introducing the variables switch	
Total number of points that students achieved for solving tasks in which they should transform coordinate system after introducing the variables switch	Pearson Correlation Coefficient	0.703
	p (2-tailed)	< 0.0005
	N	72

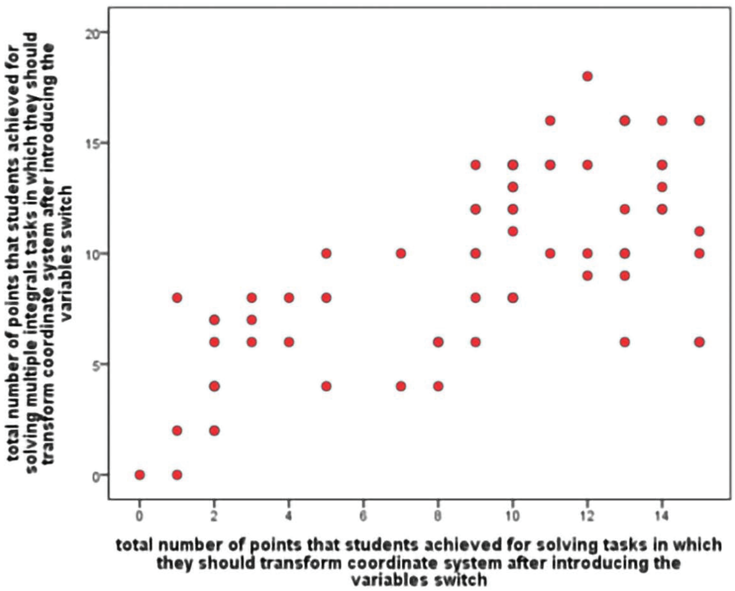

Fig. 5. Correlation between the number of points achieved by students when solving the tasks in which they needed to introduce change of variables

These results imply that as long as students better understand the geometrical property of various 2D and 3D objects and their interrelations they better solve double and triple integral tasks. Moreover, as long as they better understand in which way the change of variables maps the Cartesian coordinate system (Oxy when we introduce the change to polar coordinates or $Oxyz$ when we introduce the change to cylindrical or spherical coordinates) the better they solve multiple integral tasks in which the change of variables should be introduced.

After determining the existence of a significant, strong, positive correlation between students' success in defining sets of points in space or plane and solving multiple integral tasks, we wanted to examine the equality of correlation coefficients calculated for the group of students who used mobile applications for visualization of the aforementioned sets of points and the group of students who did not use mobile applications (who visualize those figures by drawing by hand).

Table 5. Correlation statistics between the number of points that two groups of students achieved by solving the first five tasks and the second five tasks on the test

	Total number of points that students achieved for solving second five tasks			
	Control group		Experimental group	
Total number of points that students achieved for solving first five tasks	Pearson Correlation Coefficient	0.828	Pearson Correlation Coefficient	0.761
	p (2-tailed)	< 0.0005	p (2-tailed)	< 0.0005
	N	37	N	35

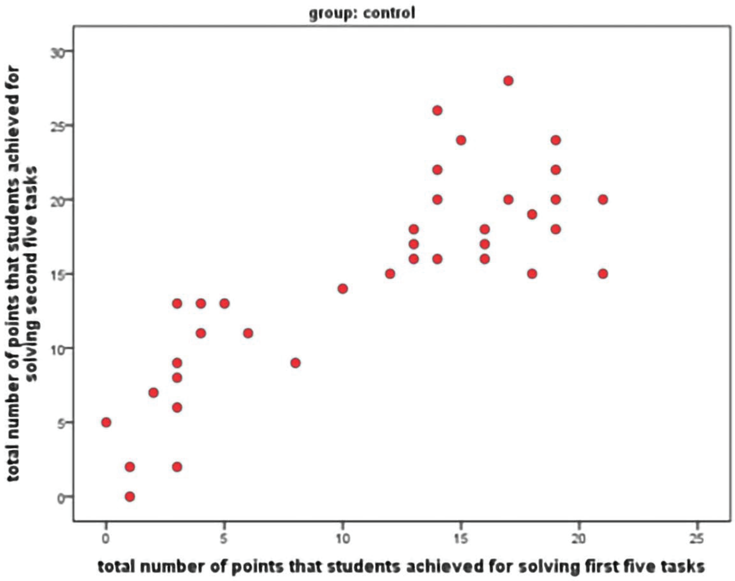

Fig. 6a. Correlation between the number of points that control group students achieved by solving the first five tasks and the number of points achieved by solving the other five tasks on the test

