MILOSAV MARJANOVIĆ

Curriculum Vitae

Milosav (Milo) Marjanović was born on August 24, 1931 in Nikšić, Montenegro, where he completed both his primary and secondary education. He received a diploma in mathematics from the Faculty of Natural Sciences, University of Belgrade in 1955 where he began to work as a teaching assistant in mathematics in 1957. He received his Ph.D. degree in mathematics from Belgrade University in 1964 with the thesis *Moore-Smith Convergence in General Topology* (supervisor Professor Đ. Kurepa). He spent his entire professional career affiliated with the Department of Mathematics of the Faculty of Natural Sciences, University of Belgrade where he was appointed as an assistant professor in 1964, associate professor in 1969, and where he attained the position of a full professor in 1980.

He spent the 1959/60 school year at the Mathematical Institute of the Polish Academy of Sciences in Warsaw, joining the research seminar led by distinguished Polish mathematicians J. Mikusinski and R. Sikorski. During the 1967/68 school year he had a visiting position at the University of Florida at Gainesville where he joined the research seminar of the outstanding Dutch topologist J. de Groot.

His main results are related to the topologies on collections of closed subsets. He gave a complete classification of topological types of the spaces of closed subsets in the case of zero dimensional, compact, metrizable spaces. His "beautiful theory of accumulation orders and spectra" (words of S. Todorčević) led him to discover some subtle properties of the products of these spaces (see the overview on the page 68), which allowed him to extend and complete some results of G. Choquet, V. Ponomarev, A. Pelczynski, P. Halmos, and others.

As a university professor he substantially contributed to the mathematics education at all levels, from the elementary school to the university, by producing highly influential textbooks and research papers, and by having an important role in the innovation of school and university curricula.

M. Marjanović was elected in the Serbian Academy of Sciences and Arts as a corresponding member in 1976 and as a full member in 1991.

AN OCCASIONAL TALK WITH PROFESSOR M. MARJANOVIĆ

The occasion for this talk was Professor Milosav (Milo) Marjanović's eightieth birthday. At the beginning of our meeting we agreed to refer to each other as usual, by our first name. As the reader will see, my simple "secret" strategy was to pose short questions, allowing the professor's narration to proceed uninterrupted.

Rade: Can you, please, tell us something about your school-days?

Milo: Since my school-days more than sixty years have passed. Some images of people and events still exist in my memory and I will try to evoke them as authentically as possible.

I spent all my school-days living in Nikšić (Montenegro), where I graduated from the Gymnasium in 1951. It was a time when school teachers did their jobs with more dignity and their sense of duty also combined a feeling that they worked for the good of society. Some of them had impressive personalities what was projected to their students to form strong characters. Expressing my gratitude to all of them, I would give special attention to two of them, Vasilije Adžić was my mathematics teacher. He was a big good-natured man. An anecdote was told about his father, a Montenegrin general and the man of gigantic strength. When a cadet and when he was taking an exam, he was asked what he would do with his heavy gun if Turks were overcoming. The expected answer was to take out the breech and run away. But the young man said that he would put the gun over his shoulders and run away. "Oh, yes, you passed the exam", were the words of the examiner.

Vasilije graduated from Mathematical Department of Belgrade University somewhere in the early 1930's. But from some reasons he never took the so-called state examination and he stayed unemployed until the years after the Second World War. During that time, as he sincerely admitted it, he had forgotten all his mathematics. Starting to teach after so many years, he had to learn school mathematics anew and, judging according to his lectures, he did it very successfully. His lessons were carefully prepared and his teaching clear and systematic. I was his favorite student and he was particularly proud of me when in the last issue of Matematičko-fizički list (a monthly edited in Zagreb and dedicated to the school mathematics and physics), I was praised as being by far the best solver of problems for 1950/51 school year.

Vladimir Mijušković taught "Serbo-Croatian", a subject that we shortly called "Serbian". He was inducing us to develop our own reasoning and imagination by reading and analyzing the literature. His estimation of our written compositions surpassed everything else, while he was slightly ignoring the grammar and the preaching of moral lessons.

My high school was a very good and demanding school of general education and we had to put a lot of effort into studying all subjects we had.

Rade: The years you spent studying at the university are considered to be very difficult for this country. What do you have to tell us about your own experience in that time?

