THE TEACHING OF MATHEMATICS
2010, Vol. XIII, 2, pp. 113–124

THE INFLUENCE OF THE GEOMETER'S SKETCHPAD ON THE GEOMETRY ACHIEVEMENT OF GREEK SCHOOL STUDENTS

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Abstract. The aim of this study is to investigate the question: "Does the use of a Geometer's Sketchpad help secondary students to improve their geometry performance?" The respondents were students enrolled in a high school (grade 7) in a northern suburb of Athens. The experimental group consisted of approximately 40 students, who spent at least one hour per week doing computer explorations for the first six weeks of the last semester. There were 39 students in the control group, which was not exposed to the computer explorations. Students in both groups were pre-tested and post-tested for their geometry performance. The results of the study indicated that the use of technology is needed for students to make significant progress in geometry.

ZDM Subject Classification: U73 *AMS Subject Classification*: 97U70.

Key words and phrases: Proof; geometry; van Hiele model; Geometer's Sketchpad; educational software.

Introduction

It has been established that the mathematics reform movement encourages the use of technology in the mathematics classroom. How should it be implemented though? How much technology should be incorporated? What effects might such usage have on teaching, learning, and the curriculum? What is the risk of disregarding current technology within the mathematics education community? Leitzel ([8], p. 6) wrote: "We risk limiting students' mathematical power by divorcing mathematics from technology".

Technology should play an important role in the learning and teaching of mathematics [18]. In order to derive the maximum benefit from technology it should be used on a regular basis [21]. It should be used to make classrooms more active and dynamic, as well as to enable students to explore realistic applications where they can focus on important concepts rather than routine calculations [16]. All classrooms should have computers installed permanently, and there should be sufficient computer laboratory facilities for student use outside the class as well [11].

The NCTM [13] suggests in their geometry standards for grades 9–12 that computer software should be used to promote inductive reasoning among geometry students. "Computer graphics software that allows students to create and manipulate shapes provides an exciting environment in which they can make conjectures and test their attempts at two-dimensional visualization".

In the geometry standards of the *Principles* document, the NCTM [15] identifies four aims for all students in pre-kindergarten through to grade 12:

- Analyze characteristics and properties of two and three dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems.
- Apply transformations and use symmetry to analyze mathematical situations.
- Use visualization, spatial reasoning, and geometric modelling to solve problems.

It seems reasonable to conjecture that appropriate technology might help to make all of these goals a reality. Dixon [4] found that a *Geometer's Sketchpad* does help improve a student's ability to visualize in two dimensions. The NCTM quickly acknowledges the important role of technology in achieving the aims outlined above. Besides improving spatial reasoning and visualization, interaction with computer software also helps promote inductive reasoning by allowing students to quickly construct many accurate examples of a certain phenomenon [15]. The authors also point out that using such software can help students understand that these examples alone do not constitute a proof.

Must we choose between a technologically enhanced inductive curriculum and the traditional deductive curriculum? The pedagogies are certainly different, but the NCTM [15] does not seem to envision the scenario as mutually exclusive. An entire section of *Principles* is devoted to the "Technology Principle". Several essential ideas emerge from this section of the document:

- Electronic tools are essential for teaching, learning, and doing mathematics.
- Technology can help students learn mathematics.
- Technology should support effective mathematics teaching
- Technology influences not only how mathematics is taught, but also what mathematics is taught.

Technology should not be seen as a panacea. It should not replace the mathematics teacher. However, it can help to change the role of the teacher, as well as the curriculum and instruction in the classroom. The burden of implementing the technology available today falls on the shoulders of our teachers.

Technology and the Geometer's Sketchpad

Mathematics educators are calling for the implementation of technology in the classroom at all grade levels, ability levels, and in different areas of content. The NCTM [13] has been endorsing the use of technology for well over a decade. The influence of technology has grown to the point that it has become one of the guiding principles of mathematics education (NCTM, [15]).

Zaranis and Ntziahristos [24] found that the use of dynamic computer software (*Geometer's Sketchpad*) helped improve students' development of geometric thought when used in conjunction with instruction based upon the van Hiele levels. They highly recommend the use of *Geometer's Sketchpad*, arguing that it could make geometry instruction effective, enhance students' interest, and help students overcome their learning difficulties.

