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ISDET 2013 CONFERENCE

Abstract. Abstracts of the plenary lecture and the four invited, introductory lectures in the section Mathematics with Informatics of ISDET 2013 Conference are presented.

MathEduc Subject Classification: A99

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The Committee for Education of the Serbian Academy of Sciences and Arts organized the conference

IMPROVEMENTS IN SUBJECT DIDACTICS
AND EDUCATION OF TEACHERS

which was held on October 24 and 25, 2013 (presented at the site:

`www.sanu.ac.rs/English/Odbor-obrazovanje/ISDET.aspx`).

Here we cite the following passage from the ISDET 2013 Conference's First Announcement:

> "In many cases education of teachers leaves them without a personal grasp of generating processes through which scientific concepts are gradually synthesized. Thereby, they are left without a deeper understanding of didactical transformation of the subject matter of the discipline they teach. The teachers' education seems to be a strategically critical period during which the improvements in this understanding can be attained. Therefore, the goal of this conference will be the raising of the quality of courses of subject didactics, focusing on a thorough reconsideration of all main teaching themes and imparting relevant historical facts as well as pedagogical, psychological and techno-logical basics essential for a specific discipline."

The conference worked in three sections: Mathematics with Informatics, Science and the Humanities. We present here abstracts of the plenary lecture (F. Arzarello) and the four invited, introductory lectures in the section Mathematics with Informatics.

The Editors

EXPERIMENTAL MATHEMATICS: A CULTURAL AND COGNITIVE CHALLENGE

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Nec manus nuda, nec intellectus sibi permissus, multum valet;
instrumentis et auxiliis res perficitur;
quibus opus est, non minus ad intellectum, quam ad manum.
Atque ut instrumenta manus motum aut cient aut regunt;
ita et instrumenta mentis intellectui aut suggerunt aut cavent.
(F. Bacon: Novum Organum, 1620)

Introduction. Many national curricula at all grades suggest involving students in the manipulation of (real or virtual) materials. The links between mathematics, natural sciences and technology, as well as the role of basing mathematics teaching on intuitive and empirical stances are in the foreground from the early documents of the International Commission on Mathematical Instruction (Bartolini Bussi et al., 2010; Ruthven, 2008; see also Smith, 1913). This represented and still represents a foundational theme for the ICMI: for example, the role of new technologies in mathematics education have been the focus of two ICMI Studies within the last 30 years and the International Community of Teachers of Mathematical Modeling and Applications (ICTMA, `http://www.ictma.net/`) has been an ICMI affiliated study group since 2003 (see `http://www.icmihistory.unito.it/ictma.php#9`). "In mathematics education, the availability of Information and Communication Technologies (ICT) has changed the landscape, including the belief that digital objects can substitute for the references to the concrete world where we live" (Bartolini Bussi et al., 2010, p.20; see also `http://nlvm.usu.edu/en/nav/vLibrary.html`). However, these changes in the landscape do not mean that we have to throw away all the past: we should risk throwing the baby out with the bathwater. In other words, modelling and applications can be pursued within "an approach that does not neglect, but rather emphasizes, the cultural aspects of mathematics, going back to the prominent founders of modern mathematics and taking advantages of the ICT support" (ibid.). This program is widely present in many researches all over the world (for a summary see: Bartolini et al., 2010). My claim is that in order to design suitable learning situations in the classroom, where manipulative materials and artefacts can be used to support learning, it is necessary carefully investigating the cultural, epistemological, and cognitive roots of mathematical concepts (Tall, 1989; Boero & Guala, 2008). This investigation will clarify how manipulative materials, instruments and ICT can help students to grasp those concepts, basing learning on what today, grounding on fresh research results (see Hall & Nemirovski, 2012), is called an embodied approach to mathematics learning. This was also present in the old documents of early ICMI recalled by Ruthven (2008) and in other coeval researches (e.g. in Enriques, 1906), generally supported more by pure speculations than by scientific research or empirical evidence. The lecture will exemplify such an approach illustrating "the generating processes through which scientific concepts are gradually synthesized", and will make this route available for teachers, "focusing on a thorough reconsideration of all main teaching themes and imparting relevant historical facts as well as pedagogical, psychological and technological basics essential for a specific discipline", as written in the program of the conference. More precisely, I will discuss how some basic geometrical concepts can be approached in the classroom, first focusing on their cognitive and cultural roots (see Tall, 1989, for the former and Boero & Guala, 2008, for the latter); second designing a consequent learning situation based on the mediation of suitable instruments.

