THE TEACHING OF MATHEMATICS

2021, Vol. XXIV, 2, pp. 55–75

INVESTIGATING STUDENT KNOWLEDGE IN MATHEMATICS AT THE END OF THEIR FIRST YEAR OF POST-PRIMARY EDUCATION IN IRELAND: A CASE STUDY

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Abstract. This paper reports on performance data obtained as an integral part of a major research study on the effect of the transition from primary to post-primary education on Irish students' mathematics performance [V. Ryan, *Making the Transition: A Students Mathematical Journey from Primary to Post-Primary School in Ireland*, PhD. University of Limerick]. The data were obtained using a standardised test designed for use in Irish primary schools to assess student mathematical knowledge at the end of their first year of post-primary school. The results highlight significant underperformance across all curriculum strands of the official mathematics curriculum indicating an unsatisfactory grasp of mathematical knowledge at this stage of students' development. The item analysis also shows statistically significant differences in performance between male and female students. These findings are concerning in the context of major recent reforms of the national post-primary mathematics curriculum and targeted measures to avoid negative outcomes in mathematics as students make the transition from primary to post-primary education. In research terms, the end of year 1 in post-primary education is a neglected milestone in students' mathematics education that straddles a key period in students' mathematical journey when students negotiate academic and emotional transitions that affect their performance, attitudes and dispositions towards mathematics. This paper gives a recent appraisal of the mathematical knowledge of students entering year 2 of post-primary education in Ireland. It discusses factors affecting these student outcomes in mathematics in the context of the students' transition from primary to post-primary school/mathematics. Finally, it sheds light on a neglected mathematics milestone for students, and its potential to influence future student outcomes in mathematics.

MathEduc Subject Classification: D63

AMS Subject Classification: 97D60

Key words and phrases: Transition in mathematics; Mathematics milestone; Mathematical achievement.

1. Introduction

The convergence of government education, economic and competitiveness policies internationally has a significant influence on mathematics and STEM education. Therefore, it matters that mathematics/mathematics education is an integral part of government economic strategy and is regarded as underpinning all STEM disciplines.

International experience shows the transfer to post-primary school is often not successful for many students creating both short and long-term consequences [51]. It is acknowledged that the academic achievement of students in mathematics during and after the transition from primary to post-primary education is a cause for concern [1, 3, 4, 11, 21, 22] but nevertheless, is extremely important for students individually and as citizens, as they pursue their educational and career trajectories. Research suggests that the transition in question negatively affects students achievement in mathematics and their attitudes towards mathematics as exhibited in underperformance and lack of engagement as they continue their educational journeys [22, 29].

1.1. Issues and opportunities around academic transition

The transition from primary to post-primary education assumes added significance because it straddles a period in students' mathematical journey when negative attitudes and dispositions towards mathematics are formed which are known to be extremely difficult to change later, thus closing down career options for students and reducing the talent pool for the STEM economy. It follows, therefore, that the congruence of education and economic policy imperatives warrants a sharper focus on this important education transition, not least because of policies designed to benefit students and their mathematics education, and the economy. Therefore, it is an opportune time to intervene in students' mathematical education. It is important to act expeditiously because there is a significant opportunity cost associated with delay and inaction.

This transition is worthy of attention because successful transition to post-primary education has a direct impact on the mathematical future of students. Eccles et al. [18] found that declines in achievement, motivation and students' self-concept of ability are more evident in mathematics than any other subject following the transition to middle school in their study based on 12 school districts in the U.S. They reported that girls are more susceptible to these negative effects than boys are. In a later U.S. study, Neild, Stoner-Eby, and Furstenberg [32] show that ninth grade outcomes are major predictors of dropout, and they argue that the transition to high school is a time when a student's educational trajectory can be reshaped. Nowadays, it is an important educational goal for students that they make the transition successfully because the consequences have a significant impact on students' academic trajectory [15, 19]. It is a given that first year of post-primary education establishes a base for mathematical advancement but it is now clear that it is also a major opportunity for educators to influence their students' mathematics trajectory in a positive way. Therefore, there is an opportunity to tackle the negative impact of the transition by completely avoiding or mitigating extreme consequences. Recent studies confirm and extend our knowledge and understanding of the mathematics transition and facilitators and inhibitors of success [4, 11].

By recognising year-end attainment in mathematics in Junior Cycle (lower secondary education) as a significant milestone in students' mathematics development, we can bring a sharper focus to bear on the effect of this transition on mathematical performance and potential improvements to facilitate better onward progress. While all post-primary schools in Ireland measure student performance across the curriculum at year-end, including mathematics, there is a dearth of research on students' mathematical knowledge and attainment at these progression points (neither TIMSS nor PISA collect data for year 1). Thus, the perspective developed in this paper has the potential to offer a significant knowledge base for teachers to improve student outcomes in mathematics as they progress through post primary education. A critical look at the implemented mathematics programme in year 1 (in this case, the Common Introductory Course (CIC) [31] in first year of secondary education) may point to inhibitors to progress, if they exist.