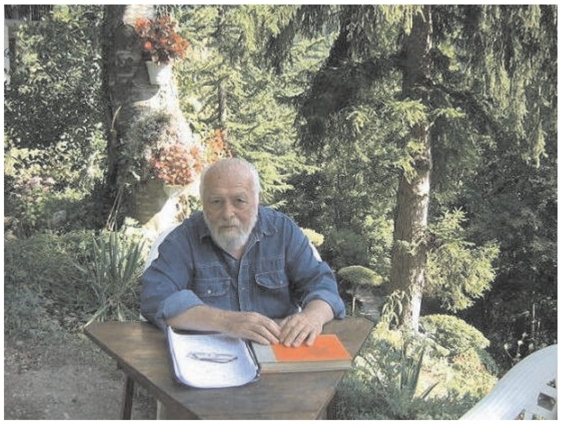
Milo: My student's days began with some problems. Namely, at the very beginning of the school year, a meeting was called where I was strongly attacked by the communist youth activists and accused of being a reactionary. In some similar cases, a number of students were expelled from the university with a verdict of permanent loss of rights to continue studies. In my case, everything was over with that meeting and I did not experience such bad consequences.

As for the curriculum, it stayed almost the same as once fixed (in 1905) according to the dominating tradition of the 19th century university education. In order to describe the character of that curriculum, it is enough to say that our mathematical courses did not comprise any ingredients of set theory, linear algebra, theory of groups, etc. As you have said it, it was also the time of heavy poverty caused by the destruction during the Second World War, augmented by the sanctions that Soviet Union and the other satellite countries imposed on Yugoslavia. Once very cheap Russian math books were not available any longer in the book shops and, due to the lack of foreign currency, nor those from the western countries. To illustrate this situation, I must mention that a collection of mathematical problems in Russian for students of engineering was the only foreign book that I managed to buy in a secondhand book shop up to 1957. At that time the students were taking notes while the lecturers were speaking. An exchange of those notes was a common practice. Was it not the worst period for studying at the Belgrade University since its foundation in 1905?

While a student, I was able to learn the content of the simple courses we had, just by attending the lectures. So I had a lot of spare time to use in this or that way. Since I was living in the student's dormitories, a part of that time I was inevitably wasting on watching my student mates playing cards or on "infinite" talky talks about football and the other sports. In order to also use the part of that time for some extra learning, I asked my favorite professor Nikola Saltykov (born in 1864) to recommend some books to me that could be ordered thanks to the opportunity offered by French Embassy in Belgrade. He suggested some books being evidently highly praised in the time of his youth. Thus, I purchased Goursat's two volumes of analysis, Darboux's geometry, etc. The language and the spirit of those books were rather old fashioned and quite different from the one I encountered when peeping into some scientific journals. Of course, a right recommendation would have been to order some contemporary university text-books if there had been anybody to suggest it.

Once already retired, Professor Saltykov belonged to the group of professors being put back to service in the post-war period. He had all necessary conditions to achieve a double pension, one for the period he worked in Russia until 1920. But due to the rather slow Soviet administration, he did not succeed before he died in 1959. Professor Saltykov was a good traditional mathematician and being quite a character, he was liked very much by his students.

Two books—Kurepa's Set Theory (in Serbo-Croatian) and Sierpinski's Nombres Transfinis arose my interest in set theory. Thus, I graduated in 1955 with a written graduation work on transfinite ordinals.



Milo at his mountain retreat

Rade: And then?

Milo: The following year I spent in the army as a part of my compulsory military service. In the autumn 1956 I applied for the post of a university assistant. Told that everything should be ready in a month or so and that they would inform me sending an official letter, I went back to my home town. I was waiting to receive that letter but it did not arrive. In January 1957 I was offered a four month job. Namely, all workers intending to have so called high qualification and not having the eight year elementary school finished, were obliged to take a course having a shortened and compressed curriculum of the upper classes of elementary school. My classroom was a nearby big garage and my students some fifteen heavy truck drivers of age 25–40. Their work-day consisted of making two tours on a poor, 40 km long, macadam road, transporting timber. Every afternoon they had their lessons and they were coming to school with their Wellington boots on. I was their mathematics and physics teacher and they were my first "pupils". The desire of those hard-working men to learn and understand was a pleasant surprise for me as well as it was their behavior in the class—exactly the same way of mutual competition that I remembered from my school days.

