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THE BEHAVIOR OF A SATELLITE TRAJECTORY NEAR THE EQUILIBRIUM POINTS OF SUN-EARTH SYSTEM AND ITS CONTROL

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ABSTRACT. In this paper, the behavior of a satellite trajectory near the equilibrium points of the Sun-Earth system is studied. The equations describing the motion of the satellite in the circular restricted three body problem for the Sun-Earth system, are discussed for their ordinary differential equations form, and Lagrange points are determined. Then, the stability is studied at each Lagrange point. The trajectories of a satellite starting its motion near Lagrange points are illustrated, showing the stability and instability behavior. Finally, the unstable trajectory is controlled by using l_2-method at L_1 as an example.

1. Introduction

The motion of a satellite in the Sun-Earth system is often described by the well known circular restricted three-body problem (CR3BP). A significantly small body (such as a satellite) is considered to be moving under the gravitational influence of two large masses (such as Sun-Earth and Earth-Moon, etc). These two large finite masses are called primaries. The equations of motion of the satellite are given by a set of non-linear autonomous ordinary differential equations (ODE). The procedures in analyzing a nonlinear system of ODEs are to find its equilibrium points, then the linear stability is applied due to each point. Euler and Lagrange deduced that there are five equilibrium points for the third body in the system. This equilibrium resulted from a balance in the centrifugal force and the gravitational attraction of the larger bodies. In [1–3], the orbits of solar sails are studied, while their importance for navigation is introduced in [4].

The equilibrium points of the dynamical system are called Lagrange points. The stability of equilibrium points are studied for CR3BP, see for example [5, 6]. For the unstable Lagrange points, there were many studies to control the trajectory such as those in [7, 8]. In this paper, we study the motion of satellites in the Sun-Earth system.

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The attraction of the infinitesimal particle on the primaries has been neglected (as if it had zero mass), so the primaries orbit in a circle around their common center of mass. The dynamical system in dimensionless coordinates is autonomous and has only one dependent parameter denoted by μ (the mass parameter introduced in the next section), which makes it attractive to be studied.

The CR3BP is a very old model used in celestial mechanics. In this paper, we have a different view for the system which focuses on its behavior. We have presented detailed implementations. They showed the local stability of some equilibrium points while others were unstable. Then, the unstable chaotic trajectories are an interesting source for applying control methods to change their behavior for a certain duration with minimum negligible values added to the system. Here, we have used Quadratic Programming to control the chaotic trajectory at one of the unstable equilibrium points (the most useful point to be controlled, that is between Earth and Sun).

In the following section, the equations of motion are presented. Then, the determination of Langrange points is discussed. After that, the stability at each point is studied theoretically. In the last section, the trajectory of a satellite near each point is shown numerically. All implementations are carried out by using Maple and Matlab programs.

2. Equation of motion

We perform our studies in the synodic reference frame, where the origin coincides with the common center of mass of the two primaries (barycenter) denoted by m_1, m_2, where $m_1 > m_2$. The x-axis connects m_1 and m_2 in direction of the lower mass m_2, and the y-axis completes the frame as a right-handed coordinate system (see Fig. 1).

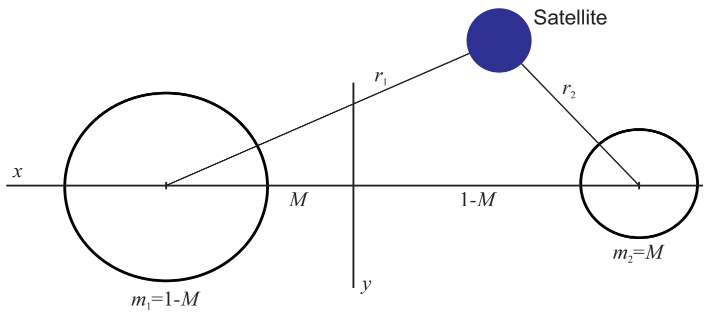


FIGURE 1. The bodies m_1 and m_2 in plane presentation.