1.2. Purpose and contribution of paper

The item analysis central to this study is one pillar of a larger doctoral study on student transition in mathematics from primary to post-primary education in Ireland [38]. Unexpected results from the main study highlighted significant underperformance in mathematics at the end of the first year in post-primary education compared to their performance at the end of primary education, and greater underperformance than that reported in other international studies. The quantitative analysis showed that on average, students' raw scores decreased by 7% from sixth class to first year despite an additional year of instruction. The results indicate statistically significant losses in each curriculum content strand area and in each process skill, and statistically significant female underperformance compared to male students [38].

These surprising results warrant further attention as no obvious explanations are available from existing research data. While *The National Strategy to Improve Literacy and Numeracy among Children and Young people 2011–2020*, PISA and TIMSS testing highlight the importance of the transition from primary to post-primary education, they do not specifically examine student knowledge following transition, at the end of their first year of post-primary education. TIMSS testing in second year and PISA testing for students aged 15 do not show the impact of transition on student mathematical performance. However, these studies do confirm that regression in post-primary mathematics is an issue and point to the problematic nature of the transition from primary to post-primary mathematics as a contributory factor [19].

Consequently, the authors reasoned that a closer look at the actual performance as measured by the test instrument (Level 5 SIGMA-T standardised test) at the end of year 1, item by item, would yield important insights. Therefore, the focus in this paper is on the test data generated as part of the main study, using it to develop an understanding of the mathematics knowledge of students entering their second year of post-primary education in the study's 14 post-primary schools.

A previous Irish study on transition measured computation scores in September and May (beginning and end) of first year and was carried out 17 years ago [42]. The educational landscape has changed extensively since Smyth's study with the introduction of a new post-primary mathematics curriculum. The main study [38] referred to above presented an opportunity for an item analysis that uses a large representative sample, and represents the first evidence-based assessment of student mathematical knowledge at the end of their first year of post-primary education. This analysis assumes added significance because it relies on data collected after the introduction of the new mathematics curriculum, and in particular, the CIC [31] in Junior Cycle (lower secondary education). The CIC [31] is a bridging course designed to improve the link between primary and post-primary mathematics and ease the transition. The authors use a standardised test (Level 5 SIGMA-T) to measure student performance in mathematics for a sample of 301 students. This data is analysed for all students grouped by syllabus sections/strands (Number, Measures, Shape and Space, Data and Algebra), process skills (Concepts and Facts, Computation and Word Problems), and by gender.

While transitions in mathematics education are being treated more comprehensively [24], a gap remains in the international research that quantifies the impact of this particular transition on mathematics achievement. Research suggests that this is attributable to difficulties associated with finding valid measures of mathematical achievement that reflect the first-year post-primary curriculum [11]. The data in this paper was generated as part of a major transition study in Ireland that deals with those issues by quantifying the impact of the transition on students' achievement in mathematics using a fit-for-purpose measure of achievement that relates specifically to the first-year post-primary mathematics curriculum as taught. Thus, this data offers an opportunity to explore an extra hitherto unexplored dimension of a specific focus on students' mathematical progress in year 1. Further, the authors examine the role of the teacher and pedagogical and curriculum continuity across the transition in the context of the quality of teaching experienced by students, as sub-optimal teaching has been identified as a possible contributory factor requiring further research [11].

The paper's unique contribution is that it brings a research focus to an important and hitherto under-researched milestone in Irish post primary students' mathematics journey including an important academic transition from primary to post-primary education. The authors offer insights based on an analysis of students' mathematical knowledge at this stage in their education, all with a view to improving student outcomes in mathematics. A fine-grained analysis of the mathematics transition from primary to post-primary education focussing on students' performance at the end of their first year in post-primary education sits well with international research in this area [11]. Such an investigation is warranted because the mathematics transition is important in its own right and is recognised internationally as such because of the intrinsic value of mathematics, and its importance as an underpinning discipline for all STEM disciplines and others besides. In addition, it is widely acknowledged that this academic transition is not negotiated well by many students as international research shows and is marked by underperformance and poor dispositions towards mathematics with potentially lifelong consequences for students. Despite negative outcomes in mathematics for students, there is optimism that a better knowledge and understanding of this challenging transition and its impact on students will lead to successful interventions and better outcomes for students.

Consequently, this work is important in the Irish context but is no less important for colleagues in the international community of mathematics education researchers for the reasons enunciated.

2. Background and context for the research

2.1 Brief sketch of the Irish education context

The Irish school system is highly centralised and comprises primary and post-primary schools. Virtually all schools are state-funded and follow the relevant centrally devised curriculum. The period of compulsory education is from 6–16 years but many children start school c. 5 years of age, and after a period of 8 years transfer to post-primary school aged 12–13 years.

Post-primary education is organised academically and administered as a compulsory Junior Cycle (3 years) followed by a Senior Cycle (2 years) where curriculum subjects are offered at Higher and Ordinary levels. A non-academic sixth year, called Transition Year, may be taken by students at the end of Junior Cycle.

While mathematics is not compulsory, virtually all students (90%+) study mathematics throughout their school careers up to and including Leaving Certificate mathematics. This is largely because a satisfactory performance in mathematics in the Leaving Certificate examination is an entry requirement for many degree and other programmes in the Higher Education sector.