Yousef [23] claims that early, unrestricted use of technology improves student motivation. He suggests exposing kindergarten students to computer software as part of geometry instruction.

Wertheimer [22] lists six positive outcomes that are related to the incorporation of technology in the classroom:

1. Technology motivates students to be more interested in exploring, investigating, conjecturing, creating, and discovering principles and making generalizations.
2. Technology helps students produce connections between different branches of mathematics.
3. Technology helps students become mathematical problem solvers and gives them a chance to solve problems in real life situations, rather than just doing routine problems.
4. Technology enhances students' conceptual understanding of geometry.
5. Technology encourages teachers to involve students in various instructional activities that facilitate the learning process.
6. Technology enables teachers to focus their attention on students in need of extra assistance or additional stimulus.

Melczarek [12] used *Geometer's Sketchpad* to explore the relationship between computer technology and a student's readiness for self-directed learning. He found that a positive correlation did exist between these variables due mainly to the students' (positive) attitudes towards the computer software.

Margaret Lester [9] also used *Geometer's Sketchpad* in her doctoral studies by examining whether using an inductive pedagogy together with the computer would improve geometric achievement. The results of her study indicate that students learn geometry skills with greater efficiency and understand geometry concepts at higher levels as a result of creating and manipulating dynamic visualization of geometric objects on the computer screen.

However, Foletta [5] found that students often use *Geometer's Sketchpad* as an extension of pencil-and-paper calculations rather than using it as a new resource, as the developers intended. Nicaise and Bames [17] take this a step further, indicating that technology has not radically changed the way mathematics is currently taught; instead most technology usage mirrors traditional instructional pedagogy. Some would argue that this is not inherently flawed. Surely it is possible to retain some merits of traditional pedagogy while experimenting with the use of technology? This author believes that such a compromise is both prudent and attainable.

Weaver and Quinn [21] describe how *Geometer's Sketchpad* can be used to construct figures, measure segments and angles, calculate expressions, and manipulate figures. They also give examples of how *Geometer's Sketchpad* can be gradually incorporated into the students' classroom experience.

Manouchehri, Enderson, and Pugnucco [10] also provide a framework for gradually structuring students' explorations with *Geometer's Sketchpad*. Students are introduced to the software through free explorations, they are then given semi-structured activities, and finally the opportunity to perform independent explorations.

Extensive research has been undertaken to study the effects of the use of *Geometer's Sketchpad* on geometry curriculum, pedagogy, and learning. It has been shown to assist student understanding of geometric relationships (Bonsangue [1], Dimakos, et al. [3]), in making mathematical generalizations, and benefits students with special needs (Shaw, Durden, & Baker [19]). It promotes student autonomy (Nicaise & Barnes [17]). Dixon [4] has highlighted the speed and ease in which students can manipulate figures using *Geometer's Sketchpad* and how slow and tedious and perhaps not even feasible these same tasks are when done with pencil and paper. This frees students from rote practice of routine calculations and allows them to focus on concepts and problems that are more meaningful to them. Thus, technology can promote higher-level thinking because students spend more time reflecting and analyzing (Nicaise & Barnes [17]).

While the positive contributions are well documented, the use of technology is not a panacea. For instance, it has been pointed out that *Geometer's Sketchpad* does not seem to improve students' abilities to visualize in three dimensions (Dixon [4]). This is possibly a shortcoming of modelling three-dimensional objects in only two dimensions. Another more practical concern is that technology can be prohibitively expensive (Nicaise & Barnes [17]). There is also the possibility that *Geometer's Sketchpad* will serve as a distraction rather than an effective tool for students.

Advances in technology certainly raise questions about pedagogy, curriculum, and even assessment (Leitzel [8]). In order to address these questions, more research on the effects of technology is required. In particular, Hannafin and Scott [6] indicate that more research is needed on interventions that will help students succeed at school. In practical terms, this means further research should investigate whether the use of technology will help improve students' understanding in high school geometry classes.

