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DOES THE PROBLEM COMPLEXITY IMPACT STUDENTS' ACHIEVEMENTS IN A COMPUTER AIDED MATHEMATICS INSTRUCTION?

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Abstract. The paper deals with two aspects of a computer-based analytic geometry instruction: students achievements, and achievement decay rates. A new mathematics learning environment—a dynamic and interactive one—is being built with introduction of computers and GeoGebra in the classroom. The subject material was introduced to the students mostly through GeoGebra dynamic worksheets. Students of the experimental and the control groups went through three tests within an eight-month period. The tests were different in terms of technology used by the experimental group and the two aspects we took into consideration. In the first two tests we observed students' achievements in different learning environments. The experimental group students showed a lower performance when solving problems in a non-computer environment than they did in a computer-based environment. However, we identified types of problems that are easier to solve in a computer-based environment, and others that do not necessary require computer usage to be quickly solved. This helped us formulate better strategies while choosing types of problems and appropriate ways and technologies to solve them. Parts of the third test helped us estimate achievement decay rates for both groups. Results indicate very close achievement decay rates for both, experimental and control groups.

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Key words and phrases: GeoGebra; dynamic worksheets; analytic geometry; students' achievement decay.

1. Introduction

Since 2008, we have been applying computers in the plane analytic geometry instruction in "Nikola Tesla" high school in Leposavić, Serbia. The instruction process built in this way evolved from the computer-aided instruction to a Dynamic and Interactive Mathematics Learning Environment - DIMLE [17]. The instruction process was developed by both teachers and students, using GeoGebra software.

The first stage when organizing this type of instruction were teachers' preparations for the new way of teaching. Several studies [22, 26] indicate that using ICT in instruction process helps teachers improve and diversify their teaching techniques and, doing so, help their students achieve better results. Having in mind that the majority of the teachers included in the process initially had little or no experience with the computer-based instruction, we had to pay special attention to this segment of the process. As stated in [1], in-service trainings can help teachers show higher achievements and attitudes towards the ICT use in the instruction process than a pre-service instruction deprived of real-life contact with students in the classroom. The initial teachers' preparations were carried out during the first semester of school year 2008/09, but we did not stop doing our best to improve teachers abilities after the computer-based instruction began. Moreover, the information we gathered after first experiences in the classroom were far more useful than all preparations and arrangements we made before the instruction started.

Problem of research

Among authors and researchers there is no undisputed idea about the impact of ICT use in Mathematics instruction. Opinions diverge from positive [3, 7, 13, 19] to undecided [21], or even to mostly negative [14].

There are number of studies [6, 16, 20, 27] that support the idea that students who learn analytic geometry with GeoGebra score better achievements than students learning it in a traditional learning environment. Though mostly positive, findings of these studies do not consider the technology that students use during the testing process, i.e., there are no data comparing students achievements when testing is done in an ICT-empowered or ICT-deprived environment.

Along with students' achievements, it is important to assess achievement decay rates demonstrated by students in the ICT-empowered environment and compare it to the corresponding rates students show in the ICT-deprived environment. It is stated in [11] that computers had a short term impact on mathematics achievements which was not maintained a month later. Similarly, in [23], no significant relation is found between method of instruction (computer assisted versus traditional) and knowledge retention. On the other hand, in [18], a decrease is indicated in retention tests for both groups and that a significant difference in favour of the experimental group was found in two units: Multiplication and Division of Natural Numbers, but there was no significant difference for Fractions. In [2], a significant difference is reported in retention performance in favour of the students taught with ICT comparing to those taught using conventional method of instruction.

The idea of our research was to assess possibilities to improve the Plane analytic geometry instruction in an environment empowered with computer and GeoGebra usage in our high schools. This means we had to:

1. Specify the extent and quality of the influence that computer and GeoGebra usage can have on students' achievements in learning new mathematical concepts and procedures of the Plane analytic geometry.
2. Assess students' achievement decay rates of both groups engaged and find out if it is influenced by the technology used in the instruction process.

To accomplish this, we define the research problems to be:

1. *Outlining and assessing possible influence of a computer-based environment on students' academic achievements in the Plane analytic geometry instruction.*
2. *Outlining and assessing possible influence of a computer-based environment on the achievement decay rates in the Plane analytic geometry instruction.*

Research focus

In order to compare the academic achievements of the experimental and control groups before the research period, we will statistically evaluate the difference between their grades average at the beginning of the Plane analytic geometry instruction.