2.2 Mathematics education context

The school mathematics landscape in Ireland has changed dramatically in recent years because of a major ongoing government reform agenda in education [15]. A new primary school mathematics curriculum introduced in 1999 is currently under review. The primary mathematics curriculum is organised in five strands: Number, Measure, Shape and Space, Algebra, Data. Each strand is associated with year-appropriate outcomes for Irish children [14]. A new post-primary mathematics curriculum known locally as Project Maths was introduced in 2010. This new curriculum aims to develop greater student understanding of mathematical concepts, and their problem solving abilities in real contexts [12]. The new curriculum content is organised in five content strands: Number, Measures, Algebra, Shape and Space, and Data. This content organisation sits well on top of the primary school mathematics curriculum that is similarly organised in similar content strands. In addition, the curriculum design includes a common yearlong bridging programme, the CIC [31], to address content continuity issues in the transition from primary to post-primary mathematics. A major national programme of professional development for existing post-primary mathematics teachers accompanied the mathematics reform.

3. Relevant literature

Key factors identified in the literature review as impacting negatively on academic performance in mathematics in the transition include the role of the teacher, pedagogical and curriculum continuity (or rather discontinuity), the ‘fresh start’ approach and quantum of instruction time [38]. For example, Attard’s study of Australian students found that student engagement in mathematics is based on positive teacher-student and student-student relationships in the middle years [3].

The quality of mathematics teaching is also an important factor in successful transition [13]. In addition, the teacher and quality of their teaching affects student attitude towards mathematics, which ultimately affects their engagement with the subject in both the short and long-term [22, 29]. Bicknell and Riley [8] reported that problems arise around curriculum continuity when there is a 'fresh start' approach. The authors discuss these factors later when they revisit the research aims in the context of this investigation and findings. In addition, the authors introduce a gender perspective on student performance at the end of their first year of post-primary school.

4. Research design

In this paper, the authors examine data for 301 students who sat the Level 5 SIGMA-T standardised test as part of the main transition study. They analyse the data with a view to learning more about the mathematical knowledge of these students at this point in their mathematics education, and explore what this might tell us about the mathematical knowledge of Irish students in general at this milestone.

The research aims are to:

- Investigate the mathematical knowledge of students at the end of year 1,
- Examine the data for gender differences,
- Identify potential inhibitors to progress in year 1 mathematics.

4.1 Sample

The sampling design used for this study was modelled on the sampling design used for PISA assessment [34]. The sampling frame for the main transition study was the official list of 723-post-primary schools in Ireland, which accounted for 367,178 students and approximated to 61,196 first year students starting post-primary school in September 2015 [38]. Schools were stratified by school type, namely: secondary, vocational, community, comprehensive, and this division formed new sub-frames for the sampling. The projected sample size for the main study was 382 students, assuming a population of 61,196 students, a 95% confidence level and a 5% margin of error.

The initial sample consisted of 20 schools and 11 agreed to take part in the study. 9 replacement schools were then selected and 4 of these replacement schools agreed to participate. Subsequently, one of the original schools selected refused to participate. Finally, 14 selected schools agreed to participate. The researcher selected participating schools using probability proportional to size systematic sampling. Replacement schools were selected when the initial 20 schools were identified. All 14 schools had multiple first year mathematics classes. A simple random sample of first year mathematics classes in a chosen school was used to select a class, and

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[1] Secondary schools are privately owned and managed. Vocational schools are governed by the state through Education and Training Boards (ETBs) while Boards of Management manage community and comprehensive schools

all students from the selected class were included in the main sample. This process resulted in a final sample size of 323 students. Due to absenteeism on the day, 301 students sat the test (Level 5 SIGMA-T) at the end of first year. The mean age of these 301 students on the day of testing was 12.54 years. Ethical approval was granted by the appropriate ethics committee at the University of Limerick (Code: 2015_09_01_S&E).

4.2 Research instrument

The Level 5 SIGMA-T test is the research instrument used to generate the results reported in this paper. The SIGMA-T is a standardized mathematical attainment test purposely designed for use within the Irish school system. Form A and Form B represent parallel forms of the Level 5 SIGMA-T test and are used to minimize the possibility of copying.

This test measures student achievement in number, measurement, geometry, elementary algebra and data and statistics, and is mostly based on the curriculum from the final two years of primary school. These topics feature in the post-primary Junior Cycle CIC [31] but are subsumed under easily recognisable strand headings used for the post-primary mathematics curriculum. It is important to note that the primary mathematics curriculum is bridged to the Junior Cycle curriculum through the CIC [31].

Each of the questions from the Level 5 SIGMA-T assesses a process skill and strand area for the primary mathematics curriculum, and similarly for the post-primary mathematics CIC (NCCA 2016) in this investigation. The validity of this approach is discussed in the appropriate section below. The 119 test questions also require students to perform several mathematical procedures and solve word problems related to the content studied [52]. Table 1 summarises how the distribution of the 119 questions by strand area and process skill.