All units of the system are set to be one (i.e. the units are dimensionless), such that the sum of primaries' masses and the distance between them equal one, the unit of time is arbitrarily chosen and not interesting in calculations since the system is autonomous. In our studied system of Sun-Earth, the mass parameter μ, can be calculated by

$$\mu = \frac{G \cdot m_E}{G \cdot m_E + G \cdot m_S} = \frac{m_E}{m_E + m_S}$$

where G is the gravitational constant, m_S and m_E are the masses of the Sun and Earth, respectively. Since $m_S = 1988500 * 10^{24}$ kg and $m_E = 5.97 * 10^{24}$ kg, then we get

$$\mu = \frac{m_E}{m_E + m_S} = 0.000003002253999.$$

The location of the Sun is at $(-\mu, 0)$, while the Earth is located at $(1 - \mu, 0)$.

The equations of motion of the satellite in the Sun-Earth system are described by,

$$\begin{aligned} \ddot{x} =& 2\dot{y} + x - \frac{(1-\mu)(x+\mu)}{r_1^3} - \frac{\mu(x-(1-\mu))}{r_2^3} \\ \ddot{y} =& -2\dot{x} + y - \frac{(1-\mu)y}{r_1^3} - \frac{\mu y}{r_2^3} \end{aligned} \tag{2.1}$$

where $x = x(t)$, $y = y(t)$ are presenting the location of the satellite in the cartesian coordinates, $\dot{x}, \dot{y}$ are the velocities, $\ddot{x}, \ddot{y}$ are the acceleration components, and $r_1 = \sqrt{(x+\mu)^2 + y^2}$ and $r_2 = \sqrt{(x-1+\mu)^2 + y^2}$ are the position vectors of the satellite with respect to the primaries m_1 and m_2.

In the next section, we will determine the equilibrium points and the system is solved for the stationary state.

Table 1. The x and y coordinates of the five Lagrange points in the synodic reference frame for the Sun-Earth system.

	x	y
L_1	0.9900280279	0
L_2	1.010032778	0
L_3	-1.000001251	0
L_4	0.4999969977	0.8660254040
L_5	0.4999969977	-0.8660254040

3. Lagrange points

Lagrange points are named in honor of Italian-French mathematician Josephy-Louis Lagrange. Their locations are in space, creating enhanced regions of attraction and repulsion. These can be used by a satellite to reduce the fuel consumption needed to stay in a position for some time to complete its mission. There are three Lagrange points, denoted by L_1, L_2 and L_3, which lie along the line connecting the two primaries. The two other Lagrange points, labeled L_4 and L_5, form the apex of two equilateral triangles that have the large masses at their vertices.

The point L_1 of the Sun-Earth system provides an uninterrupted view of the Sun and is currently home to the Solar and Heliospheric Observatory Satellite

(SOHO). The point L_2 of Sun-Earth system had been home to the WMAP spacecraft, then it became home of Planck, and currently it is home of the James Webb Space Telescope. L_2 is ideal for astronomy because it allows a spacecraft to be close enough to easily communicate with the Earth. NASA will probably not find any use for the L_3 point as it remains hidden behind the Sun all the time. By studying the real part of eigenvalues corresponding to the points vs the values of μ, it is easy to find that for $\mu < 0.0385$, the eigenvalues of L_4 and L_5 are purely imaginary, otherwise the real part would take place as well as L_1, L_2 and L_3. So that, L_4 and L_5 host stable orbits as long as the mass ratio between the two large masses exceeds 24.96 (i.e. $m_1/m_2 > 24.96$). This condition is met for both Sun-Earth and Earth-Moon systems, and for many other pairs of bodies in the solar system, so L_4 and L_5 can be used in many applications and studies such as in [9–11].