Besides, it is essential to draw clear conclusions concerning the influence of the computer usage on students' results during the evaluation process. For that reason, we conducted two tests during the Plane analytic geometry instruction: one, where the students of the experimental group were allowed to use computers and GeoGebra, and another, where both groups enrolled the test without computers.

Apart of these two tests, students went through an initial test at the beginning of the next school year. There were analytic geometry problems included in the third test, and we assessed students' achievement decay rates comparing their results at solving the problems with the ones they showed in earlier tests.

2. Method

Participants

We composed the experimental group sample by joining seven consecutive generations of the third grade (16–17 years old) gymnasium students of "Nikola Tesla" high school in Leposavić, Serbia. In a similar way, the control group sample was composed of consecutive generations of the third grade electrical engineering students of the same high school. In table 1 there are shown data concerning the marks in mathematics in both, experimental and control groups. Students with the lowest level of knowledge were given mark 1 and those with the highest level of knowledge—mark 5.

Table 1. Marks in Mathematics of experimental and control group students

Experimental group						
	1	2	3	4	5	Count
2008/9.	2	5	5	5	3	20
2009/10.	2	6	7	3	3	21
2010/11.	0	4	4	5	3	16
2011/12.	0	4	5	3	2	14
2012/13.	1	8	5	4	5	23
2013/14.	3	5	6	5	4	23
2014/15	3	6	6	4	3	22
Total	11	38	38	29	23	139

Control group						
	1	2	3	4	5	Count
2008/9.	4	4	6	6	3	23
2009/10.	1	3	5	2	3	14
2010/11.	0	6	5	2	2	15
2011/12.	1	5	7	3	1	17
2012/13.	0	5	1	1	0	7
2013/14.	3	3	2	0	0	8
2014/15.	1	3	3	2	1	10
Total	10	29	29	16	10	94

Based on the data shown in Table 1, one can see that the mean value of the experimental group marks is $M_1 = 3.11$ and for the control group it is $M_2 = 2.86$. The results of the two-sample homoscedastic t-test confirmed that there was no statistically significant difference of the grades average of the research groups ($t = 1.56$, $p = 0.12$, $df = 231$).

Software

GeoGebra is a software that can be defined as both Computer Algebra System (CAS) and Dynamic Geometry System (DGS). It offers great possibilities for a computer-aided instruction of mathematics topics that link geometry with algebra [8, 12, 15].

With computers and GeoGebra introduced into the mathematics instruction, several new aspects came to light. In the traditional way of instruction, most of the sketches were too difficult to be drawn within a class, and teachers used to bring already done sketches and drawings or sketch them on the writing board. These sketches are static and students can see their final shape only, which can make them neglect the genesis of the mathematical concept they learn. The GeoGebra dynamic worksheets composed by teachers helped us avoid the static approach to the subject material taught. They also enabled us repeat the very process of genesis of the concept or procedure explored during the lesson. Furthermore, the dynamic worksheets can be varied and used in different ways. Thus, one can have many different points of view at the same problem. In this way, students can easily imbed the mathematical concept into a broader context, i.e., it can easily be incorporated into their cognitive structure [4, 9, 16]. On the other hand, computations that are of less importance, can consume lots of precious time. Computers, if used in a right manner, can quicken the computations and give students an opportunity to devote more time to understand the mathematical problem they deal with.

Treatment

Both, experimental and control group were instructed the same lessons within the same period of time. The instruction covered lessons: Distance between two points; Division of a segment in a given ratio; Area of a triangle; Line, angle between two lines and the distance of a point from a straight line; Circle; Ellipse; Hyperbola; Parabola (properties, equations, interrelations, tangents). Since the instruction was planned according to the official syllabus, it lasted 50 lessons, each taking 45 minutes. The instruction was carried out from late January to late March.

The experimental group students were included in a computer-based instruction in a classroom equipped with 12 computers, a data projector and a blackboard, too. Due to the lack of room and computers, students worked in pairs. When assigning students to a pair, we kept in mind that they (at least roughly) supplemented each other in regard to their mathematics knowledge and computer skills.