Table 1. Number and percentages of questions in Level 5 SIGMA-T by strand and process skill

Strand	Understanding concepts and recalling facts	Performing computations and procedures	Solving word problems	Total
Number	21 (17.7%)	17 (14.3%)	9 (7.6%)	47 (39.5%)
Measures	5 (4.2%)	9 (7.6%)	22 (18.5%)	36 (30.3%)
Shape and space	11 (9.2%)	2 (1.7%)	0 (0%)	13 (10.9%)
Algebra	2 (1.7%)	4 (3.7%)	0 (0%)	6 (5.0%)
Data	6 (5.0%)	8 (6.7%)	3 (2.5%)	17 (14.3%)
Total	45 (37.8%)	40 (33.6%)	34 (28.6%)	119 (100%)

While each of the process skills are assessed, there is not an even distribution with more attention given to understanding concepts and recalling facts (37.82%)

and less attention given to solving word problems (28.57%). Given the focus the new curriculum places on problem solving, it is important that a considerable part of the assessment test (almost 30%) focus is on this skill. The test assesses each of the strands, but the SIGMA-T places a major emphasis on the Number and Measure strands. Number and Measures constitutes 69.75% of the questions on the SIGMA-T (Table 1). Shape and Space only accounts for 10.92% of questions, Algebra accounts for only 5.04% of questions and Data accounts for 14.29% of questions.

Level 5 SIGMA-T gives the following scores: raw score, standard score, percentile rank and STEN score. The raw score is the number of questions answered correctly from the total number of questions in the test. Standard scores, percentiles and STEN scores are derived from raw scores. A STEN score (Table 2) is a score from 1-10 that compares a student's result to that of the standardised sample [30]. The authors use caution and cite relevant limitations when they analyse the results reported in this paper, and inferences drawn in this context.

Table 2. STEN Scores

STEN score	What does STEN score mean?	Proportion of children with this score
8–10	"well above average"	1/6
7	"high average"	1/6
5–6	"average"	1/3
4	"low average"	1/6
1–3	"well below average"	1/6

Source: [30]

4.2.1 Test marking. The author (VR) manually corrected all test items for all students. It is worth noting that all testing was carried out without the use of a calculator. One mark was awarded for a correct answer and no marks were awarded for a partial or incorrect answer. One test from every 10 marked, was randomly selected and re-marked in order to counter human error in the marking of the attainment tests. If the combined results of the re-marked scripts showed average errors of more than 5 marks in individual tests, all scripts were re-marked. Accuracy was promoted by using a system of running totals on each page of the attainment test and checking the total on the last page with the overall total of the script.

4.2.2 Validity of research instrument. The Level 5 SIGMA-T test was used with first year students in this study because the learning outcomes for the target groups are a very good match, and the test was purposefully designed for use in Irish schools. 82% of the learning outcomes of the sixth class mathematics curriculum are repeated in the first year CIC [31]. This correspondence was established by the construction of a curriculum map to check the level of repetition between the sixth class and first year learning outcomes [38]. First year students in post-primary education follow the CIC [31], which was introduced under the new mathematics curriculum to link the primary mathematics curriculum to the Junior Cycle (year 1-3 of second level) mathematics curriculum. The SIGMA-T was standardised using a nationally representative sample of over 13,000 students, and it allows for comparisons to be made within schools and also nationally [52].

5. Results

The results of the first-year item analysis are considered in detail in terms of the basic mathematical knowledge and skills students have at the end of their first year of post-primary education. 301 first-year students sat the test on the day producing 35,819 individual test items for marking distributed across the mathematics curriculum. The test items were marked and analysed by strand area and process skill. Gender-related data were analysed and included to give a more rounded view of the findings.

5.1 Key indicators of performance

Raw data is presented in Table 3.

Table 3. Questions answered by Strand Area and Process Skills – all students

	No. of questions		No. of correct answers		% of correct answers		% of incorrect answers		
	Form A	Form B	Form A	Form B	Form A	Form B	Form A	Form B	Overall
Strand Area									
Number	6862	7285	4277	4401	62%	60%	38%	40%	39%
Measures	5256	5580	2408	2612	46%	47%	54%	53%	54%
Shape & Space	1898	2015	938	1073	49%	53%	51%	47%	49%
Algebra	876	930	548	594	63%	64%	37%	36%	37%
Data	2482	2635	1674	1699	67%	64%	33%	36%	34%
Process Skills									
Concepts & Facts	6570	6975	4246	4465	65%	64%	35%	36%	36%
Computation	5840	6200	3240	3462	55%	56%	45%	44%	44%
Word problems	4964	5270	2359	2452	48%	47%	52%	53%	53%

54% of questions on Measure and 49% of the questions on Shape and Space were answered incorrectly. Over one third of the questions on each of the other 3 strands were answered incorrectly. Similarly, 53% of Word problems, 44% of Computation questions and 36% of Concepts and facts questions were answered incorrectly. In only one Strand area (Measures), and one Process skill (Word problems) were scores above 50% recorded, and notably these were the highest scores achieved.

