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LONG TERM EFFECTS IN LEARNING MATHEMATICS IN FINLAND—CURRICULUM CHANGES AND CALCULATORS

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Abstract. Two similar tests to measure the skills of the Finnish school children in mathematics took place in 1981 and 2003. The tests are compared to a test measuring the knowledge of basic concepts in mathematics after the student examination. The results of the tests reflect the changes in the mathematics curriculum and teaching practices in Finland.

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1. Introduction

Curricula changes in the Finnish school system have taken place in 8–10 year intervals. The official curriculum texts are rather short in details. Schools are free to choose their textbooks and there is neither an official inspection nor an official approval of the textbooks in Finland. The free market principle prevails. Hence textbooks and teaching practices should be studied in order to understand the mathematics curriculum. A similar system is used in many countries. A rather detailed description of mathematics and science teaching in Finland can be found in [1]. This collection of articles also contains an account of the teacher training system used in Finland.

Almost everybody finishing the high school (gymnasium) participates in the matriculation (student) examination at the age of 18. Hence this test provides an opportunity to study the final effectiveness of the Finnish school system. The mathematics test is not obligatory although most students take it. The matriculation test is 150 years old and its mathematics part has essentially remained the same, except for the problems, for the last hundred years. In mathematics a student may choose a basic or an advanced test independently of which courses the student has followed at school. The basic test is more common. Both tests consist of 15 problems written on an A4 sheet. A student can choose at most 10 problems out of 15. In practice, solving two problems, or slightly less, he or she is able to pass the test. Eight or nine correctly solved problems is the standard requirement for the highest grade but this varies annually. The students are graded using seven grades whose distribution is the same each time. Because of this the grades in matriculation tests cannot be used to compare changes in mathematical skills of the students. The test problems have changed considerably during the

last decades. The problems are based on the aforementioned, rather loosely stated, official curriculum.

A survey of the Finnish matriculation test in mathematics is in [2].

The purpose of this article is to study the changes in curriculum and teaching practices that have had the most serious long term effects in learning mathematics in Finland.

2. Mathematics curriculum—changes and effects

The changes in the mathematics curriculum in Finland have followed the international trends. Since 1970 three major revisions have taken place. The first was influenced by the so-called New Math. This created a lot of discussion but had a relatively small effect. The second revision can be labelled "Back to basics". The last change "Problem solving" had a much greater impact. It was very much influenced by the demand that the applications of mathematics are all important—mathematics as such has little value. The influence of calculators was also profound. It was thought unnecessary to teach those skills which can be performed by a calculator. Similar changes were experienced in other OECD countries.

In Finland these trends had the following effects on the mathematics curriculum.

- Mathematics at school became descriptive - exact definitions and proofs were largely omitted.
- Geometry was neglected.
- Computations were performed by calculators and numbers and not on a more advanced level.

Students also experienced difficulties when moving from elementary school mathematics to secondary school mathematics and especially to high school mathematics. Little has been done to ease this friction.

A rather recent test problem in a basic mathematics matriculation examination demonstrates these effects. "Why is the sum of the angles in a triangle 180 degrees?" Nobody knew although the problem was explained in some textbooks (a line cuts two parallel lines in equal angles). This shows that teaching of mathematical principles has declined, at least on the basic course, and replaced by a list of facts given without reasoning. Many teachers are content to demonstrate this property of all triangles with scissors and paper.

L. Näveri [3] has studied the effects of the curriculum changes in Finland. Two similar tests were performed in mathematics in 1981 and in 2003. Participants belonged to the age group 15–16 year old (9. grade); this corresponds to the age group in the PISA survey since the school starts at the age of seven in Finland. The tests were participated by more than 350 students. The problems were identical and supposed to be solved without a calculator. In the following only samples of the test questions are presented.

The first samples of questions concerns multiplication and the percentages show the correct answers.

Multiplication	1981	2003
$5 \cdot 5 \cdot 5 \cdot 5 = 5^4$	95.2%	90.1%
$(-3)^2 = 9$	67.8%	47.5%
$18 \cdot 4 \cdot 32 \cdot 15 = 15 \cdot 32 \cdot 4 \cdot 18$	93.2%	85.9%
$0{,}015 \cdot 248 = 0{,}15 \cdot 24{,}8$	66.8%	62.3%
$0 \cdot 8436 = 0 \cdot 0{,}536$	79.0%	65.6%

In the questions concerning rational numbers the performance drop from 1981 to 2003 was the highest, 20%.

Rational numbers	1981	2003
$26 + 17 =$	98.5%	89.8%
$(1/2) \cdot (2/3) =$	56.4%	36.9%
$(4/3) \cdot 5 =$	66.3%	44.4%
$(1/6) \cdot (1/2) =$	56.5%	28.3%
$(1/5) : 3 =$	49.2%	27.5%
$1278/2 =$	55.1%	36.8%

Also in the algebra section the results did not give a healthy picture of the effects of the curriculum changes.

Algebra	1981	2003
$10^3 \cdot 10^2 =$	72.5%	43.3%
$x^4 \cdot x^5 =$	71.7%	47.3%
$(59^2)^3 = (59^3)^2$	61.1%	31.7%

If calculators were allowed in the test, the results would have most likely shown different figures.