Methodology

The preceding paragraphs gave rise to some interesting questions. How can technology be integrated into the current secondary geometry curriculum? How does it affect that curriculum? How can the dual goals of technology implementation and deductive proof writing ability be pursued concurrently? Does the use of technology help improve student achievement in high school geometry classes? The focus of this research will be on the effects of inductive *Geometer's Sketchpad* activities on high school geometry student achievement and proof writing ability. The following research question encompasses the issues raised above:

Does the use of *Geometer's Sketchpad* help secondary geometry students improve academic achievement in their secondary geometry class?

This research was conducted during the 2003–04 school year in a public secondary school with approximately 250 students enrolled (7th–9th grade), located in the suburbs of Athens (Greece). The school has a computer laboratory, which is available for classroom use daily.

The four sections of the class (grade 7) were divided so that two sections served as the experimental group ($n = 40$) and the remaining two sections served as the control group ($n = 39$). All the students in a given section were exposed to the same treatment and the group assignment was chosen so that each of the two teachers involved taught at one experimental section and one control section. Thus, the study had a quasi-experimental design since previously formed classes served as the experimental and control groups without random assignment.

A pre-test was given to the class during the first week of the last semester to isolate the effects of the treatment by looking for inherent inequities in the geometric achievement potential of the two groups. The test was based on the van Hiele test, which has been shown to be a good indicator of proof-writing achievement (Senk [18], Usiskin [20]). The content of the test was about triangles and quadrilaterals. This 35-minutes duration test consisted of 3 subtests, each containing 15 questions. Each correct answer was awarded with 3 points, giving a total sum of 135 points for the whole test. Some exercises of the test concerning the triangle are given below:

Q1. Is the right triangle an amblygon triangle? Circle the correct answer.

YES NO

Justify your answer.

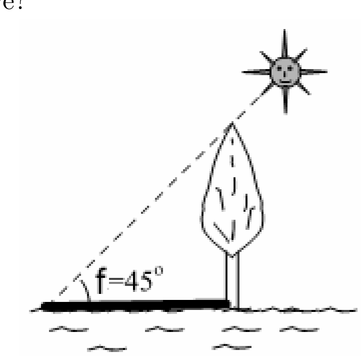
Q2. What is the amblygon triangle?

Q3. Build the triangles ABC and EFG which can be designed with the following side lengths:

a) Triangle ABC: $AB = 2\,\text{cm}$, $AC = 3\,\text{cm}$ and $BC = 4\,\text{cm}$.

b) Triangle EFG: $EF = 2\,\text{cm}$, $EG = 3\,\text{cm}$ and $FG = 6\,\text{cm}$

What do you observe?



Q4. In an isosceles triangle ABC, with $AB = AC$, the point M is the mid-point of BC. Prove that AM is perpendicular to BC.

Q5. A tree has 5 m shade and the rays of sun with the horizon shapes an angle f ($f = 45°$), as it appears in the next picture. What is the height of the tree? Justify your answer.

The control group functioned as this class has in previous years. Emphasis was placed upon a deductive development of plane geometry. Homework was assigned and collected daily, there were quizzes given periodically, to be carried out individually and in small groups. The experimental group covered the same material at roughly the same pace, but spent one or two class hours per week in the computer lab. In order to help keep students in the task, the computer activities served as a quiz (Hellenic Mathematic Society [7]). The activities were selected to complement what students had previously studied or were about to study in class. The two groups (experimental and control) received the same number of class hours, with the computer hours (experimental group) being included within the same total number of teaching hours as in the control group. The content of the six week syllabus was about triangles and quadrilaterals. These are the units of geometry content that have been investigated in this study.

The independent variable was the extent of *Geometer's Sketchpad* exposure. The control group received no exposure to the program, while the experimental group spent over 25% of their class time in the computer lab for the six weeks of the last semester.

The dependent variable was the students' geometry achievement. This was measured by a test written by the teachers and was based on the van Hiele test. The same test was given to all students in the study as a pre-test and post-test. These tests covered the material from chapters dealing with triangles and quadrilaterals.