Instruments and mathematics. The use of instruments introduces an "experimental" dimension into mathematics, as well as a dynamic tension between the empirical nature of activities with them, which encompasses perceptual and operational components and the deductive nature of the discipline, which entails a rigorous and sophisticated formalization. Many scholars have positively pointed out this dichotomy already in the past: for example, C. S. Peirce (C.P. 3.363), from a philosophical standpoint, and W. Lietzmann (1959), basing on F. Klein, from a didactic standpoint. In the lecture I will show the pedagogical possibilities offered by the tension between these two aspects with a couple of emblematic examples. Specifically, I will illustrate how the designing work can be developed in three main steps: 1. Cultural and cognitive analysis of the content to be taught. 2. Didactic analysis of teaching, possibly based on the mediation of artifacts. 3. Design of a consequent didactic situation.

Example 1. The planimeter and the area concept (grades 10–11). Drawing a cultural and cognitive analysis, a didactic gap is detected for the concept of area within the Italian secondary school curriculum (and possibly in other curricula), namely the big distance between the elementary notion of area (based on the formula "base times height") and the Riemann integral. To bridge the gap, the notion of swept area can be used, basing on an analysis of this notion in the works of Archimedes, Cavalieri and Kepler. The idea of swept area can be introduced in the classroom through the mediation of a suitable artifact, the polar planimeter, both as a concrete physical tool and as a virtual object (which simulates the other one in a virtual environment). The mediation of the artifact triggers and supports the semiotic productions of the students so that they can grasp the new concept, both conceptually and operationally. Finally the notion of didactic cycle (Bartolini and Mariotti, 2008) can be used for analysing the evolution of the didactic situation in the classroom and the interactions of students (between them, with the instrument, with the teacher).

Example 2. Walking straight on and the curvature concept (grades 9-13). Basing on an historical and epistemological analysis, three cognitive and cultural roots of the notion of straight line are pointed out: a) symmetry; b) walking straight on; c) the shortest line. All three aspects are useful when one feels immersed in an unknown space and tries to understand how she/he can produce a straight line. A didactic situation is so built, introducing suitable models and manipulative materials (world globes, cones, cylinders, paper strips, ropes ...) and solving the following problem "What does it mean going straight on familiar surfaces?". The notion of geodetic line is so introduced, and the concept of Gaussian curvature is approached in an intuitive manner. Successively (in the last years of secondary school), a physical pervasive phenomenon is investigated: the so-called Berry phase shift (Gil, 2010). This is pursued considering some concrete experiments, made with a variety of manipulative materials, and concrete or virtual artefacts: (i) slicing strips of peel from a grapefruit, crushing them on a plane and asking intriguing questions about the got configurations; (ii) studying the Foucault pendulum; (iii) using the so called Chinese south-seeking chariot (ibid.). The curvature concept allows to suitably modelling the three different phenomena in a unified manner, also developing suitable numerical computations with students.

All these experiences involve both sensory-motor and highly symbolic activities: the mediation of artefacts allows intertwining the two so that the the one can constantly be built on the other. It is worthwhile observing that such activities entail not only a cognitive behaviour, consistent with our biological being, but also cultural aspects of us as social beings. Indeed, the practices mentioned above show a deep intertwining between our cultural and cognitive components.

