THE TEACHING OF MATHEMATICS
2021, Vol. XXIV, 1, pp. 12–35

# PROBABILITY AND STATISTICS — TEACHING DRIVEN BY REPRESENTATION

**Jasmina V. Milinković and Ivica Radovanović**

**Abstract.** The newest reform of the educational system in Serbia brings forward the introduction of elements of probability and statistics in the elementary school curriculum. The paper explores possibilities for finding an optimal approach for developing basic probability and statistics concepts. Our research aimed to document the effects of alternative teaching methods of instructions based on the representational teaching approach, anchored to problems' representation (enactive, iconic, or symbolic). On a sample of 392 pupils, ages 11 to 14, three variants of the experimental program were compared using a nonparametric statistical procedure. The findings indicate that the iconic representational approach had better effects with younger pupils while the symbolic representational approach was the most effective representational method for higher grades of elementary school. A major implication of the study is that the representational teaching approach can be adjusted to the grade of pupils.

*MathEduc Subject Classification*: C73, K73

*AMS Subject Classification*: 97K60, 97K70

*Key words and phrases*: Representation; probabilistic reasoning; statistics reasoning; teaching method; elementary school.

## Introduction

The way teacher teaches determines the way pupils learn. Indeed, the teaching approach is one of the key factors affecting learning in a formal educational setting. But how exactly we decide what teaching method to use? Researchers are searching the alternatives strongly influenced by general trends in education and reform movements, such as New Math in 1950' or Constructivist Teaching Experiment.

One of the significant developments in mathematics education is the recognition of statistical literacy as an important part of mathematical literacy. Probability and statistics are domains of mathematics whose methods can be applied in different areas, from social sciences to medicine to engineering which produces the call for introducing these topics in the school mathematics curriculum [26]. We explore teaching methods in the domain of probability and statistics with a not-so-long presence in the Serbian school curriculum. Despite the strong interest of the educational community in introducing probability and statistics in schools, there is no consensus about when and how pupils should start to learn about reasoning under uncertainty [8, 10, 29, 31]. Here, we attempt to determine when should learning startup be and what could be the most beneficial method at a particular grade level. Green's large-scale research as well as some others showed that pupils age 11 to 16 develop intuitions about non-deterministic phenomena, but this intuition includes various misconceptions which become obstacles to the learning of formal theory [18–20]. It seems plausible to think that it would be great if formal learning on probability could start before pupils develop misleading intuition, but not earlier than the time when they are cognitively ready to deal with uncertainty.

## Bruner's theory of representations

Acknowledging the importance of using various external representations in process of learning is a widely accepted idea [4, 11]. Along the line, Hiebert and Carpenter [22, p. 67] argue that "a mathematical idea or procedure or fact is understood if it is part of an internal network". Representations are considered to be the final (internal) product of processing and coding information from the outside world [7]. Understanding increases with building up connections and widening the network as a learner encounters novel external representations. Bruner argued that no matter how complicated concepts were, they could be successfully grasped at a level that would be adequate for the child's capacities and experience. Generated by the dual theory of knowledge presented through action and abstraction, he identified 'images' as an alternative way of presenting external information. Accordingly, there are three ways in which people represent the world: (1) enactive (action or operation), (2) iconic (pictorial), and (3) symbolic. Each representation influences processes of constructing and deepening knowledge. Initially, Bruner's view was that the order in which these types of representations are formed is set orderly from enactive, across iconic to a symbolic one. Later, he realized, that this order is not a consequence of developmental progress in the selection of the representations, but, above all, is culturally conditioned behavior.

Let us consider three modes of representation in more detail. The enactive representation involves an instrumental operation on a structure with a distinct aim and selected means. So, it is not about random acts that lead to knowledge but rather planned exploration leading to knowledge. The basis of knowledge is a procedure, i.e., a physical action which is carried out to come to a conclusion or solve a problem. Another type of representation, the so-called iconic (or pictorial), shows "the selective organized structure of perceived phenomenon via transformed images" [7]. An object is replaced with an image that represents a relatively faithful through selective representation. Bruner stressed that the image is not only able to catch the uniqueness of events and objects, but also a source of prototype for the event category. It becomes a benchmark for the comparison of potential examples which could be included in that category [7].

Finally, the symbolic representation involves the use of abstract forms and symbols which do not themselves reflect "directly" represented object or phenomenon. The symbolic representation implies removal from concrete and arbitrariness. The symbol does not reflect directly the meaning of the represented object nor visually reminds of what it signifies. Symbols are created by people and their meaning is assigned by agreement. Using mathematical symbols is the beginning of pupils acquainting formal mathematics language. Enactive and iconic representations are predominantly universal, whereas symbolic representation is principally the result of cultural competence and entails a conscious effort to grasp it. This type of competence is acquired dominantly through formal education and other forms of adopting culture.

Bruner's theory has influenced the way mathematics education researchers explain mathematical knowledge. Consequently, teachers are encouraged to use enactive and iconic representations in instructions across grade levels. The importance of enactive representations is particularly emphasized in the initial learning of mathematics concepts. For example, the research of Beythe-Marom and Dekel [5], indicates positive effects of the action-based approach in solving realistic problems, related to probabilistic reasoning.

When speaking about representations in mathematics teaching, we need to acknowledge that most often educators stress the importance of making connections between different representations of a concept. It is certainly interesting to examine ways for improving the recognition of connections. Our research proposal acknowledges that it is necessary to establish an understanding of each particular representation to comprehend relations among them. There is research evidence that it is not always easy to move from one representation to another. Lesh observed children's difficulty in going from one to another type of representation (image-symbols-words) in solving classical "pizza problems" (discussed in [25]). He pointed out that children spontaneously use iconic representation as to the most appropriate for this type of problem. Yet, iconic representation does not facilitate manipulation to express the dynamics of a situation. Therefore, children are compelled to move to symbolic representation without ever using enactive representation.

If we consider learning abstract mathematical concepts as a process of formalization, we are directed to a presumption about the order of representations that we attempt to introduce [16]. Accordingly, enactive representation would be considered to be the most promising for introducing concepts, being "the least abstract", hence, cognitively the easiest to grasp. Curriculum documents worldwide as well as the dominant research community promote an implicit belief that young children would benefit more from active experimenting (for example, conducting surveys or playing probability games) and computer simulations than from learning ready-made formulas [??, 9, 46, 52]. The ultimate value of experimenting is intuitively and officially accepted in contemporary elementary school mathematics. Nonetheless, we still need evidence to confirm that this is the most fruitful method across grade levels. For us, one major issue is whether it is possible to determine the most appropriate form of representation for introducing a specific math concept at a given grade level. So, we asked ourselves could we find empirical evidence for the benefits of the teaching process substantially grounded in one specific representation. Moreover, we wondered whether the advantages of using a particular representation are linked to the grade level of learners. We focused on the three forms of representations in the context of developing judgment under uncertainty in middle school pupils.