When solving the equations of motion of a satellite to find Lagrange points, it is known that it will be at rest. This means that the velocity and acceleration of the third body are zero in both directions x and y. So that the equations of motion will become

$$x - \frac{(1-\mu)(x+\mu)}{r_1^3} - \frac{\mu(x-1+\mu)}{r_2^3} = 0 \tag{3.1}$$

$$(1 - \frac{1-\mu}{r_1^3} - \frac{\mu}{r_2^3})y = 0 \tag{3.2}$$

Observing equation (3.2), there are two groups of solutions. The first group has $y \neq 0$; therefore, equation (3.2) will become

$$1 - \frac{1-\mu}{r_1^3} - \frac{\mu}{r_2^3} = 0 \tag{3.3}$$

Solving the equations (3.1) and (3.3), we find the two solutions of the system corresponding to L_4 and L_5.

$$L_4 = \Big(\frac{1}{2} - \mu, \frac{\sqrt{3}}{2}\Big)$$
$$L_5 = \Big(\frac{1}{2} - \mu, \frac{-\sqrt{3}}{2}\Big)$$

The second group has $y = 0$. That is why equation (3.2) will vanish and equation (3.1) will become

$$x - \frac{(1-\mu)(x+\mu)}{((x+\mu)^2)^{3/2}} - \frac{\mu(x-1+\mu)}{((x-1+\mu)^2)^{3/2}} = 0 \tag{3.4}$$

By simplifying Eq. (3.4), we get three cases due to x-values :

- If $x > 1-\mu$, then equation (3.4) will become

$$\begin{aligned} x^5 + (4\mu - 2)x^4 + (6\mu^2 - 6\mu + 1)x^3 + (4\mu^3 - 6\mu^2 + 2\mu - 1)x^2 \\ + (\mu^4 - 2\mu^3 + \mu^2 - 4\mu + 2)x + (-3\mu^2 + 3\mu - 1) = 0 \end{aligned} \tag{3.5}$$

- If $-\mu < x < 1-\mu$, then equation (3.4) will become

$$x^5 + (4\mu-2)x^4 + (6\mu^2-6\mu+1)x^3 + (4\mu^3-6\mu^2+4\mu-1)x^2 + (\mu^4-2\mu^3+5\mu^2-4\mu+2)x + (2\mu^3-3\mu^2+3\mu-1) = 0 \tag{3.6}$$

- If $x < -\mu$, then equation (3.4) will become

$$x^5 + (4\mu-2)x^4 + (6\mu^2-6\mu+1)x^3 + (4\mu^3-6\mu^2+2\mu+1)x^2 + (\mu^4-2\mu^3+\mu^2+4\mu-2)\,x + (+3\mu^2-3\mu+1) = 0 \tag{3.7}$$

Each of the last three equations has five roots, four of them are complex numbers and one is real. So, in order to find the three real solutions of equation (3.4), we substitute by μ in equations (3.5), (3.6) and (3.7).

Then the five Lagrange points are as presented in the table below.

4. Stability of Lagrange points

When studying the stability of nonlinear second order ODEs (2.1), we first need to reduce their order.

Let $Z = \begin{pmatrix} x \\ y \\ u \\ v \end{pmatrix}$, $\dot{Z} = \begin{pmatrix} u \\ v \\ \dot{u} \\ \dot{v} \end{pmatrix}$, $f(Z) = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix}$, where$f_1 = u = \dot{x}$,$f_2 = v = \dot{y}$, $f_3 = \dot{u} = 2v + x - \frac{(1-\mu)(x+\mu)}{r_1^3} - \frac{\mu(x-(1-\mu))}{r_2^3}$, $f_4 = \dot{v} = -2u + y - \frac{(1-\mu)y}{r_1^3} - \frac{\mu y}{r_2^3}$. So that the system can take the form,

$$\dot{Z} = f(Z) \tag{4.1}$$

A solution $\phi(t)$ to the system (4.1) is said to be stable if every solution $\psi(t)$ of the system close to $\phi(t)$ at initial time $t = 0$ remains close for all future time. If at least one solution $\psi(t)$ does not remain close, then $\phi(t)$ is said to be unstable. Expressed mathematically, resembling the definition for a limit: For each choice of $\varepsilon > 0$ there is a $\delta > 0$ such that $|\phi(t) - \psi(t)| < \varepsilon$ whenever $|\phi(0) - \psi(0)| < \delta$.