5.2 Analysis by gender

The independent samples t-test is used to assess if there is a statistically significant difference between male and female student performance at the end of first year. Raw score, Strand Areas and Process Skills are each analysed and Levene's Test shows homogeneity of variances for all comparisons. A student's raw score is the number of questions the student answered correctly from 119 test questions. Students' raw scores are summarised by gender in Table 4.

Table 4. Raw score by gender (summary)

	Gender	N	Mean	Std. deviation	Std. error mean
Raw score	Female	122	62.56	20.31	1.84
	Male	179	70.35	20.30	1.52

There is a statistically significant difference in raw scores of female students (M = 62.56, SD = 20.31) and male students (M = 70.35, SD = 20.30), $t(299) = -3.27$, $p < .001$ (two-tailed) and on average, male students' raw scores are 7.79% higher than female students' raw scores.

The analysis by syllabus section (strand) and process skills further underscore female underperformance. The category Process skills includes separate scores for Concepts and facts, Computation and Word problems and these are summarised in Table 6. Strand area scores are summarised in Table 5.

Table 5. Strand Areas scores by gender (summary)

	Gender	N	Mean	Std. deviation	Std. error mean
Number	Female	122	59.57	19.80	1.79
	Male	179	67.11	19.85	1.48
Measures	Female	122	42.39	17.42	1.58
	Male	179	49.02	18.01	1.35
Shape and space	Female	122	48.39	21.65	1.96
	Male	179	53.43	19.87	1.49
Algebra	Female	122	56.57	22.21	2.01
	Male	179	67.74	21.20	1.58
Data	Female	122	63.34	18.05	1.63
	Male	179	67.65	18.66	1.39

There is a statistically significant difference in the all strand area scores between female students and male students with male students scoring higher in all strand areas (Table 5). The highest difference occurs in the Algebra strand (11.16%) with the Data strand showing the least difference (4.30%).

Process skills includes scores for 3 sub-categories, Concepts and Facts, Computation, Word Problems. A summary of students' Process Skills scores are presented in Table 6.

Table 6. Process Skills by gender (summary)

	Gender	N	Mean	Std. deviation	Std. error mean
Concepts and facts	Female	122	59.45	18.92	1.71
	Male	179	67.65	18.05	1.35
Computation	Female	122	52.56	17.85	1.62
	Male	179	58.19	17.58	1.31
Word problems	Female	122	43.73	17.44	1.58
	Male	179	49.21	19.31	1.44

There is a statistically significant difference in the Concepts and Facts scores between female students (M = 59.45, SD = 18.92) and male students (M = 67.65, SD = 18.05), t(299) = −3.80, p < .001 (two-tailed). The mean difference in raw scores is −8.20 with a 95% confidence interval ranging from −12.46 to −3.95. On average, male students' scores are 8.20% higher than female students' raw scores. There is a statistically significant difference in the Computation scores between female students (M = 52.56, SD = 17.85) and male students (M = 58.19, SD = 17.58), t(299) = −2.71, p < .007 (two-tailed). The mean difference in raw scores is −5.63 with a 95% confidence interval ranging from −9.72 to −1.55. On average, male students' scores are 5.63% higher than female students' raw scores. There is a statistically significant difference in the Word Problems scores between female students (M = 43.73, SD = 17.44) and male students (M = 49.21, SD = 19.31), t(299) = −2.51, $p < .013$ (two-tailed). The mean difference in raw scores is −5.48 with a 95% confidence interval ranging from −9.77 to −1.19. On average, male students' scores are 5.48% higher than female students' raw scores.

6. Discussion

The data presented here document in detail the mathematical performance of a sample of first-year students at the end of year 1 of their post-primary education in Ireland. The analysis sheds light on the state of their mathematical knowledge and competence at this time. However, the authors recognise that mathematical performance is but one indicator, albeit an important one, of the state of students' mathematical knowledge at any time that is shaped by many other factors. While school factors obviously affect the transition from primary to post-primary mathematics and the nature of the mathematical performance recorded in response to the test, they were not specifically included in the research focus of this study. However, the authors are concerned with inhibitors to success in negotiating the transition, and a number of obstacles and facilitators are identified in the literature, in particular, in the cited mathematics studies relevant to this work. The authors address a small number of obstacles that they consider to have a significant bearing on the findings reported here based on the literature review and their experience.

6.1 Mathematical knowledge of students at the end of first year

It is difficult to associate a once-off test result with the level of mathematical knowledge and command of that knowledge students' actually have. However, in this instance the authors' believe it is possible to paint a plausible picture for the group as a whole using a broad brush strokes' approach. A number of factors are relevant here. We have argued that the taught curriculum (CIC [31]) matches very well the primary mathematics curriculum on which the test instrument is validated (82% of the learning outcomes are repeated in the CIC [31], and there is a strong content match via strand areas). The test addresses all five strands of the taught curriculum (but not equally) and related process skills, and the test is a repeat test for all students in the sample (in a different version). Returning to the task in hand, we use the test performance as one important element of the mathematical profile of these students as a group, since end-of-year assessments are widely used as indicators of readiness and preparation for the subsequent year's work in mathematics. As we have established, this group/sample shows consistent underperformance in the respective domains across all strand areas and process skills, and a pronounced gender disparity in performance favouring boys.