When my teaching duties were over and with some money earned, I went to Belgrade to see what happened with my application and why my letters had not been answered. First, I contacted Professor Tadija Pejović who was the chairman of the Mathematical Department at that time. He told me that he had written a report suggesting my choice the year before and he sent me to see the administrative secretary of the Faculty. Sitting at a table in his office, that man said briefly that he knew nothing about it. When I asked him politely to look in his files, he showed me the way out. Professor Pejović was sure that the report was lying in the secretary's files, but he evidently felt some uneasiness to enter that office himself. Thus, he directed me to contact Comrade Vojin Dajović, then an assistant at the Department and probably a party functionary of some importance. Dajović promised me that he would see what was going on and, in two weeks, everything was settled down. That was the way how my university career began which lasted until 1998.

At the end of this tale, I have to say that in the time that followed I had to have more contacts with the administrative secretary and my impressions of that man were not as bad as it could be drawn from a single episode. As for Professor Dajović, he was a respectable member of our Department, particularly known as a good planner and organizer.

Rade: What was the way that you, young assistants, were helped by your professors to start with some research?

Milo: There was no help at all from their side and we were left to care after ourselves. I certainly was not good enough at finding a right way. I continued to purchase classical mathematical books in French: Frechet's topological spaces, Denjoy's books on transfinite enumeration, etc. I expected that the reading of those books would be an opening for doing some research. But on the contrary they blocked my advancement for a number of years. Although two things happened that were of some importance for my professional development. I finished two tape recorder courses of English, each lasting four months. Thus, mathematical books in English became quite approachable for me. In the same time, I tried twice to get a scholarship to Moscow University, but I failed. After the second try, I was informed that a number of scholarships were left and I was invited to check if some of them might be of interest for me. Knowing that Poland is a country of excellent mathematicians, I decided to get a scholarship provided by that country. Thus, I joined the seminar of J. Mikusinski and R. Sikorski intending to study their original approach to the theory of distributions. Not being adequately prepared, I had to learn some contents that now undergraduate courses comprise and not knowing the Polish language was also an extra disadvantage. It was already the time when once existing enthusiasm for the theory of distributions began to somewhat decrease. The reason for it is the fact that a distribution, which is also called generalized function, need not have a value at a point of its domain. The Lojaszewicz's condition for the existence of the value of a distribution at a point was a topic of research at the mentioned seminar. I also wrote a paper related to that matter, but my interest for this mathematical discipline did not last.

Rade: What was the turning point? When and what did happen changing such situation in the Mathematical Department?

Milo: In 1960, the Government passed a law according to which all universities had to organize two year post-graduate studies. In the case of mathematics, and in the early stage, these studies consisted of four subjects: measure theory, functional analysis, algebra and topology that all students were supposed to take and, after passing the exams, they had to write a master thesis. In a few years this simple model of the post-graduate studies was modified and those subjects were scheduled as the undergraduate courses for the third year students. That was also a stimulus for innovation and modernization of the courses that already existed on the curriculum as well as a motivation for planning new ones. That was the way how the contemporary mathematical contents arrived at our Department. I also have to mention that starting with the seventh decade of the 20th century the country (former Yugoslavia) was more and more open to the transfer of goods and the communication of people. Remembering that time I can't afford not to mention all those cheap math books in Russian found again on the shelves of the bookshops. A part of them was an excellent choice of translated courses and monographs being several times cheaper than their originals in English, French, German, etc. We all who were buying and using them have to pay tribute to that opportunity.

Our university courses have always been consisted of two parts—one theoretical taught by a professor and the other practical carried out by an assistant. During the first years of our post-graduate studies I was terribly burdened by carrying out practical courses in measure theory, functional analysis and topology. At that time the books collecting exercises in these areas of mathematics did not exist which made carrying that task out more difficult. As a positive side of such circumstances was the fact that I had to study basics of those subjects thoroughly.

Rade: How did you acquire a more intensive interest in topology?