The stability of the equilibrium solution L of the autonomous nonlinear system (4.1) is related to that of its linearized system. Since the equilibrium solution of this system is a constant vector L for which $f(L) = 0$. Then the system can be linearized about L by using Taylor's Theorem,

$$f(Z) = f(L) + Df(L) \cdot (Z - L) + R(\bar{Z})$$

Since $f(L) = 0$. Then

$$f(Z) = Df(L) \cdot (Z - L) + R(\bar{Z})$$

where $Df(L)$ is the matrix of first-order partial derivatives of $f(Z)$ evaluated at L. In other words, $Df(L)$ is the jacobian matrix J evaluated at L. The remainder is $R(\bar{Z})$ where $\bar{Z}$ is some value dependent on Z and L and includes the higher order terms of the original function. Let $Z - L = W$. Then, the linearized form of the non-linear system (4.1) is

$$\dot{W} = J|_L \cdot W \tag{4.2}$$

Since we have five equilibrium solutions, we will find for each solution its Jacobian matrix such that

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial v} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial u} & \frac{\partial f_3}{\partial v} \\ \frac{\partial f_4}{\partial x} & \frac{\partial f_4}{\partial y} & \frac{\partial f_4}{\partial u} & \frac{\partial f_4}{\partial v} \end{pmatrix}$$

TABLE 2. Jacobian matrix and its corresponding eigenvalues for each Lagrange point.

L	J	Eigenvalues
L_1	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 8.88148 & 0 & 0 & 2 \\ 0 & -2.94074 & -2 & 0 \end{pmatrix}$	$\lambda_1 = -2.48441$ $\lambda_2 = 2.48441$ $\lambda_3 = 5.55112 \times 10^{-17} + 2.05707I$ $\lambda_4 = 5.55112 \times 10^{-17} - 2.05707I$
L_2	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 9.12177 & 0. & 0 & 2 \\ 0. & -3.06089 & -2 & 0 \end{pmatrix}$	$\lambda_1 = -2.53258$ $\lambda_2 = 2.53258$ $\lambda_3 = 1.11022 \times 10^{-16} + 2.08641I$ $\lambda_4 = 1.11022 \times 10^{-16} - 2.08641I$
L_3	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3.00001 & -0. & 0 & 2 \\ -0. & -0.000003 & -2 & 0 \end{pmatrix}$	$\lambda_1 = 1.30104 \times 10^{-18} + 1.000003I$ $\lambda_2 = 1.30104 \times 10^{-18} - 1.000003I$ $\lambda_3 = -0.00281$ $\lambda_4 = 0.00281$
L_4	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0.75000 & 1.29903 & 0 & 2 \\ 1.29903 & 2.25000 & -2 & 0 \end{pmatrix}$	$\lambda_1 = -8.14453 \times 10^{-16} + 0.99999I$ $\lambda_2 = -8.14453 \times 10^{-16} - 0.99999I$ $\lambda_3 = 5.48580 \times 10^{-16} + 0.00450I$ $\lambda_4 = 5.48580 \times 10^{-16} - 0.00450I$
L_5	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0.75000 & -1.29903 & 0 & 2 \\ -1.29903 & 2.25000 & -2 & 0 \end{pmatrix}$	$\lambda_1 = 3.81639 \times 10^{-17} + 0.99999I$ $\lambda_2 = 3.81639 \times 10^{-17} - 0.99999I$ $\lambda_3 = 6.62752 \times 10^{-16} + 0.00450I$ $\lambda_4 = 6.62752 \times 10^{-16} - 0.00450I$

The resultant jacobian matrices and their corresponding eigenvalues are in Table 2 at each Lagrange point.

The stability of autonomous continuous time dynamical system could be summarized to:

(1) Every solution is stable if all the eigenvalues of J have negative real parts.
(2) Every solution is unstable if at least one eigenvalue of J has positive real part.
(3) Suppose that the eigenvalues of J are all having zero real parts. Then the linear stability is not enough, and solution behavior or its approximation could indicate whether or not the equilibrium point is stable(see [12]).

From the table, we can conclude that L_1, L_2 and L_3 are unstable. However, we cannot conclude whether or not L_4 and L_5 are stable. Therefore, we have to use another method and that is the numerical solution of the ODEs (2.1) about each Lagrange point.