Consequently, we use a number of indicators to sketch a picture of the mathematical knowledge of first-year post-primary students at the end of their first year and as they enter their second year of study in post-primary education. These include the Level 5 SIGMA-T test, performance data on the test, official measures e.g. STEN scores, official learning outcomes, relevant policy statements and targets, and some international comparisons.

6.1.1 Basic mathematical knowledge and skills. The item analysis highlighted significant numbers of students lacking basic mathematical skills that are necessary for personal progression in mathematics, and for functioning in their personal lives and within society. Basic questions such as subtraction of a three-digit number from a four-digit number highlighted that for many students, the level of mathematics the CIC [31] demands of them is beyond them. Overall, the analysis shows consistent underperformance by first year students who have had a further year of instruction, on a test designed for students in sixth class. The item analysis across the strand areas and process skills shows consistent poor performance in the respective domains. Taken together as a measure of academic performance in content and process skills, the data confirm significant underperformance in mathematics of first year students in post-primary education, and in particular, female students. Almost 3 out of every 10 students are considered low average or well below average using the STEN scores categories and descriptors.

The level of mathematical knowledge indicated by the findings has a direct impact on future outcomes in mathematics for underperforming students, and a significant bearing on numeracy achievement, which is a key national educational goal [15, 16]. Many of these findings would not have been evident if students had completed the test with a calculator. While we recognise the movement towards increased use of the calculator, there is still an argument for learning certain foundational skills without the use of a calculator. The item analysis is consistent with PISA 2015 test results which found that 15% of Irish students (15 year olds) are lower-performing students who have inadequate mathematical skills to apply mathematics to real life situations or be in a position to benefit from future learning opportunities [39].

6.1.2 Indicators of mathematical knowledge. The following section gives results relating to specific questions on the test. These indicators were chosen as examples of performance in strand areas and process skills, and are used to shed light on the state of student' mathematical knowledge at the time. 8% of students were not able to draw a time on a clock face while 26% of respondents could not write the time a digital watch image provided. 36% of students could not convert from kilograms to grams. 16% of students could not subtract a three-digit number from another three-digit number while 19% of students could not subtract a three-digit number from a four-digit number. Similarly, 22% of students could not do an elementary short division calculation. 41% of students could not solve a basic fraction word problem. 44% of students were unable to draw a line at a right angle to another given line. 23% of students did not know the number of sides of a specific shape. 28% of students did not know that the angles in a triangle sum to 180 degrees. 47% of students could not get the perimeter of a rectangle given its length and width. 51% of students could not calculate how many 50 ml measuring jugs would be needed in order to fill a 1.5 litre bottle. 60% of students could not add two mixed fractions. Almost a quarter of those examined failed to find the correct answer for a word problem involving subtracting two three-digit numbers. 85% of students failed to find the correct answer for a problem that involved calculating a percentage and fraction of a total number of people. 37% of students could not write a mixed fraction in decimal form. Such results are concerning in the context of national aspirations and policies promoting improved mathematics outcomes in post-primary mathematics for the whole cohort of students.

However, they may not be very compelling if viewed as isolated data unrelated to a wider context. The authors found it instructional to look at corresponding learning outcomes for mathematics at this stage in other countries. A detailed matching exercise tabulating corresponding learning outcomes was undertaken comparing the Irish CIC [31], the relevant outcomes for the National Mathematics Curriculum in England [16], and Australian Mathematics Curriculum (Year 7) [5]. They were found to be broadly similar. An indicative sample for two topic/strand areas is given in Table 7.

As a comparison of outcomes across a number of countries shows Irish expectations in terms of published official learning outcomes are not out of line with expectations in the international arena.

Table 7. Sample of indicative learning outcomes

Number	**Percentage**	**Data/Statistics & Probability**
Common Introductory Course, Ireland	• calculate percentages • use the equivalence of fractions, decimals and percentage to compare proportions	• recognise that probability is a measure on a scale 0–1 of how likely an even is to occur • explore different ways of collecting data
National Maths Curriculum England	• define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages greater than 100% • interpret fractions and percentages as operators	• record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness fairness, equally and unequally likely outcomes, using appropriate language and the 0–1 probability scale • understand that the probabilities of all possible outcomes sum to 1
Australian Maths Curriculum Year 7	• Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157) • Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies (ACMNA158)	• Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168) • Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169) • Construct sample spaces for single-step experiments with equally likely outcomes (ACMSP167)

6.2 Gender perspective on the test results

The first-year data provides clear evidence of a gender gap in performance favouring boys. There are statistically significant gender differences in raw scores, strand areas and process skills. Considering the female under-engagement in the STEM workforce in Ireland [46] the differences seen here affect gender parity in

STEM professions in the future. The results show that the gender gap in mathematics is already firmly established in first year post-primary education in Ireland. These findings are reflected in consistent female underperformance at Leaving Certificate. Higher level when successive cohorts of students' progress through post-primary education. For example, the Leaving Certificate 2015 data shows a higher percentage of male (28.9%) than female (26.0%) candidates sitting Higher level mathematics. Males have outperformed female students in achieving A grades and A/B/C grades in 2015 and this pattern has been consistent since the new mathematics curriculum was introduced [44]. Results from 2019 indicate more males than females obtaining higher grades (H1 and H2 grades respectively) [45].