### Research on teaching probability and statistics

In educational practice, there had been a lack of tradition of teaching elements of probability and statistics. Researchers identify probability, randomness, and uncertainty as some of the most difficult to grasp mathematics concepts [31, 42]. Stanovich [51] names probabilistic reasoning as Achilles heels of humans' cognition. Garfield and Ahlgreen noticed that inadequacy in prerequisite mathematics knowledge and abstract reasoning hinder learning about stochastic processes [18]. Yet, some researchers opt for a more optimistic stance. Jones, Langrall, and Mooney [28] are declaring randomness as a key idea in elementary school, while for the middle school they are suggesting the importance of the concept of sample space, empirical estimations, and theoretical probability, and even consideration of compound and independent events. Conversely, some more optimistic findings point that children are prone to build misconceptions related to judgment under uncertainty [??, 30]. A strong rise in interest to incorporate elements of probability and statistics in schools in the last decades of the 20th century happened in parallel to the rise in research efforts in this area [13, 14, 28]. This issue has been pushed forward and investigated seriously, there has been no conclusion about the effectiveness of early start in this area. There are remarkable overviews of research in probability and statistics [??, 8, 28, ??]. Nowadays a large scope of research in this field encompasses different topics such as studying motivation to learn, studying effects of learning probability and statistics, etc. [8, 10, 29, 33, 40, 48, 56, 57, 58]. Also, statistical literacy became a visible and significant part of mathematics literacy in curriculum documents worldwide (Australia, Canada, United Kingdom, USA). Despite the interest of the educational community, only sporadic research has been done on teaching probability and statistics in a formal school setting (e.g., [15, 19–21]. Out of these efforts, only a few studies focused on teaching middle school children or younger even though probability and statistics are formally introduced in elementary school (e.g., [12, 33, 42, 43].

### Development of probabilistic reasoning

When speaking about learning about the uncertainty we cannot skip reflecting on theories of the development of the concept of chance. The results of Piaget and Inhelder's research about the development of the concept of chance [44] fit within Piaget's theory of development. Accordingly, up to age 7 or 8 children cannot differ possible from a certain event. In the second period, concrete operational stage (from 7 or 8 to 11 years) although children can recognize "chance event" and unpredictability of individual event, they still do not understand the relationship between the sets of all outcomes and favorable outcomes for the observed event. Also, they are not capable of systematic analysis of situations. In the third, formal operational stage, from 11 years comes the ability to synthesis the concept of chance and deductive reasoning. Numerous researches are exploring different aspects of learning stochastic reasoning [17, 24, 27, 39, 41, 47]. Tarr and Jones [54] identified four levels of thinking about probability and independence among middle school children following the neo Piagetian view. The first level is characterized by subjective judgments and belief in control of outcomes of an event, whereas the next is described as between subjective and informal quantitative judgments. The third is described as the one when children can use quantitative facts to make judgments with limitations in probability conclusions as they are easily confused by irrelevant features. Finally, pupils on level IV can use numerical reasoning to express probability. These findings point to limitations in middle school pupils' reasoning about uncertainty. As levels cannot precisely be linked to a particular age, the right grade to start instructions about uncertainty is yet to be defined.

A large number of studies examined the effects of different approaches in teaching probability in informal after-school settings or laboratory conditions [15]. As alternatives for researching the school, researchers opt for lab context or after-class informal programs. For example, Abrahamson [1] conducted a study using task-based clinical interviews which utilized an inventive random generator with a research focus on reasoning about random compound events through a tutorial. The results of the studies indicate that the right time to begin an exploration of topics related to non-deterministic phenomena is the 3rd, 4th and upper grades of elementary school.

There is no doubt that laboratory research or after-school experimental programs contribute to the understanding of the development of reasoning of individuals. Often researchers use computers and other technical media to simulate uncertain processes or graphical display of results of stochastic events. Participants in the studies may run, analyze, and even modify experiments [1, 24, 33, 35, 45, 53]. For example, Kazak and Konold [35] developed a simulation tool for the data exploration software Tinker Plots to study middle school pupils' understanding of chance and distribution. Kapon and others [33] on a middle school pupils' combinatorial reasoning developed in activity-centered units in the out-of-classroom setting. Without a doubt, school context is a significant and unavoidable factor informal education. It includes multiple aspects, such as are limitations in time allocation, space, resources, specific interactions, distractors, etc. These factors were not examined in prior research studies.

Recent studies related to the understanding concept of probability are focused on different aspects of understanding such are reasoning in "random draw" situations, such as flipping coins, tossing dice or spinning spinners (Jones et al, 2005), or comparing probabilities of different events [14, 42]. In some of them, researchers investigated the age of learners as a factor, in others, they examine the context of a task. Interestingly, while some researchers concluded that young pupils (age 11) have difficulties in solving probability tasks [??], others found that young children may successfully solve probability tasks [34]. Yet, Watson and colleagues claim, based on multiple research findings, that "beliefs about chance are difficult to document consistently, difficult to change with instruction, and resistant to change over long periods." [56]. Indeed, the abstractness and complexity of concepts related to probability provide inspiration and numerous possibilities for studying. But, the abstractness and complexity of concepts bring also a challenge to find the best way for transforming this matter for children in middle school or younger. Lack of uniform, or at least, a similar approach in school systems around the world, is, to a certain extent, a consequence of the absence of unified response to a question "What is the most fruitful way to teach a subject matter?".

In practice, teachers use different approaches based on their pedagogical intuition or experience in teaching concepts from other fields of mathematics. Meanwhile, expectations are that research findings are going to identify the most promising teaching method(s) for influencing the development of reasoning about uncertainty [3, 31, 49, 50]. Jones acknowledged changing trends in instructional theory which recognize the importance of connecting probability and statistics and appreciating computers as powerful tools. Anastasiadou and Chadjipantelis [2]investigated the role of different modes of representations (graphical, algebraic, and verbal) on the understanding concept of probability among pre-service teachers. These researchers found that students tend to avoid graphical representations and to use symbols instead. They also found that students who can coordinate multiple representations show better results in problem-solving.

Now is the right time to return to Bruner's representation theory. The theory implies that a representational form of problems with a particular level of abstraction (enactive, iconic, and symbolic) could serve as an anchor model when teaching particular content (e.g., concept of chance, randomness, etc.). As we remarked earlier, this idea is somewhat in contrast to the prevailing opinion that simultaneous usage of different representations helps to achieve a comprehensive understanding of a concept. There is a common-sense understanding that simultaneous usage of different representations requires time and effort to establish relationships between those representations. We consider an alternative of using a teaching approach focused on a particular form of representation over a condensed time. In this case, it is important to make a rational choice "what is the most appropriate representation to use at the start of learning a particular concept".