Milo: The content of the topology courses that were taught at that time consisted of a too long introduction covering neighborhood spaces, then abstract concept of topological spaces, separation axioms, compactness and connectedness, all being given without much of concrete material. That material was easy to be acquired but a more fascinating topology peeped out from some books when I was turning over their leaves. The content of such books became a vague and remote aim which occupied my attention for years. But the reality had its own course.

As there was nobody in our Department to teach topology, it was the idea of Professor Pejović that I would write a doctoral thesis on topology. Without having a supervisor to guide me, I bought J. Kelley's book "General Topology" and I gathered a number of research papers written by mathematicians belonging to the school of topology at Moscow State University, headed by Paul Alexandrov. I found the papers treating topologies on collections of closed subsets the most interesting, in which direction I continued to do some research years later. After getting my doctoral degree I was elected as an assistant professor and my assignment was a course of mathematics for physics students having six hours of lecturing per week.

Twelve years passed before I had a chance to teach a course for mathematics students. It was a two year course of mathematical analysis, which I modernized including for the first time some contemporary topics: inverse function theorem, definitions of curves and surfaces independently of parameterization, line integrals and surface integrals as concepts depending on these geometric objects and the functions defined on them, etc. The colleagues who took on teaching this course preserved that conception improving it further.

Fourteen years more passed before I had a chance to teach a topology course. The plan of Professor Pejović was different from the plan of some more influential members of our Department. Namely, Professor Adnađević, a member of the staff of the Faculty of Mechanical Engineering was administratively taken over to our Department and he taught topology for years.

Milo trims these trees (hornbeams, oaks, spruces)
so that they preserve their bushy shape for years

Rade: What was the conception of your topology course?

Milo: According to the existing curriculum the content of that course covered exclusively general topology. But I compressed it to the first semester and I used the second semester to cover basics of combinatorial topology—polyhedra, fundamental group and its calculation, Brower theorem, classification of 2-manifolds, etc. Simply, my opinion is that it is disappointing for a student to take a first course in topology and to not know why, say, a sphere and a tore are topologically different. Such a conception of that course still exists.

Rade: Do you feel that you were somewhat isolated and blocked from contacts with mathematics students?

Milo: Thinking of that period of my career I am easily struck by the idea that I was somewhat ostracized. Being "tempered" of solid material, patience and persistence are essential characteristics of my personality and they, together with a rather unfavorable environment, have determined my professional life. If I am talking about some troubles I had, it's not to cause pity, but to point out that the conduct of some people in some critical situations may be below academic norms. Thus, I think that we should not only be paying respect to them but also we should analyze their entire work. By the way, I like those who are the same independently of situations they go through.

Rade: Your former students hold you in high regard as a superb lecturer.

Milo: From time to time, I meet some of them and they always speak of me as a lecturer they liked. If there is something special in my “art” of teaching then it is due to a careful preparation of lessons, a profound knowledge of the subject matter, passing of historical references and, I also have to add that I am a “peripatetic teacher”, who covers quite a distance walking in front of a blackboard keeping so my audience not to fall asleep.

Rade: You are evidently a self-educated topologist. How did you acquire knowledge about the areas of topology which mostly attracted your attention?

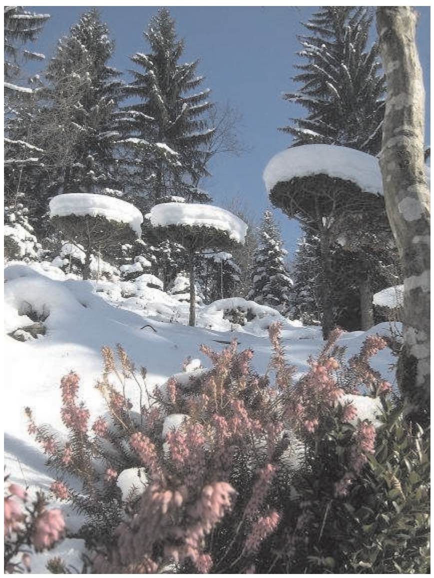
Milo: At the right time I formed an opinion that the core of that discipline is the study of topological properties of geometric objects (polyhedra, CW complexes, manifolds, etc.). And I also realized that such a study is approachable only if one is acquainted with the delicate machinery of algebraic topology. But at that time it was not an easy task. I think the situation was similar as it had been with *Calculus* in the past. Where a maitre existed, there was the possibility to think of learning such a complex branch of mathematics. Now I have to recall my tries and failures to accomplish that difficult task.