5. Trajectory of satellites

In this section, we study the behavior of different solutions of the system (2.1). These solutions are testing the satellite's trajectories left to move freely (the initial condition has zero velocity, $\dot{x}(0) = 0$ and $\dot{y}(0) = 0$) and starting from different initial positions related to each Lagrange point.

The energy of the system (4.1) has the following form

$$E = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) - \frac{1}{2}\left(x^2 + y^2\right) - \frac{1-\mu}{r_1} - \frac{\mu}{r_2}$$

For zero kinetic energy, the remaining part E_0 (the energy constant) of the total energy is given by

$$E_0 = -\frac{1}{2}\left(x^2 + y^2\right) - \frac{1-\mu}{r_1} - \frac{\mu}{r_2} = -\frac{C}{2}, \tag{5.1}$$

The contour plot of energy given by Eq. (5.1) is presented in Fig. 2. This general contour plot shows the energy for the Sun-Earth system.

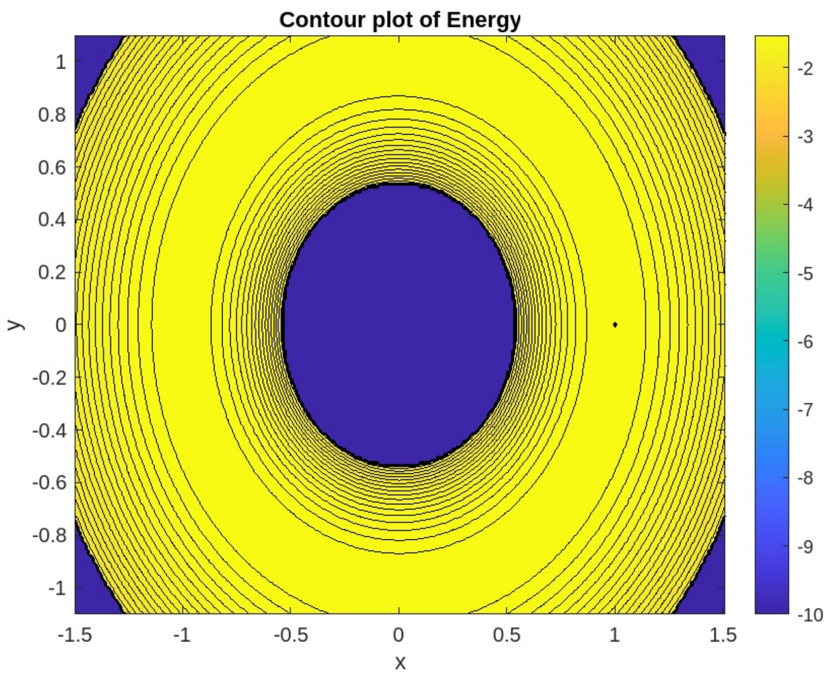


FIGURE 2. Contour plot for E_0 around Lagrangian points.