During the instruction, we made efforts to ensure that the mathematical concepts were in the focus of the process. Having in mind that the very introduction of ICT into the mathematics instruction can cause mathematics concepts adaptation to ICT [10], it was crucial not to change or miss the meaning of the subject taught while adapting it to the computer representation [16].

For instance, after defining the ellipse as the locus of points in plane so that the sum of their distances to the foci is constant, we introduced the pins-and-string method of the ellipse construction. The method was adapted to the computer-based environment using an appropriate dynamic worksheet, Figure 1, and the applet retained the core idea of the ellipse construction. Based on this applet, we derived the ellipse equation and explored its properties. Later on, teachers and students created similar applets to solve a problem or represent a solution to a problem.

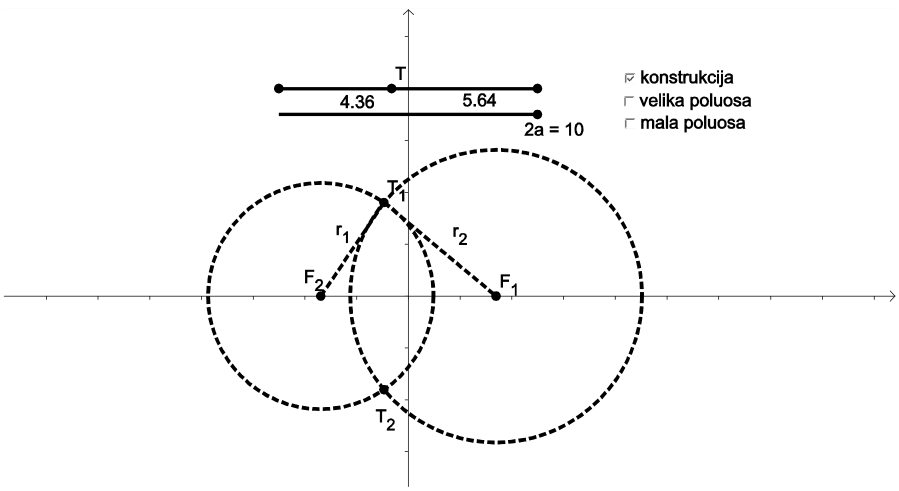


Figure 1. A detail of a GeoGebra dynamic worksheet representing the pins-and-string method for the ellipse construction

Computers and GeoGebra were used to create dynamic sketches of mathematical concepts, to represent mathematical problems and their solutions, as well as to verify the solutions to problems obtained by teachers or students.

Some of the dynamic worksheets—applets were created only as dynamic sketches of a mathematical concept or procedure. The teacher usually presented them to the students using the data projector. Their usage simplified students' efforts in understanding the subject material and quickened the instruction process.

Other applets offer their users freedom in exploration and learning mathematical concepts, rules and procedures. Students can follow steps in the Construction protocol and draw their own conclusions about the mathematical concept they explore. Another possibility is to drag geometrical objects in the geometry window, track corresponding changes in the algebra window, and test interrelations among the observed objects. In this way we encourage students' creative and heuristic activities.

The third type of the applets include those designed to help solving some specific mathematical problems. It is sufficient to adjust the input data, so the applet can be used to solve a particular mathematical problem. This kind of dynamic worksheets could also be used to verify the results students obtain in their homework or independent exploration.

Some of the applets were designed by students, either independently, or with the teacher's help. Students used some of them to represent solutions to the problems they dealt with in their homework. Many students, encouraged with the new approach to the learning process, started their own explorations with GeoGebra and the subject material. During the instruction process, all the dynamic worksheets were available to students both in school and at home. They were also available at web pages: `http://eugenljajko.info/Analytic%20geometry%20applets.html` and `http://eugenljajko.info/aplikacije%20analiticka.html`.

Though there are evidence that ICT use in mathematics instruction can lead students from visualisations to a more formal language [4, 9] , we were more cautious and insisted on using notebooks and the writing board to derive equations and prove theorems, i.e. in every situation where the symbolic and formal approach to mathematics was in the forefront. Failing to include a symbolic approach to the mathematics instruction would be contrary to basic objectives of the mathematics instruction.