As a voluntary engagement, I was using a classroom in the evening hours for introductory lectures covering basics of algebraic topology. There was always a small group of students ready to follow my lectures. I do not know if they suspected that I had to learn my teaching material just a couple of days before. Selection of a proper book was the most difficult problem. My first try was to use Hu’s book Homology Theory, but a long formal content which comes before some deeper geometric facts was a fine play with concepts and symbols not having a clear motivation. I also tried to use some big reference books on topology, but it was also a wrong choice. As it is symbolically said, such books are very good for burying your youth. After years of tries and failures, I found that Maunder’s Algebraic Topology was the kind of book that I was longing for. Along with the learning of its content I was presenting it to a small group of students for a period of two years. But then, it was also the right moment to cast a look back and to see how a long time has passed and that the role of the bearer was to be taken over by my younger colleagues who shared with me the same admiration for this fascinating branch of mathematics. Thanks to them I could say that my wish to see algebraic topology being established at our university came true. Of course, you easily recognize yourself among the younger colleagues that I have just mentioned.

At the end I find appropriate to mention two different trends that prevailed at our university—one having roots at the Mathematical Department and the other at the Faculty of Electronic Engineering. The main characteristic of the former was a permanent effort to overcome lagging behind the contemporary development of mathematics and of the latter researching in some easier approachable and less complex areas of mathematics. Comparing these trends, in the case of former a smaller number of research papers exists but some of them are of the great class, while in the latter case that number is greater as well as it is the money given by the government. Which one is really more successful only the time will tell.

Rade: Each of the natural sciences has for its object of study this or that aspect of physical reality. What would you say about the object of study of mathematics?

Milo: Some inner representations of the surrounding reality exist in our mind. Psychologists call them mental images and they are not replicas of any real object. Instead, they are representations of the classes of objects that have been experienced. That inner world of ideas is the basis upon which mathematical concepts have been created. Among them the most fundamental are the ideas of shape and number which play a prominent role in the cognitive activities of human beings. If we take shape to mean morphology type with all varieties of isomorphism that determine it, then the definitions found in encyclopedias and stating that mathematics is the study of shapes and numbers is quite satisfactory.



Erica carnea is the first to blossom (in February)

Rade: What would you consider to be the greatest achievement in the whole history of mathematics?

Milo: In my case, I would decide on number systems existing in the contemporary mathematics. From the time of the first civilizations on, the natural numbers (and positive rationals) have always been related to discrete realities. The meaning of such numbers, as it was definitively formed in the time of Pythagoreans, stayed essentially unchanged until the present day. The story of genesis of the real number system is much more complex and I will briefly present it in the way I see it. But I will present first the process leading from perception to conception in the case of such a fundamental concept as the line segment is.

The idea of shape is commonly associated with geometric solids. But the line segment is a particularly important shape. When an object extends in the space between two of its ends, then a straight stick joining these ends is a pretty good representation of this extension. Abstracting the thickness of the stick, the idea of line segment arises as a means of representing (and measuring) the extension in the space. Let us also note that when quantities of different kinds are measured, the amounts are transposed to a scale. This clearly demonstrates that the line segments are also the purest forms which represent quantities.

When Pythagoreans discovered incommensurable lengths, it became evident that pairs of natural numbers were insufficient to be corresponded to all ratios of geometric magnitudes. That crisis was logically overcome by Eudoxus' theory of proportions. Technically complicated, formulated in geometric terms, limited to the ratios of magnitudes of the same kind, this theory delayed a more abstract development of the real number system for centuries.