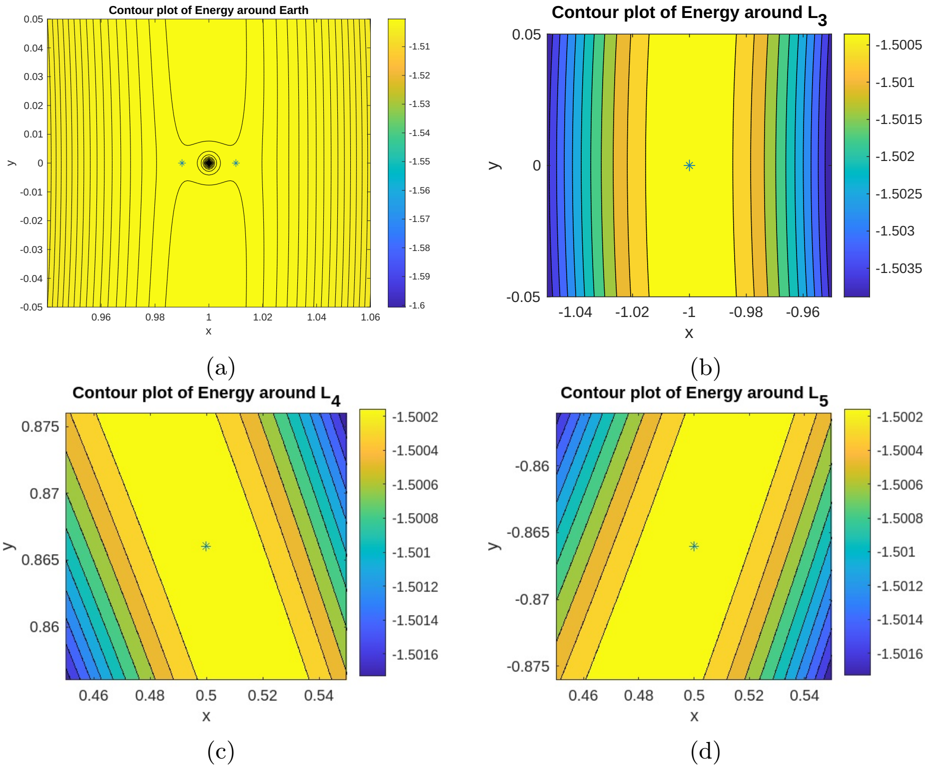


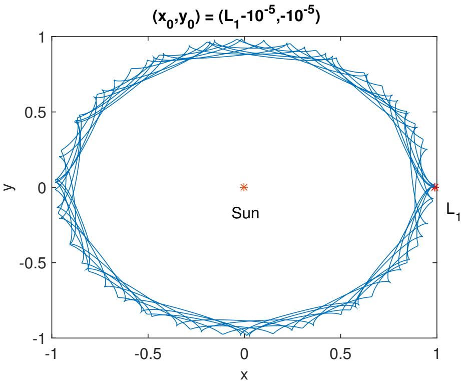
FIGURE 3. Contour plot for E_0 around a) Earth, b) L_3, c) L_4, and d) L_5.

To show the energy around Lagrange points, Fig. 3 is plotted representing the energy close to Lagrange points and close to Earth. Fig. 3a is showing the earth between L_1 and L_2. Fig. 3b is showing the energy degradation surrounding L_3. While Fig. 3c and 3d are showing the energy around L_4 and L_5. The energy has its effect on the trajectory, as they are bounded according to the energy level they are flowing inside. This is well observed in the next figures for the trajectory starting from close positions to Lagrange points.

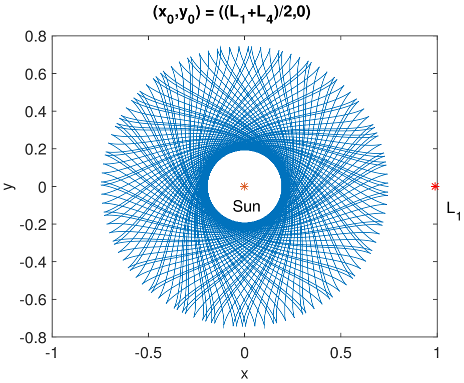
We used Matlab to plot the solutions in each case with different initial positions x_0, y_0, where the solution of System (2.1) is solved by Runge-Kutta method with step size 10^{-3}.

In Fig. 4a, 4b, and 4c, the trajectory initial positions are chosen to be close to L_1, which, if it is chosen before L_1 and on the side of the Sun, the satellite will orbit the Sun with a chaotic behavior (i.e. without any repetition for the orbit), this is the case in Fig. 4a and 4b. While for initial positions between L_1 and Earth, the orbit of the satellite would be chaotic around Earth as in Fig. 4c.