The computers were locally networked, so the teacher could have information on what the students did with their computers at any moment. A computers screen could be projected through the data projector, which is very similar to the writing board usage. In this way, students were easier to get accustomed with the new learning environment.

Students did their school activities, homework and tests in different ways: with computers, on writing board or in notebooks.

All the time we paid special attention to identify and develop the most fruitful interrelations and the most suitable instruction techniques. Since the students worked mostly in pairs, "*I – you – we*" method of knowledge building became an integral part of the instruction process [5, 25].

During the same period, the control group students were instructed in a more traditional way—without computers and GeoGebra included in the instruction process. All the sketches were drawn on the writing board and notebooks, and the mathematical concepts and procedures were interpreted and built through conversations and formulae related to the sketches. Re-sketching them was not an option due to the time needed for it. For that reason, the instruction was not as dynamic as it was in the computer-empowered environment. Students did their school activities, homework and tests on writing board or in notebooks.

After the lessons concerning: Segment, Line and Circle were taught, we conducted the first test, where the experimental group students were allowed to use computers, either as a sketching tool, or as a tool to create dynamic worksheets in order to represent a problems solution. The control group students did the test in their notebooks. After completing all of the Plane analytic geometry lessons, students of both groups did the second test in their notebooks without computer usage. The third test took place in September—at the beginning of the next school year, and it was meant to disclose the extent of the knowledge students retained six months after the instruction was over. Both groups did it in their notebooks, too.

Variables and measurement

In order to disclose possible impact that the technology used in the instruction can have on students' achievements, we compared results the groups scored at two consecutive tests—one during the experimental period, and another at the end of it.

To assess achievement decay rates we took into consideration results both groups scored at three tests—the third one being the initial test at the beginning of the next school year.

In all tests both groups had to solve the same problems. The problems included in the tests were based on the official syllabus and the tests were reviewed by 7 educators (5 Mathematics and two Informatics teachers) working in the same school where the experiment was carried out.

The first test was comprised of following problems:

1. *Find a point on x-axis with equal distances to points $A(-2,1)$ and $B(3,4)$.*
2. *Obtain the value of parameter m so that the y-axis segment of the line $3x + 2my - 12 = 0$ is twice as long as the x-axis segment of the same line.*
3. *Find the equation of the line that passes through the point $A(4,2)$ and the intersection point of lines $4x + y - 3 = 0$ and $x + y - 1 = 0$.*
4. *Calculate the distance between the centroid T of the triangle ABC and its longest side if its vertices are: $A(-6,2)$, $B(1,3)$ and $C(-1,-2)$.*
5. *Calculate the angle between the line $y = 2x+1$ and the circle $(x-2)^2+(y-1)^2 = 16$, at their intersection point located in the first quadrant.*

The control group students did it in the traditional way—in their notebooks. The experimental group students were allowed to use computers and GeoGebra with a sole condition not to use already made applets designed during the instruction process. Students of both groups were requested to write down not only solutions to a problem, but the course of the solving process, too. It means, we considered a problem to be solved correctly by a student only if the correct solution and the proper solving process were written down in his/her notebook. For a correct solution to a problem, students could score 10 points and the maximum score was 50 points.

The second test consisted of four problems:

1. *Find coordinates of the nearest and the farthest points of the circle $x^2 + y^2 - 8x - 4y + 12 = 0$ to the line $x - y + 5 = 0$.*
2. *Calculate the area of the quadrilateral that has two vertices in foci of the ellipse $9x^2 + 25y^2 = 225$ and the other two vertices are at the ends of the minor axis of the same ellipse.*
3. *Write down the equation of the hyperbola if its foci coincide with major vertices of the ellipse $9x^2 + 25y^2 = 225$ and its directrices pass through the foci of the ellipse.*
4. *Find the equations of the tangents from the point $A(0,3)$ to the parabola $y^2 =$*

$4x$. After that, find coordinates of the conjoint points B and C of the tangents and the parabola and calculate the area of the triangle ABC.

Both groups did it in their notebooks, without computer usage. The second test evaluation was similar to the one in the first test: a correct solution to a problem was assessed with 10 points and the maximum score was 40 points.