If an abstract concept also needs to have a corresponding denotation, then this story cannot evade Vieta's *logistica speciosa* in which letters were used to represent species of numbers. Vieta's algebra was up to some degree rhetoric and the operating on magnitudes was an obstruction for its further development but with that algebra a clear idea of variable appeared for the first time. Further improvements followed, among which the most significant was René Descartes' geometric model of the real number system. Taking a half-line and all segments on it having one of their ends at the origin (the half-line end) and choosing a unit length, Descartes created his model in which one line segment became the representative of an infinite class of equivalent ratios and so a unique conveyor of the meaning of a real number. Moreover, by performing simple geometric constructions (construction of the fourth proportional), products and quotients of two real numbers were again real numbers represented by the corresponding line segments. In that way, the binding condition of homogeneity, still present in Vieta's algebra, was eliminated. Adding to it Descartes contributions to mathematical symbolism, an independent system of real numbers was created producing the development of rich analytic methods in mathematics and physics. Making a comparison with rhetoric algebra, we could say that the symbolic algebra compressed our words and accelerated our thoughts very much. By the way, could we imagine the birth of Calculus without these great achievements? A particularly great effect that these achievements had is the conceptualization of the space in the form of analytic spaces. Wonderful geometric spectacles in these spaces that were opened to the human mind made a difference that could not be even mistily foreseen in the previous times.

Rade: At the end, what would you say about your life as a mathematician?

Milo: I have been somewhat drawn out from personal existence into the world of contemplation. Analyzing my own personality I find that curiosity has been the most dominant drive of all others that have shaped my life. I find this to be enough to say.

Rade: Thank you very much.

Milo: Thanks go to you for letting me talk at such length.

A REVIEW OF SELECTED PUBLICATIONS OF MILOSAV MARJANOVIĆ

[1] *Topologies on Collection of Closed Subsets*, Publications de l'Institut Mathématique, T. 6 (20), 1966, pp. 125–130.

The study of topologies on the set $2^X := \{F \subset X \mid \emptyset \neq F = \overline{F}\}$ of all nonempty, closed subsets of X has been one of important research themes of general topology, with contributions of many topologists, among them Vietoris, Michael, Ponomarev, etc. The paper provides a systematic (axiomatic) approach to the problem of generating and classifying topologies on 2^X, with a particular emphasis on topologies which are *admissible* in the sense that the map $I : X \to 2^X$, $x \mapsto \{x\}$ is a homeomorphism. The input for the construction is a function α with the domain $D(\alpha) = 2^X$ such that $\alpha(F)$ is a family of covers of F, satisfying three natural axioms (α_1)–(α_3). The associated topology $(2^X, \alpha^*)$ on 2^X turns out to have all the favorable properties, including the Vietoris-type compactness property. For particular choices of the generating function α, specifying the families of covers, one obtains lower (upper) semi-finite topology, finite (Vietoris) topology, etc.

Different topologically equivalent metrics may induce different topologies on 2^X by means of the associated Hausdorff metric. It has been shown that the supremum of all these topologies is another example of a α^*-topology, corresponding to the case when the covering families in $\alpha(F)$ are locally finite (see P.L. Sharma, Proc. Amer. Math. Soc. 103(1988)). In different direction, the compactness property of 2^X is used to reduce the proof of the Blaschke Convergence Theorem to a lemma which states that the limit set of a sequence of convex sets is again convex.

Another important idea introduced in the paper is a method of representing function spaces as subspaces of α^*-hyperspaces $(2^X, \alpha^*)$. For example, given a family $\{X_\alpha\}_{\alpha \in I}$ of compact spaces, the topological product $\prod_{\alpha \in I} X_\alpha$ can be embedded as a closed subspace of the hyperspace of the one-point compactification of the sum $\bigoplus_{\alpha \in I} X_\alpha$. This elegant result, which provides a link between two classical results, the Tychonoff product compactness theorem and Vietoris hyperspace compactness theorem, is included in the well known monograph R. Engelking, General Topology, Warszawa, 1977.

[2] *A Pseudocompact Space Having no Dense Countably Compact Subspace*, Glasnik Matematički, Tom 6 (26) - No. 1 - 1971, pp. 149–151.

A topological space X is pseudocompact if each continuous real-valued function on X is bounded and X is countably compact if each sequence in X has a cluster point. For a period of time all known examples of pseudocompact spaces had an everywhere dense countably compact subspace. This motivated S. Mardešić and P. Papić to pose the question of the existence of a pseudocompact space having no everywhere dense countably compact subspace (New Scottish Book of Problems). In this note the first example of this kind of a pseudocompact space was constructed.

[3] *Exponentially Complete Spaces III*, Publications de l'Institut Mathématique, tome 14 (28), 1972, pp. 97–109.

Perhaps the most important and far reaching work of M. Marjanović is his "*beautiful theory of accumulation orders and spectra of compact metric zero dimensional spaces*" (S. Todorčević, Topics in Topology, Springer, 1997).