Should the choice of representation depend on the grade level of pupils? According to the prevailing view of the educational community, the answer seems obvious – instructions should start from enactive forms, using manipulative and physical exploration and experiments when it comes to younger grades of elementary school. Further, in higher grades of elementary school pupils most often should deal with symbolic representations. For example, pupils in middle school will learn how to calculate with fractions but will have few, if any, opportunities to manipulate with fraction models, nor will they see many pictorial representations (with exception of the number line). Because of that, we reasoned that it would be logical to expect the better achievement of pupils in 6th or 7th grade if new concepts are introduced using an 'iconic' or 'symbolic' teaching approach rather than an 'enactive' representational approach based on manipulation and physical exploration. Therefore, comes one of our research questions: "Would older pupils (6th or 7th grade) find representations which are of a higher level of abstractness more conveying than the others?"

## Methodology

### Research questions

The study addresses the following research questions:

(1) What is the grade (encompassing age, i.e., mental maturity of pupils and prior knowledge base) is the most suitable for the introduction of the concept of chance and related concepts?

(2) What is the effect of different approaches to selected content (having in mind three types of representations)?

(3) What are the effects of choosing different 'representational' methods at each grade?

We defined three teaching methods based on particular representation: action approach, iconic approach, and symbolic approach. Our sub hypotheses are based on earlier theoretical and empirical constructs of Bruner, Piaget, Lesh discussed in earlier sections of this paper. Primarily, we hypothesized that less abstract forms of representations would be more suitable for younger pupils. We took into account strong advocacy for the benefits of enactive representations discussed earlier, which we called the 'enactive' method. With the same line of reasoning, we expected that the 'iconic' method, being less abstract, would be more suitable for older pupils than the 'symbolic', method but less beneficial than the 'enactive' method for younger grades.

### Hypotheses

The main hypothesis is: An elected representation can assure a successful start in exploring basic elements of probability and statistics in elementary school. Sub hypothesis are 1) 'enactive method brings better effect in work with 11 to 12-year-old pupils (IV and V grade); 2) 'symbolic method brings better effect in work with 13 to 14-year-old pupils (VI and VII grade); 3.1) 'iconic method brings better effect than the 'symbolic method in working with 11 to 12-year-old pupils (IV and V grade); 3.2) 'iconic' method brings better effect than the 'enactive' in the work with 13 to 14-year-old pupils (VI and VII grade). The first sub hypothesis is based on positions discussed earlier in the papers [9, 46, 52]. The second is based on cultural expectations related to Bruner's theory as discussed earlier. The last sub hypothesis is based on the previous discussion about Bruner's advocacy of distinctive advantages of iconic representations which visually convey meanings to learners.

## Method

The study took place in regular classrooms in four elementary schools (Regularly, they are modestly, if not poorly, equipped with technology tools) to explore the possibility to learn about probability and statistics in the context defined by a regular classroom setting.

**Sample**

The research sample consisted of 392 pupils, grades 4 to 7 (11 to 14 years old). The study was conducted in regular public schools and respected the regular composition of classes. The number of pupils in the classes who were present at all experimental lessons varied from 18 to 31 (Fluctuation was due to flu season). The schools involved in the experiment were located downtown, a short distance away from each other. They were selected based on the principle of similarity of pupils' population, to eliminate factors of education or lifestyle that may affect the intuitive understanding of statistical concepts of interest to us. The range of average marks in mathematics in grade 4 was from 4.2 to 4.42, in grade 5, 3 to 4.3 in grade 6 from 3 to 4, and in grade 7 from 2.5 to 3.6., on a scale from 1 to 5. (The mean score in mathematics had a declining trend with grade level.). Teachers involved in the experimental program were regular teachers assigned by principals of the schools at the beginning of the school year.

Probability and statistics were not part of the elementary school curriculum in Serbia. Thus teaching probability and statistics were new not only for pupils but also for teachers. They relied on lesson plans developed by the researcher. The intention was to secure that teachers in experimental classes implement instructions following the research ideas. The detailed lesson plans contained elaborated variants of activities according to three representational approaches. Each teacher was acquainted with only one 'representational' teaching approach (enactive, iconic, or symbolic). Note, that all math teachers had an opportunity to learn probability and statistics during the undergraduate program as they prepared to become teachers, but neither of them had an opportunity to teach probability and statistics earlier. Lesson plans included not only activities but also protocols for discussions. As a result, teachers felt confident and well prepared for the class. Also, the lessons were tailored and accomplished under the research ideas.

The same researcher (first author) observed all experimental classes, taking field notes about the learning process. The presence of the researcher ensured that teachers stuck to the particular 'representational' method as was projected in the experimental design.

**Experimental program (3 alternatives)**

The study was structured as a $4 \times 4$ factorial design – with four groups (3 experimental, one control group) at four grade levels (grades 4 to 7). We measured pupils' achievements on the test after the experimental program. Teaching sessions in each of the three experimental conditions continued over 4 school days. After that, the final test was conducted. The test had 17 items (Appendix 1). It was composed and balanced to cover content matter addressed in all three experimental conditions. The same test was distributed to the control groups (one control group for each grade level).

For three experimental programs, we designed different lesson plans following teaching methods but set in the same contexts. The following aspects of teaching were common to all classes: a unique group of elementary concepts to be studied;

general educational objectives and educational tasks; most functional tasks; time allocation; introductory activity. We stress that those aspects are (at least in our school system) set by state documents and school regulative. For example, a teacher cannot decide to extend the learning situation and keep teaching math for an extended time (e.g., 2 or 3 class hours instead of 1 class hour) since it is set by the school schedule. Three teaching approaches differed in: teaching methods, scenarios for activities, teaching resources. Short opening discussion on how we judge the likelihood of events was similar in all groups. All other activities in the program were tailored to fit a specific experimental teaching method.

During the experimental program, pupils explored problem situations with an uncertain outcome, described the problems with words and symbols, explored ways of presenting data, interpreted data given in the numerical and graphical form, solved problems. During the experimental program, pupils learned terms that can be used to describe the chance for the realization of an event. They also became familiar with the concepts of 'impossible event', 'certain event', 'equally likely events', fair game, etc. As it could be expected, the 'symbolic' teaching approach leads to the concept of theoretical probability, as it is derived from the analysis of relations between the set of possible outcomes and favorable outcomes set for the given event. The other two experimental groups were led to the empirical (often called statistical or posterior) probability concept.

We provide details of experimental activities.

1. *Activity "Cubes"*

The first of them, the "Cubes" was a game inspired by the context used in the research of Piaget and Inhelder [44]. Note that Freudenthal [16] suggested that the entire instruction regarding the exploration of the concept of chance could rest on one model – a basic context, e.g., "box". He believed that the "what will we get out of the box" issue is an ideal model for a realistic stochastic situation, which is a simple reference, suitable for different types of work as a means which some of the exemplars can explain different concepts. So, in the experimental activity, we used three boxes. In the first box, there was one white and one red cube. The second box contained one red and nine white cubes. In the third box, there were two white and two red cubes. The objective was to determine the chance to get a red cube out of the box without looking. The same request was posed for the second and the third box.

| | |
|---|---|
| The First box | □■ |
| The Second box | □□□□□□□□□■ |
| The Third box | □□■■ |

Fig. 1. Three boxes

In the 'enactive' (A) method pupils in a game-like activity of drawing out a cube from a box to determine the chance to get out a red cube. Pupils worked in groups. Each student pulled out one cube from a box. The result was recorded, the student returned the cube into the box. Then the next student in the group had a turn. The results were written down and summarized. Then each group discussed results and concluded what was the chance to get a red cube from a certain box. They were expected to come to the concept of posterior (empirical) probability. Thus, pupils were cognitively challenged to recognize uncertainty and a different likelihood of the observed event by analysis of the results of physical activity. They compared probabilities for getting a red cube from the first and the second box, etc. We also examined how much are they convinced of what they were saying. The teacher asked them to imagine a situation where they would get a prize if they predict well the color of the cube. Their answers unveiled that for many of them their earlier statements did not match their intuitive beliefs.