To examine the equality of the correlation coefficients $r_{\text{con}} = r_{\text{exp}}$ we used the z-test to compare the correlation coefficients of two samples. First, we calculate the z values for both samples according to the formulas $z_{\text{con}} = \frac{1}{2}\ln\left(\frac{1+r_{\text{con}}}{1-r_{\text{con}}}\right)$ and $z_{\text{exp}} = \frac{1}{2}\ln\left(\frac{1+r_{\text{exp}}}{1-r_{\text{exp}}}\right)$. So we get $z_{\text{con}} = \frac{1}{2}\ln\left(\frac{1+0.828}{1-0.828}\right) = 1.182$, $z_{\text{exp}} = \frac{1}{2}\ln\left(\frac{1+0.761}{1-0.761}\right) = 0.999$. Further, we have $z = \frac{z_{\text{con}}-z_{\text{exp}}}{\sqrt{\frac{1}{n_{\text{con}}-3}+\frac{1}{n_{\text{exp}}-3}}} = \frac{1.182-0.999}{\sqrt{\frac{1}{37-3}+\frac{1}{35-2}}} = 0.744$. Having in mind that $\alpha = 0.05$, critical area for the given test is $(-\infty, -1.96) \cup (1.96, +\infty)$,

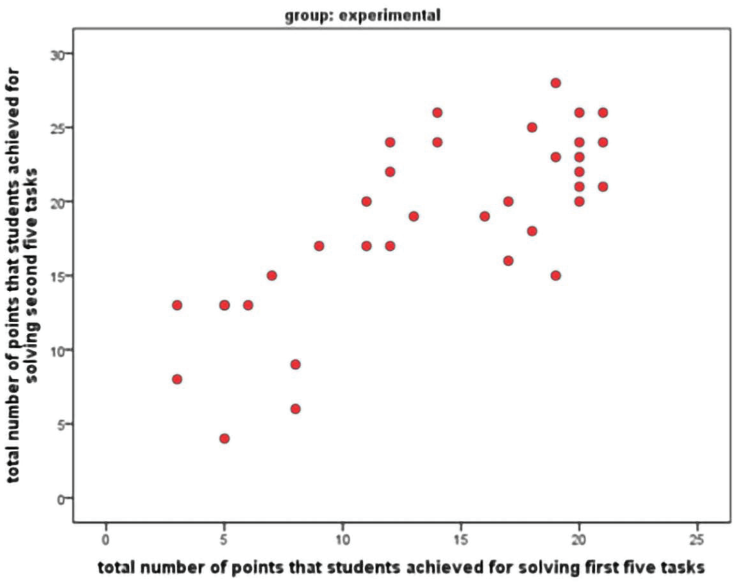

Fig. 6b. Correlation between the number of points that experimental group students achieved by solving the first five tasks and the number of points achieved by solving the other five tasks

and since $-1.96 < 0.744 < 1.96$, we can conclude that we can accept the hypothesis that there is no statistically significant difference between the correlation coefficients between the total number of points achieved on the first five tasks and the total number of points achieved on the second five tasks, in the control and experimental groups of students.

Analogous to the previous procedure, we examined whether there are statistically significant differences between the correlation coefficients of the control and experimental groups of students between the total number of points that students achieved for solving tasks in which they should transform the coordinate system after introducing the change of variables and the total number of points that students achieved for solving multiple integral tasks in which they also should transform the coordinate system after introducing the change of variables.

Using the abovementioned z-tests, we get $z_{\text{con}} = \frac{1}{2}\ln\left(\frac{1+0.660}{1-0.660}\right) = 0.793$, $z_{\text{exp}} = \frac{1}{2}\ln\left(\frac{1+0.705}{1-0.705}\right) = 0.877$ and $z = \frac{0.793-0.877}{\sqrt{\frac{1}{37-3}+\frac{1}{35-2}}} = -0.343$. Since $-1.96 < -0.343 < 1.96$, we can again accept the hypothesis that there is no statistically significant difference between the correlation coefficients in the total number of points achieved on the tasks were students transformed the coordinate system after introducing change of variables and number of points that students achieved while solving multiple integrals where the cahenge of variables and transformation of the coordinate system is done previously.