Results

In order to find baseline values for geometry achievement, all respondents in both the experimental and control groups were administered a test during the first week of the last semester. This was used to test initial geometric achievement for both groups. The test contained 45 (3×15) questions concerning the topics of triangles and quadrilaterals. Multiple-choice and short answer questions were posed on this test that was taken by 79 students. Forty-four of the students were male and 35 were female.

Within the class syllabus, the first three weeks of the last semester consist largely of a review of fundamental geometric concepts and an introduction to proof and deductive logic. Any proofs that students are required to write during this period are based solely on definitions and do not require long chains of deductive reasoning. The first real opportunity to test students on their ability to construct adequate proofs arises during the second three weeks. Here, students learn about theorems involving triangles and quadrilaterals.

Analysis of the data was carried out using an SPSS (ver. 15.0) statistical analysis computer program. An independent sample t-test was conducted. The independent variable had two levels: exposure to *Geometer's Sketchpad* computer labs (experimental group) and no exposure (control group). The dependent variable was the student's test score. Levene's Test for equality of variances was significant ($F = 5.063$, $p = 0.027$). The t-test for equality of means was not significant ($t = 1.287$, $p = 0.202$), indicating no significant differences (Table 2), initially, in geometry achievement between the experimental and control groups. Though the experimental group had a mean score higher than the control group, the mean difference in the test scores was greater than 4.352. The results of this test are summarized below:

Group/ Pre-test	N	Mean	Std. Dev.	Std. Error
Experimental	40	59.80	16.50	2.608
Control	39	55.45	13.44	2.152

Table 1. Group statistics

	t	df	mean difference	Sig. (2-tailed)
Pre-test	1.287	74.660	4.352	0.202

Table 2. Independent samples test

In order to determine if the performance of the experimental group is significant, a paired t-test was performed using the grades of this group for a comparison between pre-test and post-test scores. The mean grade for the pre-test in the study was 59.804 ($SD = 16.500$) compared to 104.952 ($SD = 0.822$) for the post-test. At $\alpha = 0.05$ and $df = 39$, the critical value of the t ratio was less than 0.001. Therefore, the post-test score was significantly different from the post-test score in the experimental group (Table 4).

Experimental Group	N	Mean	Std. Dev.	Std. Error
Pre-test	40	59.804	16.500	2.608
Post-test	40	104.952	0.822	1.711

Table 3. Paired samples statistics

	t	df	Mean	Sig. (2-tailed)
Pair 1	-20.667	39	-45.14825	0.000

Table 4. Paired samples test

Similarly, to determine if the performance of control group is significant, a paired t-test was performed using the grades of this group for a comparison between pre-test and post-test scores. The mean grade for the pre-test in the study was

55.451 ($SD = 13.441$) compared to 74.780 ($SD = 16.418$) for the post-test. At $\alpha = 0.05$ and $df = 38$, the critical value of the t ratio was less than 0.001 (Table 6). Therefore, the post-test score was significantly different from the post-test score in the control group.

Control Group	N	Mean	Std. Dev.	Std. Error
Pre-test	39	55.451	13.441	2.152
Post-test		74.780	16.418	2.629

Table 5. Paired samples statistics

	t	df	Mean	Sig. (2-tailed)
Pair 1	−10.448	38	−19.328	0.000

Table 6. Paired samples test

An independent sample t-test was conducted. The independent variable had the same two levels as in the previous test: experimental and control. The dependent variable was the student's post-test score. Levene's Test for equality of variances was not significant ($F = 2.458$, $p = 0.121$). The t-test for equality of means was significant ($t = 9.667$, $p < 0.001$) indicating significant differences, in scores between the experimental and control groups (Table 8).

Group/ Post-test	N	Mean	Std. Dev.	Std. Error
Experimental	40	104.952	10.822	1.711
Control	39	74.780	16.418	2.629

Table 7. Group statistics

	t	df	mean difference	Sig. (2-tailed)
Post-test	9.667	77	30.172	0.000

Table 8. Independent samples test

Moreover, a stratification of experimental group according to their success in the testing was divided into three equal categories (Table 9): less than 50 (33.33th percentile - *low*), 50 to 70 (33.33 to 66.66th percentile - *medium*), and more than 70 (66.66th percentile - *high*).