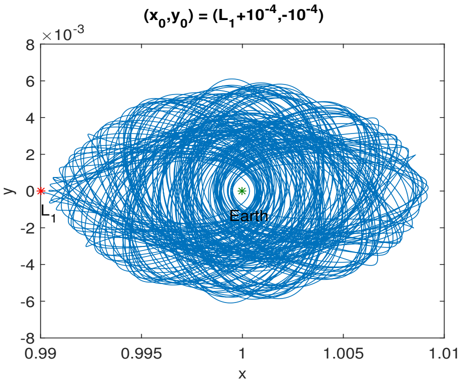
Fig. 5 illustrates the chaotic behavior of the trajectories starting from initial positions between Earth and L_2. Both Fig. 5a and 5b are orbiting the Earth. On



(a)



(b)



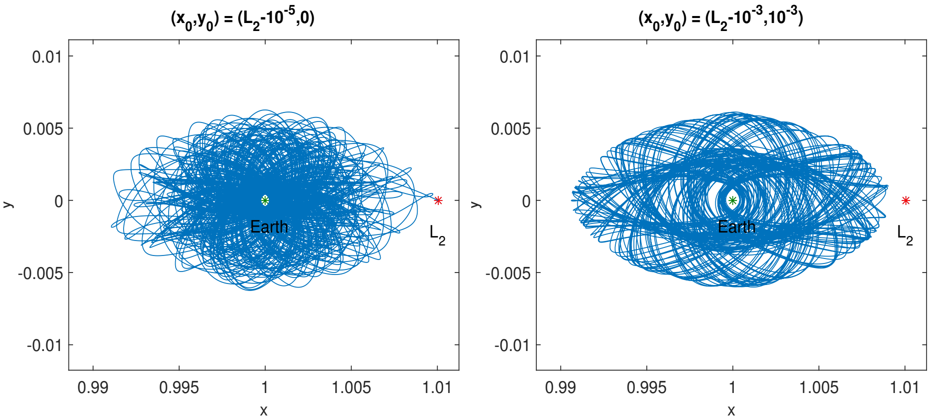
(c)

FIGURE 4. Trajectories initial positions close to L_1

the other hand, for initial positions between L_3 and the Sun, the trajectories would be also chaotic but orbiting the Sun. This is introduced in Fig. 6a, 6b, and 6c for different positions.

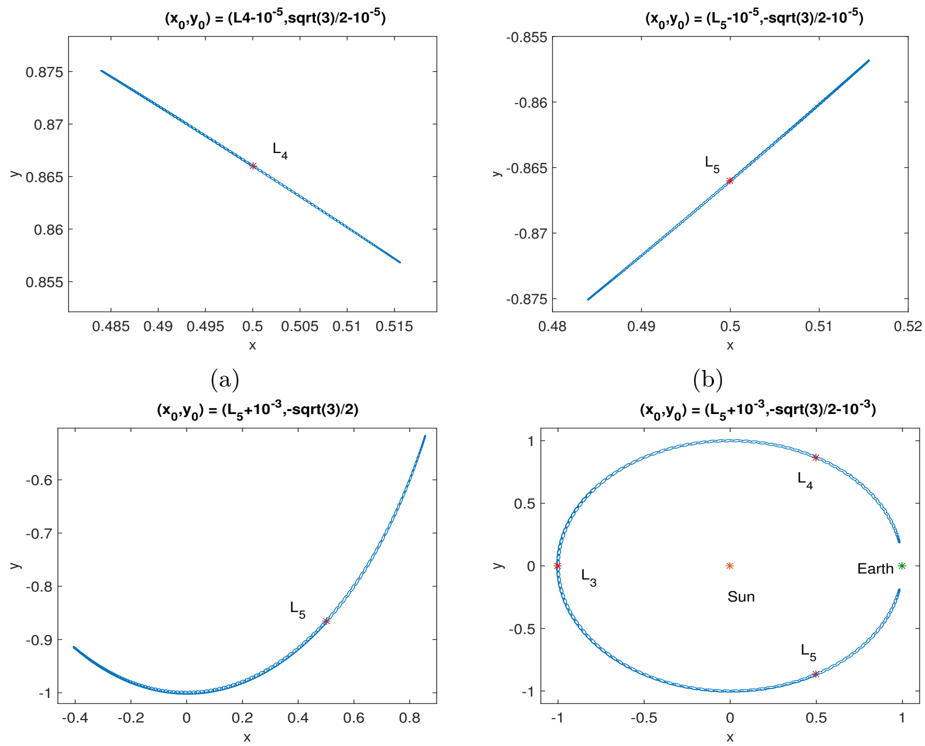
From Fig. 7, the satellite's trajectory could be considered to have local stable behavior for very close initial positions, inside a very small neighborhood to L_4 and L_5 as shown in Fig. 7a and 7b, respectively. The trajectories are forming repetitive cycles around L_4 or L_5. For further initial positions, the trajectories are extending and forming further cycles from L_4 and L_5 such as in Fig. 7c. Then, a longer cycle could lead to an incomplete circular cycle as in Fig. 7d, and this incomplete circle is bounded as shown from the energy effect around the Earth. This behavior for both L_4 and L_5 is considered to be stable, so that asteroids are kept and collected at L_4 and L_5.