Gender disparity evident in first year continues throughout post-primary education. PISA 2015 data has shown that the gender gap, on average, is more pronounced in Ireland than across OECD countries [39]. However, female underperformance is a complex issue and teacher training alone is not enough to address the issue. Gender equality must be promoted in the home, school and society as a whole as research tells us that female underperformance in mathematics is eliminated in cultures that are more gender-equal [25]. The transition from primary to post primary education is a pivotal point for female students and it is important that the stereotype of mathematics as a male domain is challenged, while students are making the transition. It is important that teachers be aware of how they treat both genders in the classroom because student self-beliefs, achievement and participation in mathematics are affected [27]. On a positive note, Spencer, Steele, and Quinn [43] found that reducing stereotype threat increased female mathematics performance and decreased anxiety in female students.

6.3 Inhibitors of successful transition in mathematics

6.3.1 Role of the teacher. The authors now return to discuss key inhibitors of successful transition identified earlier in Section 3 and localise them in the Irish context. This study was undertaken during a period when a new post-primary mathematics curriculum was being implemented in Irish schools. In terms of teaching quality, a contemporary report by Rordin and Hannigan [37] found that 48% of teachers teaching mathematics in Irish post-primary schools did not hold a mathematics teaching qualification, and in the main, these teachers were deployed in non-examination classes and confined to Junior Cycle. These teachers were referred to as out-of-field teachers of mathematics in the report following this designation in [13].

These findings speak directly to (a) the capacity of this same teaching force to deliver the new curriculum as envisaged including the CIC [31], (b) quality of mathematics education experienced by Junior Cycle students. Both issues would have a direct bearing on whether or not a successful transition would be achieved. In the event, the authorities addressed both issues comprehensively in the rollout of the new curriculum. The DES and the NCCA implemented the most comprehensive CPD programme in the history of the state for any subject, and funded a part-time Professional Diploma in Mathematics for Teaching (PDMT) to upskill out-of-field teachers of mathematics starting in 2012 and continuing [23].

Initial reports by the National Foundation for Educational Research (NFER) however, signalled that the new curriculum had not yet been associated with any improvement in achievement [26]. Gains appear to be hard to achieve still at Junior Cycle [40], but a reasonable expectation is that there will be improved outcomes when the new curriculum is well established. While necessary steps to improve teaching quality were taken, these steps alone were not sufficient to ensure a successful transition as authors' findings show.

6.3.2 Curriculum continuity. Pedagogical and curriculum continuity are identified in the research as necessary conditions for successful transition from primary to post-primary mathematics [8]. The authors acknowledge overlap between the two named constructs but first they focus on issues related to curriculum continuity and some pedagogical issues emerge.

The new post-primary mathematics curriculum in Ireland was engineered to address continuity issues in a number of ways. The mathematics content is organised in 5 key cognate strands Number, Measures, Algebra, Shape and Space, and Data, that subsume and develop primary curriculum strands. Thus, a curriculum map with learning outcomes exists that recognises students' prior learning and provides mathematics content horizons for students and teachers. The CIC [31] is deliberately embedded in Year 1 of the new mathematics curriculum to facilitate the successful transition from primary to post-primary mathematics. Viewed as a bridging framework it is intended as a tool to combat wide variations in mathematical knowledge and competency of first-year students who arrive in post-primary school from different feeder primary schools (the 14 schools in this study were served by 109 feeder schools). Taken together, these steps represent an important contribution to subject continuity across the transition but clearly the central component, the CIC [31], it is not functioning as intended [40].

The research literature also points to the importance of the quantum of instruction time allocated to mathematics classes. The TIMSS studies and other international studies such as PISA have shown a positive correlation between academic performance and instruction time [41]. In Ireland, students in sixth class spend 50-60 minutes on mathematics per day and experience a significant reduction in instruction time when they transfer to post-primary schools. These issues have been investigated by Smyth, McCoy, and Darmody [42], McCoy, Smyth, and Banks [28], and recently highlighted by O'Meara and Prendergast [33]. A diminution of class time devoted to mathematics immediately when students transition to post-primary school represents a discontinuity that affects curriculum and pedagogy. While sufficient mathematics instruction time in mathematics is important, O'Meara and Prendergast make the point that more instruction time alone will not suffice if used inefficiently.