In the 'iconic' (I) method pupils were given a graphical illustration of the "Cubes" game. They discussed the results of the game presented to them. The pupils were said that three pupils allegedly had one of the boxes (like the ones used in the already described in 'enactive' teaching method. The data record of 30 trials was presented in a form of a table. The class discussed which one of three pupils have had a better chance to get a red cube out of the box. So, by skipping the phase of experimenting, pupils directly jumped into the analysis of the results of the game presented iconically. Comparable to the enactive method, the teacher hoped that pupils would discover the idea of (posterior) probability.

In the 'symbolic' (S) method pupils discussed the "Cubes" game but only by contemplating what the design of the game has provided for. So they were asked to think about what the ratio of the number of drawn red cube and the total number of cubes in a given box is telling them. In this way, they were directed toward the classical concept of (a priory) probability.

2. *Activity "Favorite comic book hero"*

Here, pupils explored the concept of a survey. They dialed with the following questions: "How can we conduct a survey?", "Why do we do it?", "How can we use results?" and "Can we generalize results?". In this activity, the 'action' classes made a poll about which is their favorite comic book hero. Pupils were asked to choose between four heroes. After the poll, the results were recorded and displayed in tabular and graphical form. The class discussed what can be inferred from these displays. They talked about different ways of presenting data and what they can conclude from them. Then they debated whether the survey results could be generalized as well as how these results could be used. So they were making simple statistical inferences.

In the 'iconic' classes, pupils were asked to analyze graphically presented results of a survey on favorite comic book heroes run by a school magazine. They studied a histogram. The plot for the discussion was similarly contextualized to the problem situation investigated in the 'action' class.

In the 'symbolic' class pupils learned about the results of an imagined school magazine survey about "Favorite comic book hero". In this method, numerical values obtained from the survey were presented to pupils. So the pupils knew how many pupils voted for each of four characters. They discussed what they could conclude from the provided information.

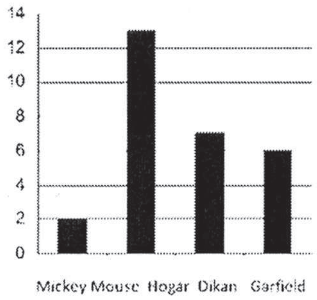


Fig. 2. Histogram "Favorite Comic Hero"

3. *Activity Fair Games: "2 Coins" Game and "3 Coins" Game*

The last two activities in the context of games were devoted to combinatorial games. Since our primary objective was to study the possibility for the initial development of the idea of chance we selected rather simple game contexts. The main task was exploring the notion of fair game and developing the ability to analyze the game situation based on the intuitive idea of equal chance" as a criterion for fairness.

In the first game, they flipped two coins. Player A would win if both coins dropped on the same side (head or tail), in other cases the winner was player B. In the second game, players were flipping three coins, two of 1 dinar and one coin of 2 dinars. Player A would win if the sum of flipped coins was less than 3. In other cases, player B would be the winner. After the introductory conversation, pupils have tried various ways to determine whether the proposed games were fair.

Pupils within the 'enactive' approach were given rules and were playing these games in pairs. Later, they talked about whether each one of the games was fair or not. In other words, they discussed whether both players had an equal chance to win. In the 'iconic' group teacher proposed the same games to be studied using the records of played rounds of games. Based on the analysis of these records, pupils discussed whether each one of the games was fair. Finally, in the 'symbolic' method, pupils analyzed the same games but they were guided to make argumentation based on a theoretical consideration of the possibilities to win considering the number of outcomes favorable for player 1 or player 2. They reasoned whether the games had uncertain outcomes and whether each one of them was fair.

*The final test*

During the last session, pupils took the final test. The test was designed so that the results indicate whether there was a positive shift in their attitude toward the reasoning about phenomena with an uncertain outcome. Among other ideas, we examined whether the pupils understand the idea of chance. Since there were three experimental teaching approaches, the test was structured so that those test items corresponding to the content matter addressed in each experimental group.

The final test had 17 items. To assure reliability and validity of the test, we used multiple questions to assess a particular concept, matching the problems addressed in all instructions to assess the same concept with clear criteria (Appendix 1). The items' formats varied from multiple choices to open-ended questions. About one-third of the test items referred to distinguishing the existence of a phenomenon with uncertain outcomes and understanding the possibility of estimating the probability of an uncertain event. In about one-fourth of the items, we questioned pupils' interpretation of data given in graphical or tabular forms. There were also two problems related to combinatorial reasoning set in realistic contexts similar to ones that were given in the experimental program. Pupils were asked to judge the fairness of rules for taking turns in using a computer; would each pupil have an equal chance to be the first on the computer.

## Results

### Descriptive statistics

First, we present the basic descriptive statistics of student achievements according to the 'representational' teaching method (Control, Enactive, Iconic and Symbolic). Then, we present results of the two-factor Kruskal-Wallis test by ranks, nested design. It is a nonparametric alternative to the two-factor ANOVA, which does not assume a normal distribution [38]. Accordingly, we have made the less stringent assumptions of an identically shaped and scaled distribution of all groups, except for any difference in medians. The null hypothesis was that the medians of all groups were equal and the alternative hypothesis was that at least one population median of one group was different from the population median of at least one other group.

Table 1 summarizes descriptive statistics for groups according to the experimental program.

| Group | Control | Enactive | Iconic | Symbolic |
|---|---|---|---|---|
| N | 98 | 111 | 93 | 90 |
| R | 16 | 19 | 17 | 19 |
| Min | 5 | 8 | 10 | 7 |
| Max | 21 | 27 | 27 | 26 |
| M | 12.8 | 17.96 | 18.42 | 18.91 |
| $m_e$ | 13 | 18 | 19 | 19 |
| $m_o$ | 13 | 20 | 20 | 18 |
| $S^2$ | 11.50 | 15.36 | 12.46 | 11.66 |

Table 1. Group statistics for different teaching approaches

Table 2 displays the mean values for each class surveyed in our study, with the range of the obtained mean values was from 11.52 to 20.11.

| Treatment/Class | Grade 4 | Grade 5 | Grade 6 | Grade 7 |
|---|---|---|---|---|
| Control | 11.52 | 11.6 | 13.52 | 14.68 |
| Enactive | 16.03 | 18.04 | 17.87 | 19.74 |
| Iconic | 18.09 | 17.80 | 18.60 | 19.35 |
| Symbolic | 17 | 18.39 | 19.88 | 20.11 |

Table 2. Mean values of groups on the final test

Looking at Table 2 we can see a considerable difference of achievements in the average of the number of winning points at the final measuring the pupils in the experimental programs (M = 16.03 to M = 20.11). We point to only two more facts observable from Table 2. The first is that the 7th grade 'symbolic group obtained the highest mean value. The second is that the mean values of the 7th grade 'action and the 6th grade 'symbolic group had mean scores above 19.50, close to the highest mean value obtained in all 16 classes.