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THE ROLE OF THE EUROPEAN PROJECTS LEMATH AND MASCIL FOR IMPROVING MATHEMATICS EDUCATION IN BULGARIAN SCHOOLS

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Till the beginning of 20th century the educational system had a leading role in the creation and dissemination of knowledge. The acquisition, on the side of young people, of key skills and competencies needed for the everyday life, was also among the major goals of this system. This was the reason for the prestigious place the educational institutions enjoyed in those days society. The schools and universities were the place where the essential part of teaching and learning was taking place. The educational process followed a well established pattern which proved to be effective over the centuries: "The teacher and/or the textbook are the bearers of knowledge and skills. The students 'absorb' the knowledge and competences by listening, reading, memorizing and reproducing". This one-way transmission of information and knowledge from teacher to student required from the latter listening, reading, writing and, occasionally, contemplating a static picture (diagram, drawing or photograph) designed to illustrate the studied topic. The learner was expected to communicate directly and mainly with the information source only (the book and/or the teacher). Today the situation is entirely different. Due to the intensive development of science and digital technologies, the educational process, seen as accumulation of facts and life experience, takes place everywhere. It embraces not only students but grown-ups as well. Beyond school, people have a relatively easy access to a huge volume of important information which is available in an attractive, easy to navigate and dynamic manner based on the use of hypertext, videos, movies and cartoons. Through their social networks students and grown-ups share, discuss and reflect on ideas and on facts refining and improving, in this way, their information and knowledge. In general, this has to be viewed as a very positive development from point of view of education. It would not be a great exaggeration to say that this is already a new way of "learning about the world". There are side effects of this development however which are far from being positive. The school is no longer the only home of educational process. The students get alienated from the traditional schooling which seems boring and repelling to them with its one-way information flow (from teacher to students mainly, without intensive interaction within the group) and the reading and writing as the basic communication channel. Though unavoidable and absolutely necessary for the development of mind and personality, reading and writing as channel of communication is rather slow. It is "linear" in nature (goes letter by letter) and inferior in capacity to the communication using pictures and dynamic images.

Another factor that contributes to changing attitude to school is the easiness by which one can get answers to various questions through popular search engines on the web or through asking other people from the own personal network. This conveys the wrong idea that the answers to most of the questions already exist and it is only necessary to retrieve them. Correspondingly, many young people are no longer firmly convinced that they have to spend a considerable part of their lives in an educational institution. Another negative consequence of this "ready answer philosophy" is that the ability to find the correct answer to a certain question by own reasoning, by using logic and own knowledge, by experimenting, is not cultivated, developed and appreciated. We should not forget however that one of the major goals of education is to learn how to find answers to questions and solutions to problems never studied before. The question how to counteract these negative trends is not an easy one and does not have a unique answer. In the talk I will present the essence, the structure and the realization of two European Projects: LE-MATH (Learning mathematics through new communication factors) [1] and MASCIL (Mathematics and Science for Life) [2] which have the potential to improve, at least to some extent, the above discussed situation in the field of mathematics education.