Table 6. Correlation statistics between the number of points that two groups of students achieved by solving the first five tasks and second five tasks in which students needed to introduce the switch of variables

	Total number of points that students achieved for solving multiple integrals tasks in which they should transform coordinate system after introducing the variables switch			
	Control group		Experimental group	
Total number of points that students achieved for solving tasks in which they should transform coordinate system after introducing the variables switch	Pearson Correlation Coefficient	0.660	Pearson Correlation Coefficient	0.705
	p (2-tailed)	< 0.0005	p (2-tailed)	< 0.0005
	N	37	N	35

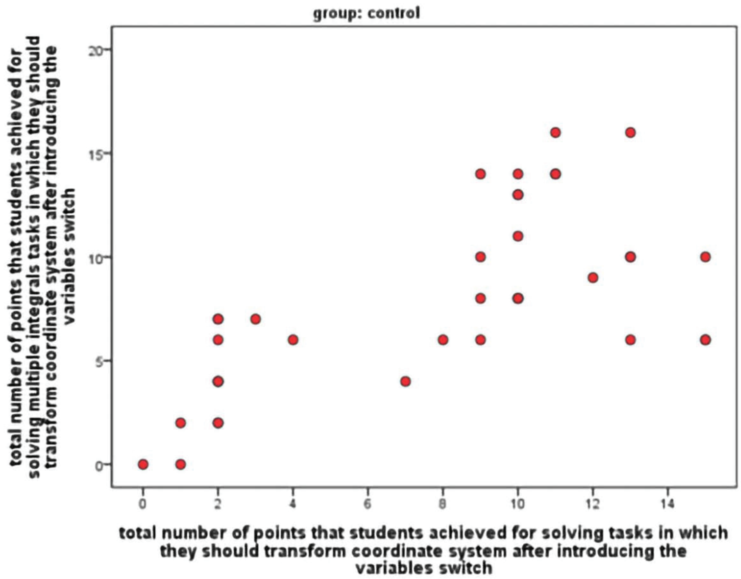

Fig. 7a. Correlation between the number of points that control group students achieved by solving the first five tasks and the number of points achieved by solving the other five tasks in which students needed to introduce the switch of variables

Since there is no significant difference in the linear correlations of students' success (for the two groups of students) in solving tasks in which they have to show knowledge about determining sets of points defined with some mathematical objects and setting boundaries for variables and theoretical and practical knowledge about solving multiple integral tasks, we can confirm that as long as we better teach students to visualize and to know, understand and analyze some concepts from analytical geometry (in plane or in space), by using different methods and

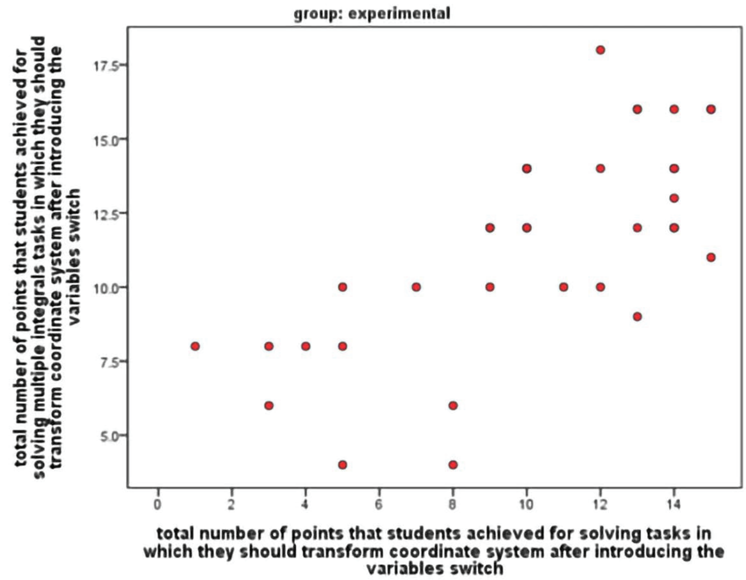

Fig. 7b. Correlation between the number of points that experimental group of students achieved by solving the first five tasks and the number of points achieved by solving the other five tasks in which students needed to introduce the switch of variables

methodological approaches, and as long as they better acquire the proper knowledge, the better results they will achieve while solving multiple integrals.