Statistics by grade are presented in Appendix 2. The observed differences support the expectation that with the age increases the ability of pupils to develop knowledge of the content matter.

**The effects of the 'representational' teaching approaches and grade level**

We examine the effect of two factors: grade (U) and teaching method (M). Grade factor has four levels: 4, 5, and 6, 7. teaching method factor has four levels: K (control), A (enactive), I (iconic), and S (symbolic). The following has been tested using the Kruskal-Wallis method for the ranks, "nested design" (scheme nests): 1) the principal effect of the teaching method, and four nest-effect of experimental teaching method at each age level, 2) the teaching method effect M at U = 4 (grade U), 3) the teaching method effect M at U = 5 (grade 5), 4) the teaching method effect M at U = 6 (grade 6), and 5) the teaching method effect M at U = 7 (grade 7). To preserve the threshold of significance at the same level as in the application of the standard approach, we conducted the two-way dispersion analysis, with $\alpha$ set at $\alpha = 0.15$, $\alpha_M = 0.05$, $\alpha_{M(4)} = .025$, $\alpha_{M(7)} = .025$.

For the main effect M (teaching method), the output tables are presented in Table 3.

| Group | N | Mean Rank |
|---|---|---|
| 1.00 | 98 | 89.25 |
| 2.00 | 111 | 221.18 |
| 3.00 | 93 | 230.35 |
| 4.00 | 90 | 247.87 |
| Total | 392 | |

Table 3. Main effect M – teaching method

Test statistics is $H = (N-1)\eta^2$ where $N$ is the total number, $\eta^2 = \frac{SS_B}{SS_T}$ is the percent of variation explained by Method, $SS_B$ is the sum of squares between groups and $SS_T$ the sum of squares total. The value of $H$ is compared with $\chi^2_{df,1-\alpha}$ [38]. Kruskal Wallis Test Statistic Chi-square is 120.565 with 3 degrees of freedom, asump. sig. .000, table critical value $\alpha_M = 0.05$, $\chi^2_{3,1-.05}$. Note that the asump. sig. value is the significance value that determines whether the difference between the groups is statistically significant or not. If this value is less than 0.05 than we can reject the null hypothesis. As sampling statistics $H$ is equal to $120.565 > 7.81$, we concluded that there is a statistically significant difference between teaching methods. If we look at the mean value rankings, it is evident that the control group notably differs from the other three experimental groups.

For the nest-effect of the experimental teaching method at grade level 4, the output tables are presented in Table 4.

| Method (grade 4) | N | Mean Rank |
|---|---|---|
| Control | 23 | 21.74 |
| Enactive | 29 | 51.47 |
| Iconic | 23 | 64.72 |
| Symbolic | 23 | 59.57 |
| Total | 98 | |

Table 4. The nesting effect M – teaching method at grade 4

Test Statistic Chi-Square for 4th grade is 31.772. The table critical value of $\chi^2$ distribution with three degrees of freedom at the significance level 0.025 is $\chi^2 + 3, 1 - 0 - .025 = 9.35$. As the sample statistic is $31.772 > 9.35$ we reject the hypothesis $H_{0M(4)}$ and conclude that there is a statistically significant difference between teaching methods at the level of the fourth grade. (The mean values of ranks suggest considerably lower achievement of the control group compared to the experimental.) The group with the iconic approach was the most successful, and then the group with symbolic approach, and the least successful was the action method group.

For the nest-effect of the experimental teaching method at grade level 5, the output tables are presented in Table 5.

| Method (grade 5) | N | Mean Rank |
|---|---|---|
| Control | 25 | 19.26 |
| Enactive | 28 | 57.88 |
| Iconic | 25 | 57.86 |
| Symbolic | 18 | 61.53 |
| Total | 96 | |

Table 5. The nesting effect M – teaching method at grade 5

Test Statistic Chi-Square for 5th grade is 37.695. The table critical value of $\chi^2$ distribution with three degrees of freedom at the significance level 0.025 is $\chi^2_{3,1-0.025} = 9.35$. As the sample statistic $37.695 > 9.35$, we reject the hypothesis $H_0$ for M(5) and conclude that there is a statistically significant difference between teaching methods at the level of the fifth grade. According to the mean values of ranks, the control group is remarkably weaker than the other three groups. Among the experimental classes, the most successful was the symbolic group, followed by the action and iconic group which obtained similar values.

For the nest-effect of the experimental teaching method at grade level 6, the output tables are presented in Table 6.

| Method (grade 6) | N | Mean Rank |
|---|---|---|
| Control | 25 | 23.64 |
| Enactive | 23 | 52.22 |
| Iconic | 25 | 55.50 |
| Symbolic | 24 | 65.56 |
| Total | 97 | |

Table 6. The nesting effect M – teaching method at grade 6

Test Statistic Chi-Square for 6th grade is 30.477. The table critical value of $\chi^2$ distribution with three degrees of freedom at the significance level 0.025 is $\chi^2_{3,1-0.025} = 9.35$. As the sampling statistic $30.477 > 9.35$, we reject the hypothesis H0 for M(6) and conclude that there is no statistically significant difference between the three experimental teaching methods at the level of the sixth grade. Note that the control group is considerably weaker than the group with experimental classes. Also, we note that the symbolic group performed substantially better than the other two groups of experimental classes (which have approximately the same mean values of ranks).

For the nest-effect of the experimental teaching method at grade level 7, the output tables are presented in Table 7.

| Method (grade 7) | N | Mean Rank |
|---|---|---|
| Control | 25 | 22.14 |
| Enactive | 31 | 60.53 |
| Iconic | 20 | 56.55 |
| Symbolic | 25 | 63.60 |
| Total | 101 | |

Table 7. The nesting effect M – teaching method at grade 7

Test Statistic Chi-Square for 7th grade is 33.216. The table critical value of $\chi^2$ distribution with three degrees of freedom at the significance level 0.025 is

$\chi^2_{3,1-0.025} = 9.35$. As $33.216 > 9.35$, we reject the hypothesis $H_{0M(7)}$ and conclude that there is a statistically significant difference between the effect of teaching methods at the level of seventh grade. Note that the analysis showed that the symbolic group achieved the best results.

Overall, we conclude that there is a statistically significant difference among the four teaching methods. Also, we find that there is a statistically significant difference between the four teaching methods at each grade level.