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INTEGRATING NEW TECHNOLOGIES IN CURRICULA AND TEACHERS COMPETENCES

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Development of science and technology, social and cultural development, and mass media development, require changes in curricula, especially in the context of teaching specific subjects and its way of realization. Research has proved that making curricula should be preceded by deep scientific analysis of more adequate adjustment of science and technology development, social community needs, school system possibilities, pupils' needs, interests and abilities. As a rule, experimental testing of new curricula is not carried out before their mass practical use in educational processes at all levels and, without it, there is no analysis of their mass applicability. The fact that in the process of teaching, when curriculum is overloaded, pupils' memory is burdened (facts description, historicism, data strict definitions, etc.), their free expressing is limited, which restrains critical thinking and more immobilizes than encourages creative learning. Wanting to realize the official curriculum under such conditions, the teacher is in a situation to give much more facts and definitions, neglecting his/her own wish to do it, possibilities and interests of pupils, their needs to inquire and to express themselves in the teaching process in their own way and to use more independently sources of knowledge which are not modelled by curriculum. We will present an analysis of curriculum changes in the case of the Faculty of Education, Belgrade University. We particularly stress conclusions concerned with research done in contact with graduate students and teachers whose attitudes are very important for changing curricula. Modern teaching aids make it easier for pupil to learn more quickly, develop critical thinking and creative abilities; form his/her own view of the world, create his/her individuality and personality. Innovations in teaching and learning require training of teachers for team work, work with small groups as well as mentor work; furthermore for preparing teaching sheets, learning packages, semi-programmed and programmed materials and application for multimedia aids (distance learning, multimedia, educational television, artificial intelligence etc.). Naturally, it is not realistic to expect that these innovations in teaching can be done by teachers themselves. A pedagogical idea that has been widely realized is that each school having 500 or more pupils has two or three teaching assistants who are at disposal of the teacher when he/she organizes group and individual work, when it is necessary to follow various independent activities of pupils, to follow their work and provide various aids and materials. In such cases, the assistants do all routine work (especially administrative), so that more time is left to teacher to do creative pedagogical work. We also propose all-day school so that pupils can do all their assignments at school. Although it is not a new idea, it requires new organization in preparing teachers and their practical work. With better professional methodical training and more pedagogical knowledge, with readiness to accept innovations and to improve his/her own work and pupil's learning, teacher can, with increased efforts and support of community, make teaching closer, more interesting and acceptable to pupils. Frontal, group and individual forms of work can be supported by the use of computer and the other teaching aids and materials, assignments which pupils have to do can be presented and checked, level of knowledge and degree of understanding assessed. Computer can be used in the process of learning, checking and evaluating of pupils work, making possible quick, precise and objective insight into what pupils know and what do not. That stimulates pupils' motivation, provides their more active participation in learning process and contributes to their intellectual growth. We intend to present results of our research on teachers' capacity in the domain of new technologies and their readiness for perfecting that kind of knowledge and skills. Integrating informational technologies in curricula should affect all subjects, methods, forms of work, evaluation and organization of teaching and learning process. This makes clear why teachers need life-long learning to improve their knowledge and skills according to imperatives of ever-changing, modern society.

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SETS IN SCHOOL MATHEMATICS

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According to the view of this author, an integrated course of didactics of mathematics consists of some relevant topics from history of mathematics, educational heritage and psychology which, together with a good knowledge of subject matter, make a basis upon which didactical analysis of the main teaching themes of school mathematics is performed. Basics of set theory in school mathematics is such an extremely provocative theme that we intend to discuss here, without going into details that project the ways of didactical transformation of this content. When in the course of the eighth decade of 19th century this theory was created (G. Cantor, R. Dedekind) as a language more general than traditional languages of algebra and geometry, its fundamental concept of set was raised at the level of being more general than all other concepts of classical mathematics. Namely, each of those concepts could be defined to be a set with its differentia specifica being added. Thereby, this theory had a great effect on logical foundation of mathematics. Taking of this theory as a basis for innovation and reorganization of school mathematics led to a far reaching reform known as "New Math". Lacking a right didactical ground that reform was doomed to failure, though the attempts made to organize systematically the subject matter and the idea of its structuring is an enduring inspiration. And what followed as a counter reaction, elements of set theory have been much reduced in or completely banished from school curricula. Having its examples at all levels of abstraction, the concept of set can be acquired only gradually and after years of learning. Fixing these levels dependently on the concrete content of school mathematics, we distinguish the following ones: sets at the sensory level, sets having conventional symbols for their elements and sets given by attributing characteristic property to their elements.

1. Sets at the sensory level. To fix a level of abstraction for a concept means to limit the kind of corresponding examples and their extent. Thus, when the concept of set is at the sensory level all corresponding examples are observable groups of objects existing in the real world or being represented iconically. Many words in natural languages as, for example, in English "flock", "brood", "shoal", "bundle", etc. mean exactly the sane as the word "set" does, except that they specify the nature of grouped objects. Today, teaching of mathematics creates that universal meaning of terms "set" and "element (member) of set" which has not been spontaneously formed in any natural language. In arithmetic, the meaning of natural numbers has always been based on the intuition of set (experience of discrete realities), no matter if the word "set" has or has not been explicitly used. To express precisely the processes of creation of numbers and arithmetic operations, the language of set theory should indispensably be the part of vocabulary of didactics of mathematics. And as for the actual classroom practice, from our point of view, the meaning of terms "set" and "member (element) of set" should be developed spontaneously by using them. Set theoretical operations should not be defined formally at this level, but they should be expressed descriptively using terms in which concrete examples are formulated. We give arguments that support and examples that illustrate this point of view.