6.3.3 Pedagogical continuity. Academic discontinuity is evident in this 'fresh start' approach where post-primary teachers re-teach much of the curriculum from fifth and sixth class in primary school. While continuity is a key factor in successful transition, continuity for first-year students based on a repetition of prior learning only, is unlikely to succeed. The importance of student exposure to challenge in mathematics is acknowledged by international research [2, 48, 49], and it is well known that student motivation and learning in mathematics can be improved by a moderate level of challenge [9, 47]. The 'fresh start' approach applied to first-year students which disregards prior learning has been shown to have negative consequences for student engagement and learning [7, 21]. A high level of repetition is particularly detrimental to students as it offers no challenge to a large number of students and affects their motivation, commitment and interest in mathematics [17, 50]. Repetition represents a flat or decreased level of challenge for students. Research tells us this affects engagement, interest, commitment, motivation and the development of reasoning processes [6, 10, 17, 20, 35, 48, 50]. Insufficient challenge occasioned through this repetition affects the mathematical trajectory of students through post-primary education and beyond. Other researchers point out that the distrust inherent in the 'fresh start' approach impedes smooth transition, and halts academic progress for students of all abilities and affects attitude towards mathematics [8, 22].

The introduction of the CIC [31] is a very positive step to support students in the transition from primary to post-primary mathematics. However, its efficacy is undermined by a common response in post-primary schools. In order to 'level-up' variations in mathematical knowledge and competency of new first-year students, post-primary mathematics teachers implement a fresh start' approach in first-year mathematics classes. This level of repetition, particularly when there is not sufficient additional challenge, undermines mathematical learning because it fails to exploit students' prior learning, and fails to engage and motivate students. Consequently, the authors believe that the 'fresh start' approach is a significant contributory factor in the lack of efficacy of the CIC [31]. Other contributory factors to academic discontinuity include lack of knowledge of their respective school mathematics curricula by primary and post-primary teachers. Prendergast et al. [36] found that approximately half of primary sixth class teachers post-primary teachers reported being either highly or slightly unfamiliar with each other's syllabi.

6.3.4 Limitations of the study. We acknowledge limitations to this study. The main transition study dictated the characteristics of the sample and the nature of data collected. The target sample size of 382 to ensure 95% confidence interval and a 5% margin of error was not achieved due to reluctance of post-primary schools to participate. Instead, 301 students completed the test instrument. Two further points are worth noting in the context of this paper. The sample is not a sample of individual first-year students chosen individually, but rather 301 out of 342 students from 14 first-year class groups from the participating schools. The researchers had no control over the gender balance that emerged in schools included in the sample, and consequently the proportion of boys and girls among the 301 students who sat the test.

Finally, cooperating class teachers administered the test following specific written instructions supplied by the researcher (VR). However, there is no way of knowing if these instructions were strictly adhered to in practice.

7. Summary and conclusions

This paper focuses on the mathematical knowledge of students at the end of year 1 in post-primary school in Ireland. It identifies a significant underperformance in mathematics and discusses factors affecting these student outcomes in mathematics in the context of the students' transition from primary to post-primary school/mathematics. The authors argue that end of year 1 in post-primary education is a significant milestone in students' mathematical education that is under-researched and offer one approach to improve matters in this regard. Academic transition in mathematics at this interface, and new insights particularly in the context of on-going curriculum reform are matters of interest internationally, and since transition happens at local level, studies such as this are important in the international debate.

The underperformance identified in this study is considered in the broader context of mathematical knowledge, skills and competencies that are foundational for school and life careers. The results highlight the possibility that students are missing an opportunity to develop some basic mathematical skills including estimation and may be over-reliant on calculators for doing simple calculations. The data warrant further observations and consideration in this regard. Undoubtedly, a majority of the students will recover from this situation and go on to do well in school mathematics, but the weaker students are unlikely to do so. It is also probable that among those who do recover, that for significant numbers of them, their recovery will fall short of what they might have achieved had their transition been more successful, and this represents an opportunity cost for them and the nation.

Looking to the future, issues remain with first year Junior Cycle mathematics. It was expected that the CIC (NCCA 2016) in mathematics would address anticipated difficulties with the transition. This has not been entirely successful as the authors' study demonstrates. Shiel and Kelleher (2017) give an excellent appraisal of the issues surrounding the CIC [31] and its implementation. In our view, the CIC [31] provides a very good mathematical framework for teachers to address the academic transition in mathematics successfully. The CIC [31] provides adequate content, and scope for selected additional content to meet the challenge but the kernel of the issue is the mathematical pedagogy employed by the teachers. To our minds, a failure to fully embrace the CIC [31] represents a lack of confidence in, and knowledge of, the mathematical outcomes of the primary school curriculum, and a reluctance to move away from traditional approaches. To improve continuity, consideration ought to be given to matching mathematics instruction time in first year of post-primary education with that of sixth class, and assigning the most experienced and qualified teachers to teach first-year classes/students. These steps are necessary to ensure that the transition is negotiated well by all students. Research has shown that the best mathematics teachers are not being deployed in first year across all schools [37]. There are some grounds for optimism in the knowledge that significant numbers of newly qualified out-of-field teachers of mathematics (c.17% of the entire mathematics teaching force in post-primary schools), who qualified through the Department of Education and Skills upskilling programme, are now deployed, mostly in the Junior Cycle classes. It is reasonable to expect that the contribution of a large cohort of qualified mathematics teachers in the early years of post-primary education, who were previously out-of-field teachers of mathematics, will lead to better outcomes in the future.

ACKNOWLEDGEMENT.

We are grateful to the learned referee of this paper whose suggestions helped to improve it.

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