In the 2003 survey it was also asked: Explain with your own words the meaning of $(4/5) \cdot 5$. The results were as follows:

Correct	6.5%
Almost correct	5.4%
Correct computation but explanation incorrect	8.8%
No explanation but computation corect	31.5%
Incorrect computation and explanation	31.0%
No answer	16.8%

Rather few reliable international surveys have been made to compare the changes of the students performance in elementary and secondary school mathematics in the time scale of 2–3 decades. It is difficult to separate the effects that are due to the changes in the curriculum from those which are due to changes in teaching practices. The survey [3] certainly shows that these effects exist. It would be interesting to survey the situation in other countries and to look for a general pattern behind the results. There is at least one reason behind the above results. It is the use of calculators.

Finland was the best country among the OECD countries in the PISA 2003 survey. This survey concentrated to the 14 year old age group. The type of questions asked in [3] were rare in the PISA test. In the TIMSS 1999 report the performance of the Finnish pupils was also above average. In the latter test the problems were closer to the questions asked in [3]. The reasons for the Finnish PISA success are analyzed in [1]. A more critical discussion can be found in the Finnish electronic journal Solmu [4] of school mathematics (`http://solmu.math.helsinki.fi/`) where two special issues have been devoted to the Pisa survey.

3. After the matriculation examination

Students, who have passed the matriculation test, do not only go to universities to study. Many of them go to professional schools (training schools for nurses, various engineering colleges etc.)—usually, but not necessarily, they are students who have got low grades in the matriculation test.

During the last ten years teachers in professional schools, and not only mathematics teachers, have complained on the level of the mathematical skills of the new students. The following sample from [5] shows that these complaints are not without basis. The test was performed for freshmen in an engineering college and the figures indicate the percentages of those who correctly answered the problems on the left hand side. "Basic test" and "Advanced test" mean students who have passed the corresponding matriculation examination in mathematics. Calculators were not allowed.

	Basic test	Advanced test
$\sqrt{3^2+4^2}=$	55%	78%
$(1/3-1/7)/4=$	25%	54%
$a^2-(a+1)^2+2a=$	17%	50%
Find R from the formula $U=E-IR$	26%	68%
$\ln(x^2)-2\ln x=$	7%	34%

The test shows that formula handling, rational numbers, logarithms and algebraic operations by hand are difficult for those who have passed the basic matriculation examination. Among the students who have passed the advanced test and

who have had much more mathematics lessons at school there are many who have not learnt basic algebraic operations.

4. Conclusions

The most serious drawbacks in the Finnish mathematics curriculum are the order and time allocated to different concepts and skills. It is outside the scope of this report to analyze the situation in detail. Some typical examples can be mentioned. In the advanced course probability is taught before the concept of an integral and sequences and series are left to the very end. As the above reports indicate there are serious defects in the secondary school mathematics curriculum. In Finland a customer cannot any more ask for 3/4 kilogram meat in a butcher's shop since the meaning is not known to a shop-assistant. The right expression is 750 g since this can be fed to a computer. Although the changes in the mathematics curriculum were made to help people to use mathematics in everyday life, this aim has badly failed. The problems now considered at school are not those people meet later on. Problem solving has been overestimated in all levels of the mathematics curriculum. Teachers at professional schools have learnt this in a hard way.

From the studies [3] and [5] a serious defect in mathematics teaching emerges. This is the incorrect use of calculators in teaching. Although number handling is learnt by pen and paper at the elementary school, many students later completely forget this skill because they have got used to calculators. This does not concern so much the best 15–20% of the students as results show in the matriculation examination. The use of calculators is overemphasized since nowadays their use is extremely limited in everyday life. Professional users of mathematics almost never use them. Hence the time spent with calculators does not follow the idea that the skills obtained at school should have some practical value later.

Calculators came to schools in 1975–1995. After a slow beginning they are now used more and more. No doubt this has been the most essential change in teaching mathematics and the effects can be seen in the reports [3] and [5].

Mathematics does not concern professional mathematicians only. Mathematics is used more and more in ordinary professions and the problems involved are different from those in the PISA survey. In Finland, as in many countries, the mathematics curriculum includes concepts and skills which once have been put there because somebody has thought them useful. In most cases time has shown that these special skills do not meet the demands of the society any more. The Finnish curriculum architecture and teaching practices require considerable changes to meet the challenge. Here Finland is not alone.

REFERENCES

[1] *How Finns Learn Mathematics and Science*, Editors: E. Pehkonen, M. Ahtee, J. Lavonen, Sense Publishers 2007, 278 pp.

[2] Lahtinen, A., *The Finnish Matriculation Examination in Mathematics*, In: Nordic Presentations (eds. E. Pehkonen, G. Brandell & C. Winslw), 2005, 64–68. University of Helsinki. Department of Applied Sciences of Education. Research Report 262.

[3] Näveri, L., *Understanding computations*, Dimensio 3/2005, 49–52 (in Finnish).

[4] SOLMU, 2 special volumes on the PISA survey 1/2005–2006, 2/2005–2006: `http://solmu.math.helsinki.fi/2005/erik1/` (in Finnish) and `http://solmu.math.helsinki.fi/2006/erik2/` (in Finnish).

[5] Tuohi, R. et al., *Fact or fiction—mathematical skills of new engineering students*, Turun ammattikorkeakoulun raportti 29, Turku 2004 (in Finnish).

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