## Discussion

Our findings are following the findings of Tarr and Jones [54] that middle school pupils can use numerical reasoning to express probability if obtaining appropriate instructions. The beginning could happen at grade five but a successful start can be expected if the concepts are introduced in sixth or seventh grade. The teaching approach should direct pupils to think about the concept of chance in a particular way, either by developing the idea of experimental (posterior) probability or by developing theoretical (a priory) probability. Teacher's choice of representational teaching method thus can effectively change the way how pupils understand chance.

While Kazak and Confrey's study [36] and Abrahamsons laboratory experimental research [1] results suggested advantages of early start at grade three, our findings indicate that later start may be more beneficial which is following findings of Borovnik and Benc [??]. The results signify that middle school pupils achieve more with the 'iconic' or 'symbolic' teaching method than with the 'enactive' teaching method. Therefore, if introducing probability in 4th grade, the 'iconic' teaching method would be the most beneficial. Starting from 5th grade, the 'symbolic' teaching method should be dominant. Notice, that typically, in many school systems worldwide, orientation toward the 'symbolic' method is domineering starting from grade 5. Yet, we believe that the 'enactive' method should not be entirely overlooked, as it brings valuable educational experience, linking theoretical knowledge to practical applications.

The "classical" definition of probability is more easily perceived among middle-grade students than the "experimental" definition of probability, based on the observed frequency of generated events. The results of our study encourage us to believe that 6th-grade pupils are ready to understand the concept of chance.

Before the end of this report, we need to address the limitations of our study. A limitation comes from the research design. We followed the position that educational recommendations for school teaching need to be based on realistic expectations and realistic contextual conditions of in-school research. Consequently, we had to adjust our research plans to be acceptable for school principals and teachers as they had very limited freedom to incorporate extracurricular activities in the school curriculum. We also had to use less powerful, nonparametric statistical procedures. But this research design reassures that the conclusions of the research are relevant for regular school conditions.

## Conclusions

In this paper, we have dealt with theoretical issues and practical implications of the 'representational' approach in teaching probability and statistics in elementary school.

The findings indicate that instructions on statistical reasoning may start as early as in the fourth grade. The results show that regardless of the approach taken in the fourth-grade and fifth-grade classrooms, pupils at these grades would have more difficulty grasping elementary ideas related to reasoning under uncertainty. With younger age, we can expect to achieve the best results if implementing the 'iconic' method. Faster progress can be expected if instructions are postponed to the sixth or seventh grade. The analysis confirms a significant difference between groups in grades six and seven.

The statistical analysis shows a significant difference between the four experimental groups. At all grade levels 'iconic' method produced better (or equal to) results in comparison with the 'enactive' method. Starting from the fifth grade, the 'symbolic' teaching method has better effects on the development of reasoning about chance events than the other two approaches. The results show that sixth-grade pupils are ready to understand the concept of chance.

A general implication for mathematics educators is that teachers should consider adjusting the choice of the 'representational' method (enactive, iconic, or symbolic) to the age of pupils. This finding should be explored further in other domains.

## Appendix 1

### Final test

### Chance on test

*In this test, you will answer questions about events that may or may not materialize.*

*In many problems, you are expected to circle the letter in front of the correct answer. You may find yourself thinking that there are two correct answers, but we ask you to decide which one you think is more "accurate". In the rest of the questions, please write on the line in block letters.*

**Part I**

1. Imagine that you are drawing out one card from a deck of cards (with 26 red and 26 black cards) with closed eyes. What do you think is going to happen?

   a) I will get a red card.

   b) I will get a black card.

   c) I cannot predict.

2. Finish the sentence to make a true statement expressing the probability that the described event is going to happen.

   ________________________ tomorrow is going to be a foggy morning.

3. Finish the sentence to make a true statement expressing the probability that the described event is going to happen

   ________________________ I am going to meet a penguin on the road.

4. Pick a statement that describes the most accurate chance that the following event will materialize.

   a) Perhaps I'll meet Vlade Divac.

   b) Chances to see Vlade Divac are small.

   c) There is no chance to meet Vlade Divac.

5. The next two questions are related to the following two sentences.

   – There is a small chance that I am going to get a bike in a lottery game.

   – I will certainly fall to the floor if I jump off the chair.

   The above sentences describe different events. What is expressed by them?

   a) Those sentences express the degree of possibility for something to happen.

   b) The above sentences show the degree of belief that something can happen.

   c) Those sentences state the conditions under which something can happen.

6. How do these sentences differ?

   a) One event is true while the other is false.

   b) For one event we are sure that it will happen while for the other we are not sure that it is going to happen.

   c) One event is not going to happen for sure, while the other may happen.

## Part II

*To answer the next set of questions read about Mira's play with beads.*

Mira had two boxes of beads. In the first of them, there were one black and one yellow bead. In the second box, there was one black and four yellow beads. With closed eyes, Mira pulled out a bead from one of the boxes. She recorded on the paper the color of that bead. Then, she returned the bead into the same box. She repeated the same action three times taking beads from the same box. Look at her record on the table.

| | 1 | 2 | 3 | |
|---|---|---|---|---|
| Black | | X | | |
| Yellow | X | | X | |

7. How many times did she get black beads?

   a) once;

b) twice;

c) three times.

8. How many times did she conducted pull-out?

a) once;

b) twice;

c) three times.

9. Out of which box did she pull out beads?

a) The first box.

b) The second box.

c) I cannot conclude based on these results.

**III part**

Here are the results of the survey "What's your favorite pet" conducted in a class of second-graders in our school. Look at the record.

| | | | | | |
|---|---|---|---|---|---|
| Mira | rabbit | Nada | dog | Sanja | cat |
| Jovan | dog | Yasna | cat | Yagoda | dog |
| Zorana | dog | Drashko | dog | Tanja | dog |
| Ellena | dog | Milosh | fish | Maja | rabbit |
| Zorana | cat | Milan | dog | Marija | dog |
| Dragan | fish | Yvan | cat | Yvana | rabbit |
| Milla | cat | Strahinja | fish | Jagosh | dog |

10. Show the results graphically.

11. What is the most popular pet in that class?

a) rabbit;

b) dog;

c) cat;

d) fish;

e) I cannot answer based on these results.

12. What is the most popular pet among children in our school?

a) Rabbit;

b) Dog;

c) Cat;

d) Fish;

e) I cannot answer based on these results.

13. What else can you deduce about the popularity of pets based on the survey results?

______________________________________________________________________

______________________________________________________________________

## Part IV

Game 1

Three children cannot decide who is going to be the next to use their computer. They decided to toss a die. They created rules: Rule 1. If the number turns to be less than 3, Tamara will sit first. Rule 2. If the number is less than 5 but does not give an advantage to Tamara then Alex is going to be the first. Rule 3. Otherwise, Jelena is going to be the first.

14. Does it matter who tosses the die first?

    a) Yes.

    b) No.

15. How many times they need to toss the die to determine who is going to be the first to sit down at the computer?

    a) once;

    b) three times;

    c) I dont know.