At the beginning of the next school year both groups went through an initial test comprised of items covering the third-grade mathematics. Both groups did it in their notebooks, without computer usage. Among the items, there were two plane analytic geometry problems:

1. *Determine the type of the curve given by equation $x^2 + y^2 - 6x - 4y + 8 = 0$ and explore its relation to the line $x - 3y - 2 = 0$.*
2. *Find coordinates of foci of the ellipse $3x^2 + 4y^2 = 48$, check if the point $A(2, 3)$ lies on the ellipse, and calculate distances from the point A to the foci of the ellipse.*

We decided to use this test results for the research in order to avoid excessive testing of students. Organizing one more separate test would disrupt students daily routine and influence validity of the instruction process.

Since these two problems were comprised of more sub-problems, they were similar to more complex problems of the first two tests—Problems 2, 3 and 5 in the first test, and Problem 4 in the second one. Taking students achievements in this part of the third test and comparing them to their results in previous tests would help us estimate their achievements decay rates.

For these two problems we also used a similar key for evaluating results the students achieved: a correct solution to a problem was assessed with 10 points and the maximum score was 20 points.

In all three tests we also took into consideration solutions that were partially correct and scored them with less than 10 points.

In order to compare results students achieved at all three tests, we represented their scores with percentage points. Students' marks in Mathematics before the experimental period were also represented in the same way using formula $a = (m-1)/4$, where a is student's achievement and m is his/her mark in Mathematics. Thus, the marks could be easily compared with the students' achievements in tests, and the mean values of the marks before the experimental period could be expressed as $M_1 = 0.53$ and $M_2 = 0.47$.

In this research we used one variable—Students' achievements, presented with percentage points of correct answers. Its mean value was $M = 0.49$, standard deviation $SD = 0.26$ and reliability $\alpha = 0.98$.

Statistical analysis

After examining the students' works at the first test, we learnt that the experimental group students scored significantly higher than the control group students (percent correct: 56% vs. 49%, $t = 2.71$, $p = 7.15E - 3$).

It is very important to consider possible differences between the points the groups scored when solving a particular problem. In Table 2 t-statistics and p-values are shown for every problem separately, along with its significance at 0.05 level.

Table 2. Statistical significance of marks the groups scored in the first test

Problem	1.	2.	3.	4.	5.
t-statistics	-0.65	4.60	2.30	9.45E-04	2.10
p-value	0.52	6.82E-06	0.02	0.9992	0.04
Significance	No	Yes	Yes	No	Yes

The difference appears to be the highest in Problems 2, 3 and 5. This fact can be explained with the types of the problems. Though computer and GeoGebra usage can make a significant simplification for any problem solution, it is more evident when solving these three problems due to the nature of the requests the problems were consisted of.

In the second test the experimental group students' scores were also higher than the control group scores, but in this case the difference was statistically less pronounced (percent correct 51% vs. 44%, $t = 1.83$, $p = 0.07$).

We will also consider the results of the second test for every problem separately. Statistical data related to the second test problems are shown in Table 3.

Table 3. Statistical significance of marks the groups scored in the second test

Problem	1.	2.	3.	4.
t-statistics	0.99	1.43	1.13	1.59
p-value	0.32	0.15	0.26	0.11
Significance	No	No	No	No

Although there is no statistically significant difference between the groups' scores for any of the problems, one can notice that the t-value is highest for the fourth problem. It is comprised of more sub-problems and, having in mind its complexity, it is similar to Problems 2, 3 and 5 in the first test.

This leads us to a conclusion that the computer usage, either during the instruction process, or while problem solving, improves the students' efficiency in solving more complex problems. On the other hand, computer usage for simpler situations can turn the students attention to parts of less importance (simple computations, drawing sketches). This can cause students misunderstanding of the problem or even mislead them to incorrect results.

In the third test the experimental group students scored significantly higher than the control group students (percent correct: 47% vs. 40%; $t = 2.27$, $p = 0.02$).

In order to assess overall achievement decay rates for both groups, we compared their results during the instruction period (the first test) and at the end of it (the second test) to the ones they achieved at initial tests (the third test) six months later. Students' marks before the experimental period presented with percentage points were used as pre-test values. Comparative results for both groups are shown in Table 4 and the achievement decay graphs can be seen in Figure 2.