2. Conventional symbols as elements of sets. This is just an intermediate level, when brackets are used as a notation for sets and when a finite number of conventional symbols (number signs, letters, etc.) are their elements. Related to the solving of the simplest inequalities in natural numbers, examples of these sets are primarily the sets of solutions of this type of inequalities We provide some examples and fix the main didactical tasks attached to this theme. Confusing two levels and the way of iconical representation, when elements of sets are circled by lines, with the syntactic way, when brackets are used, many "barbaric" blunders are encountered in school books, research papers etc.

3. The case of attributing characteristic properties. A canonical form of inequality and the set of its solutions determine each other uniquely. This correspondence is gradually extended to the case of correspondence between a formula and its truth set. When a set S is taken to be determined by the formula $p(x)$, the symbol

$$S = \{\, x \in U \mid p(x) \,\}$$

is used as a means of determination of the set S. This is the case when it is said that the property p(x) is attributed to the elements of the set S. The set U is the universal set of the variable x. Such universal sets are the number systems: $\mathbb{N}$ – the set of natural numbers, $\mathbb{Q}$ – the set of rational numbers, $\mathbb{R}$ – the set of real numbers and $\mathbb{C}$ – the set of complex numbers, while the corresponding variables are n, q, x, z respectively. The spaces $\mathbb{R}^2$ and $\mathbb{R}^3$ and the corresponding variables (x, y) and (x, y, z) are additional examples. A particularly important aspect of this theme is the correspondence between set theoretical operations and logical operations, when the former content, as being more intuitive, serves as a basis for interpretation of the latter one. Touching this issue we will provide some related details.

CURRICULA THAT BUILD THEMSELVES AND MATHEMATICAL SUMMER SCHOOLS

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We review and compare two of the most recent developments in the dynamical world of mathematical education which have attracted worldwide attention of mathematicians. The first is the Common Core State Standards Initiative as a global U.S.A. education project for reforming mathematical curricula on the national level. We focus on the Illustrative Mathematics Project, http://www.illustrativemathematics.org/, as one of the accompanying projects supported by Bill and Melinda Gates Foundation. This project, envisaged as a collective effort of many professionals and a networking service for teachers, distantly resembling Wikipedia, Facebook or Math Overflow, offers a glimpse into a possible 'online' future of mathematical curricula development. This is also an opportunity to recall some of the earlier dilemmas and controversies related to K-12 mathematics education, including the debate over modern mathematics education, textbooks and curricula in the United States known as "Math wars". The second development is the organization of summer schools for gifted young students, as a joint effort of some leading European universities, with the participation of some of the world most renowned research mathematicians. One of the latest issues of the American Mathematical Monthly (March 2013) was devoted solely to the International Mathematical Summer School at Jacobs University in Bremen. "The Monthly", as one of the Mathematics Association of America oldest periodicals (published since 1894), has a long tradition in publishing articles about teaching of mathematics, as illustrated by "ten commandments for teachers", by George Polya (Vol. 65 (1958), 101–104), "What is Teaching?" by Paul R. Halmos (Vol. 101, No. 9 (Nov., 1994)), and the humorous report of Andre Weil (Vol. 61, No. 1, Jan., 1954) for a "joint meeting of the Nancago Mathematical Society and the Poldavian Mathematical Association". Mathematical summer schools for the gifted students have also a long tradition in Serbia, as well as the reforms of the curriculum and the associated debate, dilemmas and controversies. By illustrative examples, formally from different levels of mathematical education, we try to understand what is stable and what keeps changing, what the latest tendencies are, and what may be expected to become a standard in mathematical education in the years to come.