Suppose that X is a zero dimensional compact metric space. One inductively associates to each point $x \in X$ a natural number or "infinity", $\mathrm{Ord}(x) \in \mathbb{N} \cup \{\omega\}$, called the *accumulation order* of x. The increasing sequence $\{n \in \mathbb{N} \cup \{\omega\} \mid (\exists x \in X)\, \mathrm{Ord}(x) = n\}$ is called *the accumulation spectrum* of X and denoted by $s(X)$. It can be shown that if $s(X) \neq \mathbb{N} \cup \{\omega\}$ then for some $n \in \mathbb{N}$ either

$$(1) \qquad s(X) = (0, 1, \dots, n-1, n) \quad \text{or} \quad s(X) = (0, 1, \dots, n-2, n).$$

A space is called *full* if for each n the closure of the set $X_n := \{x \in X \mid \mathrm{Ord}(x) = n\}$ is either empty of homeomorphic to the Cantor set C.

The first fundamental observation of the theory of accumulation orders is that two full spaces X and Y are homeomorphic if and only if $s(X) = s(Y)$. A transparent description of the class of full spaces with a finite spectrum is provided by the following construction. Let C_{-1}, C_0, C_1 be respectively the empty space, a singleton, and the Cantor set. Define recursively C_n as the space

$$C_n := C \cup \bigcup_{I \in \mathcal{F}} (C^I_{n-1} \oplus C^I_{n-2})$$

obtained from the Cantor set C by inserting a copy of $C_{n-1} \oplus C_{n-2}$ in each of the removed intervals I. All these spaces are full. Moreover,

$$(2) \qquad s(C_n) = (0, 1, \dots, n-2, n) \quad \text{and} \quad s(C_n \oplus C_{n-1}) = (0, 1, \dots, n-1, n).$$

Given a (zero dimensional, compact, metric) space X, the associated *hyperspace* $\exp(X)$ is the space whose points are non-empty, closed subsets $F \subseteq X$, topologized by the Hausdorff metric (Vietoris topology). M. Marjanović proved that, besides the trivial cases of the spaces $\exp(C \oplus [n])$, where $C \oplus [n]$ is obtained from the Cantor set by adding n isolated points, there exist precisely seven possibilities for the topological type of the space $\exp(X)$,

$$C_1,\ C_2,\ C_1 \oplus C_2,\ C_3,\ C_4,\ C_5,\ C_7.$$

From here he concluded that X has the property that $X \cong \exp(X)$, if and only if X is one of the following nine spaces,

$$C_0,\ C_1,\ C_0 \oplus C_1,\ C_2,\ C_1 \oplus C_2,\ C_3,\ C_4,\ C_5,\ C_7.$$

These results allowed M. Marjanović to complete earlier partial results obtained by G. Choquet ($C_1 \cong \exp(C_1)$) and A. Pelczynski ($C_2 \cong \exp(C_2)$) and to generate examples of non-homeomorphic, zero dimensional, metric, compacts X and Y such that $\exp(X) \cong \exp(Y)$ (problem of V. Ponomarev).

[4] *Numerical Invariants of* 0*-dimensional Spaces and Their Cartesian Multiplication*, Publications de l'Institut Mathématique, tome 17 (31), 1974, pp. 113–120.

In this paper M. Marjanović addresses the problem of determining the spectrum $s(X \times Y)$ if the spectra $s(X)$ and $s(Y)$ are known. This problem is equivalent to the problem of describing the function $\mathcal{M}(m,n)$ such that for each $(x,y) \in X \times Y$,

$$\mathrm{Ord}(x,y) = \mathcal{M}(\mathrm{Ord}(x), \mathrm{Ord}(y)).$$

He discovered a remarkable formula

$$\mathcal{M}(6m_1 + r_1, 6m_2 + r_2) = 6(m_1 + m_2) + \mathcal{M}(r_1, r_2)$$

where for $r_1, r_2 \in \{0,2,3,4,5,7\}$ the corresponding value $\mathcal{M}(r_1, r_2)$ is explicitly calculated. Among the consequences of this analysis is the observation that

$$X = C_{6n+2} \oplus C_{6n+3} \quad \text{and} \quad Y = C_{6n+5}$$

are non-homeomorphic, zero dimensional, compact metric spaces, such that $X^2 \cong Y^2$, which answered a well known problem posed by P. Halmos in his *Lectures on Boolean Algebras* (Van Nostrand, 1963).