Conclusion

After analyzing students' success in determining parts of space, and/or plane defined by some lines, planes, surfaces etc. and students' success in solving double and triple integral problems, we can conclude that there is a strong connection between these students' achievements. It is shown that as better the knowledge and skills students have in determining the sets of points specified with some objects in plane and determining sets of points which belong to the body bounded with planes and surfaces in space, the better results they achieve when they solve double and triple integral tasks. This is a very significant result because it highlights the importance for the teacher to be sure that students can visualize and understand in which way the integration domain is defined and how to define bounds for the variables in the concrete task. Only when the teacher is quite sure that students have achieved an appropriate level of reasoning and understanding, they can go further into the calculation process of multiple integrals.

The other, also quite significant conclusion of our research is that there is no statistically significant difference between the correlation coefficients for two groups of students (the students who used mobile applications for the visualisation of the integration area and the students who visualized those areas using chalk and blackboard, i.e., pencils and notebooks) when it comes to their success in determining sets of points defined by some mathematical objects and using changes of variables to redefine those sets of points (which should be significant in order to calculate multiple integrals) and students' success in solving concrete multiple integral tasks. As mentioned earlier, there have been some studies to determine methodological approaches that influence positively on the students' outcomes in determining the domain for multiple integrals and to set boundaries for variables in order to calculate the multiple integral task. Having that in mind, this result is quite significant because, based on the results of this research, the better students understand the geometric properties of lines, curves (in the plane), and planes and surfaces (in space), as well as how the Cartesian coordinate system is mapped in the plane (by using polar coordinates), i.e., how the Cartesian coordinate system in space is mapped (by using cylindrical or spherical coordinates), the better results they have in solving multiple integral tasks. These results provide us with feedback and give us an impuls to further investigate how the visualization of mathematical concepts and their mutual relations could be further accelerated (by meaningful use of technology, which can be used in different ways, and also perhaps by creating teaching aids and manipulatives that could help us enable students to create appropriate visual representations). Some further research could be carried out in that direction.

Appendix

Test

1. Determine the area of domain D in the coordinate plane xOy if it is bounded by the line $x = 2$, parabola $y = x^2$ and hyperbola $xy = 1$.
2. Represent the set of points $D = \{(x, y) \mid 0 \leq x \leq 2, x^2 \leq y \leq 2x\}$ in plane in the form $D' = \{(x, y) \mid \alpha(y) \leq x \leq \beta(y), c \leq y \leq d\}$, so that variable y is the independent one, and variable x dependent one.
3. Determine the set of ordered pairs (φ, ψ) that are (after the appropriate change of variables) mapped onto the subset D in coordinate plane xOy:
   a) $D = \{(x, y) \mid 1 \leq x^2 + y^2 \leq 12\}$;
   b) $D = \{(x, y) \mid 2x \leq x^2 + y^2 \leq 4\}$.
4. After the appropriate change of variables, determine:
   a) ordered triplet (ρ, φ, z) that corresponds to the point in space $M(1, -1, 4)$;
   b) set of ordered triplets (ρ, φ, z) that are mapped onto the 3D area V bounded by the cone $z = \sqrt{x^2 + y^2}$, cylinder $x^2 + y^2 = 16$ and plane $z = 0$.
5. Determine the set of ordered triplets (ρ, φ, θ) that is mapped onto the 3D area V bounded by the sets of points given by equations $z = \sqrt{16 - x^2 - y^2}$ and $z = \sqrt{25 - x^2 - y^2}$, after using change to spherical coordinates.
6. Calculate $\iint_D x \, dx \, dy$ if the domain D is determined by the x-axis, line $x = e$ and the graph of real function $y = \ln x$.

7. Calculate $\iint_D \cos(x^2+y^2)\,dx\,dy$ if the domain D is determined by the central circle with radius 3, for $y>0$.
8. Calculate the volume of the body determined by the planes $x=0$, $x=6$, $y=0$, $y=4$, $z=0$ and the graph of function $f(x,y)=xy$.
9. Calculate $\iiint_V e^z\,dx\,dy\,dz$ if the domain V is bounded by the sets of points defined by the equations $z=1+x^2+y^2$ and $x^2+y^2=5$.
10. Calculate $\iiint_V (x^2+y^2+z^2)\,dx\,dy\,dz$ if the domain V is bounded by the central sphere with radius 5.

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