Table 4. Overall grades average decay during and after the experimental period

	Pre-test	Test			Change		
		1.	2.	3.	2 – 1	3 – 2	Overall
E	0.53	0.56	0.51	0.47	- 0.05	- 0.04	- 0.09
C	0.47	0.49	0.44	0.40	- 0.05	- 0.04	- 0.09

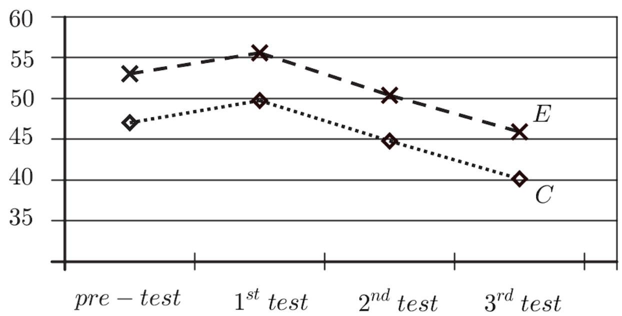


Figure 2. Overall achievement decay graphs

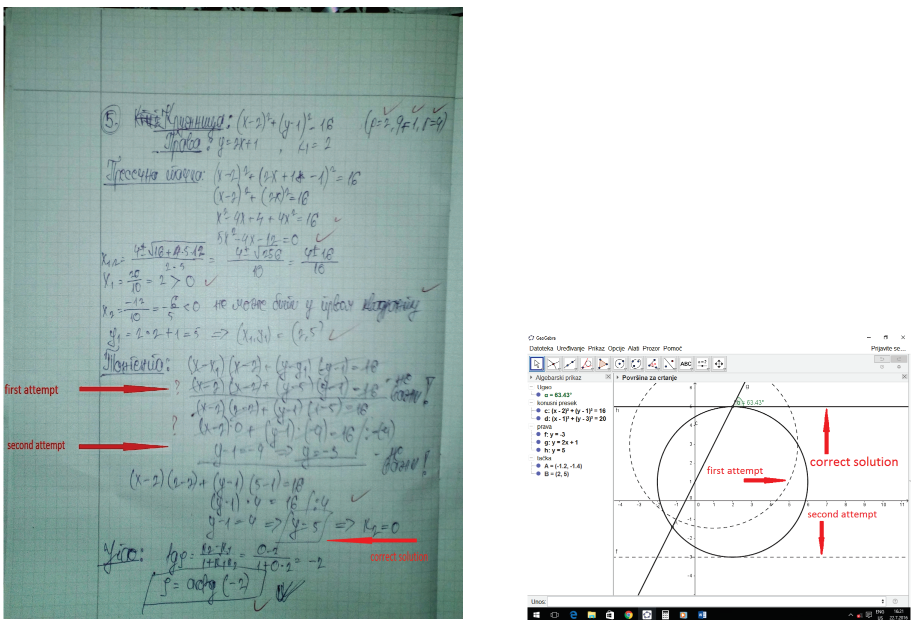
The overall changes of grades average during the experimental period are approximately equal for both groups, which means that, in this case, ICT use in the instruction period did not lead to a lower knowledge decay rate than it was in the traditional instruction process.

Discussion

In order to understand the difference in efficiency students of experimental and control groups demonstrate while solving problems, we will examine and contrast their approaches to the fifth problem of the first test.

Students of the experimental group, that were not sure about the formulae needed to solve the problem, used computers to check the solution they have come to. This helped them to correct errors at earlier stages of the problem solving process, Figures 3 a) and b), and come to the proper solution after several attempts. Though it was obvious that after the first attempt the student obtained a second order equation, she was convinced that the result was incorrect only after the GeoGebra applet showed a conic as a representation of the obtained equation. In the second attempt she came to a linear equation, but the corresponding line did not pass through the first quadrant, which was easy to notice in the applet. That made her check the correctness of the formula she used and, finally, obtain the correct solution. On the other hand, students of the control group had limited possibilities to recognize incorrect results obtained by using a wrong formula, Figure 4.

The results also bring to the focus a question: Why the computer usage did not enable students achieve similar results while solving the first and the fourth problem? A close examination of the nature and the results at the first problem shows that the computer usage became an aggravating factor, mostly due to simplicity of the requests. In fact, the experimental group students, assured in the "computers supremacy" and led by their habit of using the computer first, tried to solve the problem with computers [24], instead of doing simple calculations that did not require the computer usage. On the other hand, the control group students solved the problem in the only way they were accustomed to, and scored better results. A similar situation took place while solving the fourth problem.