16. Is this a way of determining the fair?

    a) Yes;

    b) No.

Game 2

Two players toss two dice. If the sum of the numbers that show on the dice, is an even number, the first player gets a point. In other cases, the second player receives points. The game is over after 15 throws. The winner is the one who gets more points.

17. Is this a fair game? Do both players have the same chance of winning? Explain the answer.

    Answer:

## Appendix 2

Statistics by Grades 4 to 7

| Grade | Grade 4 | Grade 5 | Grade 6 | Grade 7 |
|---|---|---|---|---|
| N | 98 | 96 | 97 | 101 |
| R | 19 | 22 | 19 | 18 |
| Min | 6 | 5 | 8 | 9 |
| Max | 25 | 27 | 27 | 27 |
| M | 15.62 | 16.37 | 17.43 | 18.52 |
| $m_e$ | 16 | 17 | 18 | 19 |
| $m_o$ | 18 | 20 | 20 | 19 |
| $S^2$ | 19 | 19.39 | 19.29 | 9.57 |

ACKNOWLEDGEMENT.

We are grateful to the learned referee of this paper whose suggestions helped to improve it.

### REFERENCES

[1] D. Abrahamson, *Rethinking probability education: perceptual judgment as epistemic resource*, In: E.J. Chernoff & B. Sriaman (Eds.), *Probabilistic Thinking: Presenting Plural Perspectives*, pp. 239–260, Springer, New York, 2014.

[2] T. Anastasiadou, T. Chadjipantelis, *The role of representations in the understanding of probabilities in tertiary education*, 2011, Retrieved from http://www.ethikkommission-kaernten.at/ICME11/p18_ICME11_TSG13_paper_18_role_of_repres_in_understanding_ANNOT21A_mb_EE_ana_EE.pdf

[3] A. Ahlgren, J. Garfield, *Analysis of the probability curriculum*, In: R. Kapadia & M. Borovcnik (Eds.), *Chance Encounters: Probability in Education*, pp. 107–134, Dordrecht, Netherlands: Kluwer Academic Publishers, 1991.

[4] A. Arcavi, *The role of visual representations in the learning of mathematics*, Educational Studies in Mathematics, **52**, 2003, 215–241.

[5] R. Beythe-Marom, S. Dekel, R. Gombo, M Shaked, M. (Eds.), *An Elementary Approach to Thinking under Uncertainty*, Hillsday, NJ: Lawrence Erlbaum Associates Inc., 1983.

[6] J.S. Bruner, M.H. Bornstein, *On interaction*, In: M.H. Bornstein & J.S. Bruner (Eds.), *Interaction in Human Development*, Hillsdale, NJ: Lawrence Erlbaum Associates Publ., 1989.

[7] J. Bruner, *The Culture of Education*, Harvard University Press, 1996.

[8] P. Bryant, T. Nunes, *Childrens Understanding of Probability – A Literature Review*, Nuffield Foundation, 2012.

[9] G. Burrill, T.A. Romberg, *Statistics and probability for the middle grades: examples from Mathematics in Context*, In: S.P. Lajoie (Ed.), *Reflections on Statistics: Learning, Teaching and Assessment in Grades K-12*, pp. 33–62, Mahwah, NJ: Lawrence Erlbaum, 1998.

[10] E.J. Chernoff, Bh. Shriaman (Eds.), *Probabilistic Thinking – Presenting Plural*, Perspectives, Advances in Mathematics Education, Springer, 2014.

[11] R. Duval, *Representation, vision and visualization: cognitive functions in mathematical thinking, basic issues for learning*, In: F. Hitt (Ed.), *Representations and Mathematics Visualization*, PME proceedings. North American Chapter of the International Group for the

psychology of mathematics education, pp. 311–337, 2002. Retrieved from `http://www.er.uqam.ca/nobel/r21245/varia/Book_RMV_PMENA.pdf.`

[12] L.D. English, *Promoting statistical literacy through data modelling in the early school years*, In:E.J. Chernoff&Bh. Sriraman (Eds.), *Probabilistic Thinking: Presenting Plural Perspectives*, pp. 441-457, Netherlands, Springer, 2014.

[13] R. Falk, F. Wilkening, *Children's construction of fair chances: Adjusting probabilities*, Developmental Psychology, **34** (6) (1998), 1340–1357.

[14] R. Falk, P. Assouline, A. Elstein, *Children's concept of probability as inferred from their binary choices – revisited*, Educational Studies in Mathematics, **81** (2) (2012), 207–233.

[15] E. Fischbein, I. Pampu, I. Mînzat, *Comparison of ratios and the chance concept in children*, Child Development, **41** (1970), 377–389.

[16] H. Freudenthal, *Didactical Phenomenology of Mathematical Structures*, Dordrecht, Netherlands: Reidel Publishing Co., 1983.

[17] J. Garfield, *The challenge of developing statistical reasoning*, Journal of Statistics Reasoning, **10** (3) (2002).

[18] J. Garfield, A. Ahlgreen, *Difficulties in learning basic concepts in probability and statistics: Implications for research*, Journal for Research in Mathematics Education, **19** (1) (1988), 44–63.

[19] D.R. Green, *The chance and probability concepts project*, Teaching Statistics, **1** (3) (1979), 66–71.

[20] D.R. Green, *School pupils' probability concepts*, Teaching Statistics, **5** (2) (1983), 34–42.

[21] D.R. Green, *Probability concepts, putting research into practice*, Teaching Statistics, **9** (1) (1987), 8–14.

[22] J. Hiebert, T.P. Carpenter, *Learning and teaching with understanding*, In: D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*, pp. 65–97), Mcmillan, New York, 1992.

[23] B.L. Huber, O. Huber, *Development of the concept of comparative subjective probabilities*, Journal of Experimental Child Psychology, **44** (1987), 304–316.

[24] K. Iversen, P. Nilsson, *Students' reasoning about one-object stochastic phenomena in an ICT-environment*, International Journal of Computers for Mathematical Learning, **12** (2) (2007), 113–133.

[25] C. Janvier, *Problems of Representation in the Teaching and Learning of Mathematics*, Lawrence Erlbaum Associates Publ., London, 1987.

[26] V. Jevremović, *Probability Theory and Mathematical Statistics* (in Serbian), Građevinski fakultet, Beograd, 1999.

[27] G.A. Jones, C.W. Langrall, C.A. Thornton, A.T. Mogill, *A framework for assessing and nurturing young childrens thinking in probability*,Educational Studies in Mathematics, **32** (1997), 101–125.

[28] G. Jones, C.W. Langrall, E.S. Mooney, *Research in probability: responding to classroom realities*, In: F. K. Jr. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics*, pp. 909–957, Information Age Publishing, Charlotte, 2007.

[29] G. Jones (Ed.), *The challenges of teaching probability in school*, Retrieved from `http://www.mathunion.org/fileadmin/ICMI/files/Digital_Library/ICMEs/Bulletin_Jones.pdf.`

[30] D. Kahneman, A. Tversky, *Judgment under uncertainty: Heuristics and biases*, Science, New Series **185** (4157) (1974), 1124–1131.