Let us remark that M. Marjanović and A. Vučemilović (Comm. Math. Univ. Carolinae, vol. 26 (1985)) have obtained analogous results in the case of countable metrizable spaces.

[5] *On Topological Isometries*, Indag. Math. 31, No. 2, 1969, pp. 184–189.

A homeomorphism of a metric space X onto itself is called a topological isometry if there exists a topologically equivalent metric with respect to which that homeomorphism is an isometry. In this paper the concept of an evenly continuous family, introduced by J. Kelley, is used to characterize homeomorphisms which are topological isometries.

[6] *Symmetric Products and Higher Dimensional Dunce Hats* (coauthors R.N. Andersen and R.M. Schori), Topology Proceedings, Vol. 18, 1993.

Given a space X, the associated symmetric product $X(n)$ is defined as the hyperspace of n or fewer points in X, topologized by the Hausdorff metric. The symmetric product was introduced by Borsuk and Ulam who proved that $I(n) \cong I^n$ if and only if $n \leq 3$, which left open the topological characterization of these spaces if $n > 3$. It was known (Schori) that $I(n) \cong C(D^{n-2}) \times I$, for a $(n-2)$-dimensional polyhedron D^{n-2}. In this paper the spaces D^n are precisely determined as polyhedra. For n even D^n is a contractible but not collapsible polyhedron, refereed to as a higher dimensional dunce hat. For n odd the reduced homology groups of D^n are all trivial except in the case of dimension $n-1$ when $H_{n-1}(D^n; \mathbb{Z}) \cong \mathbb{Z}$.

[7] *Topological Types of Some Symmetric Products* (in Russian) (coauthor S. Vrećica), DAN 349, 2, 1996, 172–174.

In this paper a general quotient model is established, providing immediate proofs of the relations $C(C(X)) \cong C(X) \times I$ (R. Schori's result, Fund. Math. 63 (1968)) and $C(S(X)) \cong S(C(X)) \cong C(X) \times I$ (the authors' result), where $C(X)$ and $S(X)$ are the cone and the suspension on X, respectively. In particular, very short proofs are given of the relation $I^n(2) \cong C(\mathbb{R}P^{n-1}) \times I^n$ (R. Schori, loc. cit.) and its generalization that the space of all k-dimensional balls contained in the n-dimensional unit ball B^n is homeomorphic to $C(G_{n,k}) \times I^n$, where $G_{n,k}$ is the Grassmann manifold of k-planes in $\mathbb{R}^n$.

[8] *An Iterative Method for Solving Polynomial Equations*, Topology and its Applications, Budva 1972 (Beograd 1973), 170–172.

Dragoljub Marković and subsequently Slaviša Prešić were preoccupied with the problem of finding an iterative method for solving algebraic equations of the degree n with real coefficients. In this note a sequence is defined which either converges to the greatest root of an algebraic equation or shows that the equation has no real roots (which is possible in the case of an even n).

[9] *A Topological Approach to Recognition of Line Figures*, (coauthors R. Tomović and S. Stanković), Bulletin T. CVII de l'Académie Serbe des Sciences et des Arts - 1994, Sciences mathématiques No 19, pp. 43–64.

The general concept of shape is a typical "Platonic" idea insurmountable to be precisely defined. In this paper a semi-topological idea of shape is defined for a class of patterns in the plane consisted of arcs. As a generalization of the so called cross numbers, the matrices with entries 1's and 0's are used to scan these patterns.

[10] *Semi-topological Classification of Line Patterns in the Plane*, Bulletin T. CXXXIX de l'Académie Serbe des sciences et des arts - 2009, Science mathématiques, No 34, pp. 17–42.

In this paper the invariant matrices with respect to the semi-topological classification of the line patterns are defined. An algorithm consisted of three elementary transformations is given which transforms a matrix scanning a pattern into the corresponding invariant matrix. Let us remark that the invariant matrix of a pattern is its unique arithmetic code which determines the pattern up to the semi-topological equivalence.

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