N	valid	40
	Missing	0
Percentiles	33.333	49.63
	66.667	69.70

Table 9. Stratification of Experimental group

In the following Table 10 the students' performance is presented including both groups (i.e. the experimental and the control group) before teaching.

Pre-test	Experimental Group		Control Group	
Grading	N	$f\%$	N	$f\%$
Low	15	37.5	10	25.6
Medium	12	30.0	24	61.6
High	13	32.5	5	12.8
Total	40		39	

Table 10. Frequencies of the two groups in the pre-test

Table 10 shows that 32.5% of the students of the experimental group exhibited high grading, 30% exhibited medium grading, whereas 37.5% exhibited low grading. Likewise, 12.8% of the control group exhibited high grading, 61.6% medium and 25.6% low. In other words, students' performance in the medium category of the control group appeared to be superior (i.e. 61.6% compared with 30% of the experimental group).

The differences between the students' improvement (post-test minus pre-test score) of the three experimental groups (low, medium, high) were statistically significant with one-way analysis of variance ($F = 11.950$, $p < 0.001$, Table 11).

Experimental group	Sum of squares	df	Mean square	F	Sig.
Between groups	2921.535	2	1460.767	11.950	.000
Within groups	4523.002	37	122.243		
Total	7444.537	39			

Table 11. ANOVA

The Bonferroni post hoc tests (Table 12) indicated that students' improvement of the low group differed significantly from students' improvement of the medium and high group ($p < 0.05$).

However one-way ANOVA (Table 13) yielded no significant differences for low, medium and high improvement of the control group ($F = 1.616$, $p = 0.213$).

(I) level	(J) level	Mean difference (I–J)	Std. error	Sig.	95% Confidence interval	
		lower bound	upper bound	lower bound	upper bound	lower bound
low	medium	12.57150(*)	4.28211	.017	1.8331	23.3099
	high	20.17733(*)	4.18962	.000	9.6709	30.6838
medium	low	−12.57150(*)	4.28211	.017	−23.3099	-1.8331
	high	7.60583	.42609	.282	−3.4936	18.7053
high	low	−20.17733(*)	4.18962	.000	−30.6838	−9.6709
	medium	−7.60583	4.42609	.282	−18.7053	3.4936

*The mean difference is significant at the .05 level.

Table 12. Multiple comparisons in experimental group
Dependent variable: improvement. Bonferroni

Control group	Sum of squares	*df*	Mean square	*F*	Sig.
Between groups	417.800	2	208.900	1.616	.213
Within groups	4653.668	36	129.269		
Total	5071.468	38			

Table 13. ANOVA

Conclusions

This study has attempted to answer a basic research question. Does the use of *Geometer's Sketchpad* help secondary geometry students improve academic achievement in their secondary geometry class? The evidence provided in the previous paragraphs support this conclusion. Initially, there was no significant difference in the pre-test scores for experimental or control group achievements. However, throughout the study, the experimental group had higher geometry achievement than the control group, and the difference was statistically significant. The research question has clearly been answered positively.

Moreover, the results of the study agree with other researches (Zaranis and Ntziahristos [24], Yousef [23], Melczarek [12]) which state that ICT helps students' understanding of geometric relationships, making mathematical generalizations and allowing them to focus on concepts of the problems. In addition, the current research shows that the stratification of students groups according to their improvement in the testing (*low, medium, high*) is inversely proportional to the level of their success.

The *Geometer's Sketchpad* provides a unique way of investigating geometric notions that will undoubtedly assist some students. It might take the form of enrichment or a classroom demonstration but students should have some exposure to technology in their high school geometry class. Failing to use such powerful technological tools in a society, where computers permeate the culture is a missed opportunity in showing the relevance of geometry to modern life. It is an ongoing challenge for the reflective teacher to decide how this technology can be best utilized in education, especially in light of the current research on the effects of such an implementation. This study is simply one small piece in the puzzle of geometry education.

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