[31] R. Kapadia, M. Borovcnik (Eds.), *Chance Encounters: Probability in Education*, pp. 73–105, Kluwer Academic Publishers, Dordrecht, 1991.

[32] R. Kapadia, *Developments in statistical education*, Educational Studies in Mathematics **11** (1991), 443–461, In: R. Kapadia and M. Borovcnik (Eds.), *Chance Encounters: Probability in Education*, pp. 213–254, Kluwer Academic Publishers, Dordrecht, 1991.

[33] S. Kapon, G. Ron, R. Hershkowitz, *Perceiving permutations as distinct outcomes: the accommodation of a complex knowledge system*, Educational Studies in Mathematics, **88** (1) (2015), 43–64.

[34] S. Kazak, J. Confrey, *Elementary school students' informal and intuitive conceptions of probability and distribution*, ICOTS-7, 2006. Retrieved from `http://iase-web.org/documents/papers/icots7/6B1_KAZA.pdf`

[35] S. Kazak, C. Konold, *Development of ideas in data*,In: C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society*, Proceedings of the Eighth International Conference on Teaching Statistics (ICOTS8, July, 2010), Ljubljana, Slovenia, International Statistical Institute, Voorburg, 2010. Retrieved from `http://iase-web.org/documents/papers/icots8/ICOTS8_8D2_KAZAK.pdf`

[36] C. Konold, *Tinker Plots: Dynamic data exploration: Overviewempowering students as data analysts*, 2004. Retrieved from `http://www.umass.edu/srri/serg/projects/tp/tpovervw.html`

[37] C.W. Langrall, E.S. Mooney, *Characteristics of elementary school students' probabilistic reasoning*, In: G. Jones (Ed.),*Exploring Probability in School: Challenges for Teaching and Learning*, pp. 95–119,Kluwer Academic Publ., Dordrecht, 2005.

[38] L. Marascuilo, R. Serlin, *Statistical Methods for the Social and Behavioral Sciences*, W.H. Freeman and Company, New York, 1995.

[39] K.E. Metz, *Emergent ideas of chance and probability in primary grade children*, In: S.P. Lajoie (Ed.), *Reflections on Statistics: Agendas for Learning, Teaching, and Assessment in K-12*,Lawrence Erlbaum Associates, Hillsdalle, NJ, 1998.

[40] J. Milinković, *Intuition about concept of chance in elementary school children*, In: K. Krainer & N. Vondrov (Eds.),*Proceedings of the Ninth Conference of the European Society for Research in Mathematics Education* (CERME9, 4–8 February 2015), pp. 722–726, Charles University in Prague, Faculty of Education and ERME, Prague, 2015.

[41] D. Moore, *Uncertainty*, In: L. Steen (Ed.), *On the Shoulders of Giants: New Approaches to Numeracy*, National Research Council, Washington, D.C, 1990.

[42] I. Papadopoulus, D. Deslis, *Equiprobable events and probability comparison in primary grades*, In: J. Novotna, (Ed.), *Proceedings SEMT 15*, 2015, pp. 257–266.

[43] E. Paparistodemou, *Children's construction of sample space with respect to the law of large numbers*, In: E.J.Chernoff& Bh.Sriraman (Eds.),*Probabilistic Thinking: Presenting Plural Perspectives*, pp. 599–612, Springer, 2014.

[44] J. Piaget, B. Inhalder, *The Origin of the Idea of Chance*, Routledge & Kegan Paul, London, 1951.

[45] D. Pratt, *Making sense of the total of two dice*, Journal for Research in Mathematics Education, **31** (5) (2000), 602-625.

[46] D. Pratt, *How do teachers foster students' probability?*, In: G.A. Jones (Ed.), *Exploring Probability in School: Challenges for Teaching and Learning*, pp. 171-189), Kluwer Academic Publishers, 2005.

[47] J. Randi, L. Corno, *Teaching and Learning Variation. Pedagogy/Learning for Teaching*, BJEP Monograph Series II, 3 Q, pp. 47–69, The British Psychological Society, 2005.

[48] P. Rasfeld, *The Role of Statistics in School Mathematics Teaching Today*, ICOTS6, 2004.

[49] R.W. Scholz, *Psychological Research in Probabilistic Understanding*, In: R. Kapadia & M. Borovcnik (Eds.), *Chance Encounters: Probability in Education*, pp. 213–254, Netherlands: Kluwer Academic Publishers, Dordrecht, 1991.

[50] A. Schoenfeld, *Solving the problem of powerful instruction*, In: C. Csikos, A. Rausch & J. Sztanyi (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education*, 3-7 August, 2016. Szeged, 2016.

[51] K.E. Stanovich, *How to Think Straight About Psychology*, Peachpit Press, 2013.

[52] H. Steinbring, *The theoretical nature of probability in the classroom*, In: R. Kapadia & M. Borovcnik (Eds.), *Chance Encounters: Probability in Education*, pp. 135–168, Kluwer Academic Publishers, Dordrecht, 1991.

[53] H. Stohl, J.E. Tarr, *Developing notions of inference with probability simulation tools*, Journal of Mathematical Behavior, **21** (3) (2002), 319–337.

[54] J.E. Tarr, G.A. Jones, *A framework for assessing middle school students' thinking in conditional probability and independence*, Mathematics Education Research Journal, **9** (1997), 39–59.

[55] J.M. Watson, R. Callingham, *Statistical literacy: A Complex hierarchical construct*, Statistics Education Research Journal, **2** (2) (2003), 3–46.

[56] J.M. Watson, A. Caney, B.A. Kelly, *Beliefs about chance in the middle years: Longitudinal change*, MERGA Proceedings, 2004, pp. 581–588. Retrieved from: https://www.researchgate.net/publication/255597115_Beliefs_about_Chance_in_the_Middle_Years_Longitudinal_Change?enrichId=rgreq-d726213c-ac7d-4071-ac55-927cb8279bdb&enrichSource=Y292ZXJQYWdlOzI1NTU5NzExNTtBUzoxMzA2NDcxMjEyMDcyOTZAMTQwODE2MDEwOTM0MA%3D%3D&el=1_x_2

[57] J.M. Watson, J.B. Moritz, *Fairness of dice: A longitudinal study of students' beliefs and strategies for making judgments*, Journal for Research in Mathematics Education, **34** (2003), 270–304.

[59] J. Way, *The development of young children's notions of probability*, CERME 3 Proceedings, Retrieved from http://www.dm.unipi.it/~didattica/CERME3/proceedings/Groups/TG5/TG5_way_cerme3.pdf

J.V.M.: University of Belgrade, Teachers Training Faculty, Belgrade, Serbia

*E-mail*: milinkovic.jasmina@yahoo.com

I.R.: University of Belgrade, Teachers Training Faculty, Belgrade, Serbia

*E-mail*: ivica.radovanovic@uf.bg.ac.rs