(a) (b)

Figure 3. A student's work corrected after checking the solution with GeoGebra

Conclusions

One should always keep in mind that, in the computer-based instruction, students tend to come to a "solution" to a problem even if they do not understand the subject or concepts they deal with [24]. Most of the computations are easier to do by computer and this can contribute to the behaviour the students demonstrate.

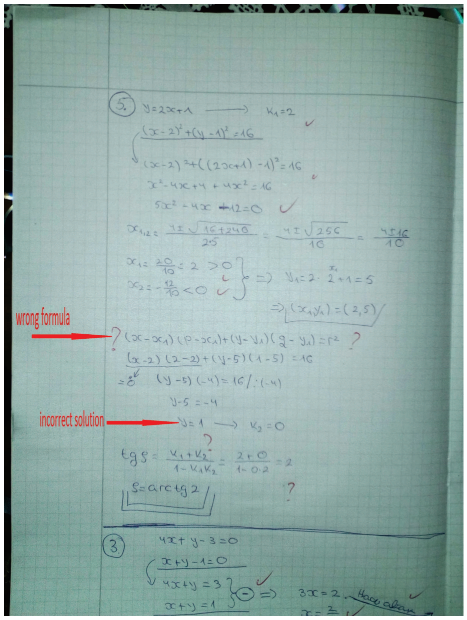


Figure 4. A wrong formula easily leads to an incorrect solution if not checked with GeoGebra—work of a control group student

Use of dynamic worksheets can strengthen understanding of concepts [4, 9], but it can also make students develop a habit of "letting the computer do the job" [24]. For all these reasons students tend to lessen their cognitive activities. Teachers should, therefore, organize the instruction process in a way that ensures students to be in its focus [26]. The students' activities should be directed at understanding the concepts and procedures rather than solving problems at any cost.

In our research it is noticed that the computer usage in simpler problems does not bring expected improvements as it does with more complex problems. There could be more reasons for this, but several can be outlined:

- Relatively quick computations and sketching can reduce time needed for solving more complex problems. On the other hand, computer usage can be distracting in cases where building an applet takes more time than solving by "paper-and-pencil" method. That used to happen while solving Problems 1 and 4 in the first test.
- Students used computers not only to verify a solution to a problem, but also to confirm and/or justify reasoning used in a problem solving. Though this aspect is important regardless on the problem complexity, it can be decisive at solving problems composed of more sub-problems.

Therefore, it makes sense to pay more attention at initial stages of the computer-based instruction and teach students how to recognize situations where the computer usage can be helpful and others that are easier to deal without computer.

In other words, teaching students to consider problem complexity before deciding whether and to what extent they will use computers would help them select a more efficient way to solve a problem.

Although results of the statistical analysis seem to be inconsistent, it is not difficult to disclose their background. Results of the second test, that was done by both groups in their notebooks, without computer usage, show that there is no statistically significant difference between the groups' mean values they scored. Anyway, it is evident that the experimental group students had better scores.

However, the results of the first test, when the experimental group students used computers and GeoGebra, show that there is a statistically significant difference between the groups mean values they scored, which is in line with [6, 16, 20].

This, further, means that if one wants to work with the experimental group in a computer-based environment, then it is impossible to counterbalance the groups in regard to conditions they work and results they achieve. If we consider a complete computer-based instruction (i.e. the testing process takes place in the same kind of environment, too), a significant improvement of the instruction is achieved, but in this case its comparison with the traditional instruction is inappropriate.

A very important issue for future researches will be specifying the ways to organize all phases of such an instruction and to assess the validity of tests done in the computer-based environment and corresponding results.

Results of the third test indicate a significant difference in experimental vs. control group scores in favour of the experimental group. Comparison of results of the three tests suggest that there is no significant difference in achievement decay rates, regardless on the way the instruction was organized—with or without ICT use, which supports findings of [23]. Though the students' achievement decay rates are similar for both groups, the difference in the third test suggests longer maintenance of higher scores for the experimental group, which is opposite to what [11] suggests.

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