On the other hand, the chaotic behavior is attracting researchers to control such cases. In the next section, we will give a brief example to control the trajectory at L_1 by using quadratic programming ($l-2$ technique).



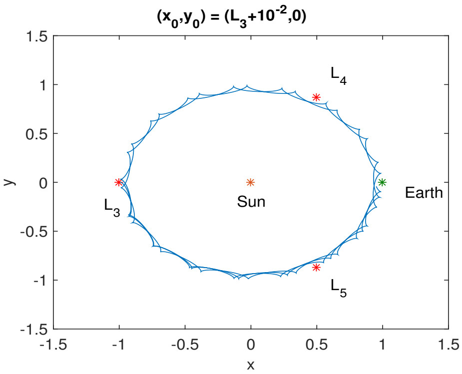
(a) (b)

FIGURE 5. Trajectories starting from close positions to L_2.

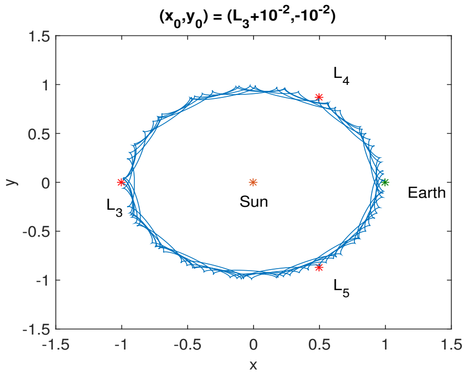


(a) (b) (c) (d)

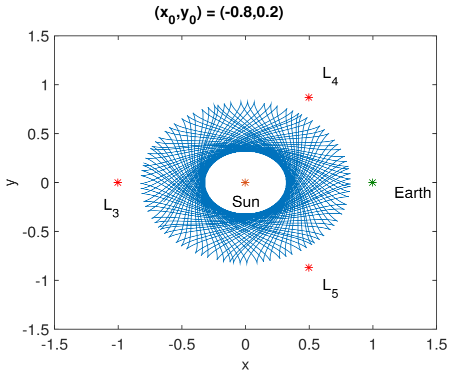
FIGURE 7. Trajectories starting from near positions to L_4 and L_5.



(a)



(b)



(c)

FIGURE 6. Trajectories plotted from initial position close to L_3

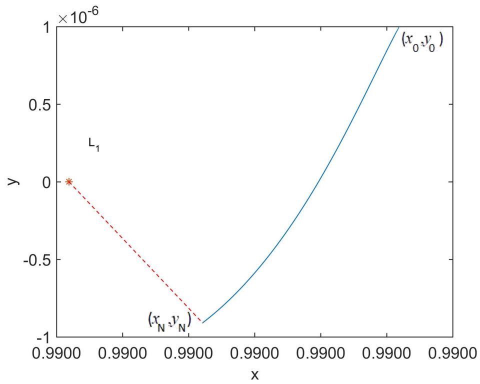


FIGURE 8. The controlled trajectory (colored in blue) at L_1 (marked as red star), and the distance between the final position of satellite and L_1 is the dashed line.

6. l_2-technique to control trajectory at L_1

We assume that there is a satellite, supplied by two jet engines. One engine is directed horizontally and the other is vertically. Suppose that the inserted control components are $U^{(1)}, U^{(2)}$, added to the right hand side of the last two equations of the System (2.1). We will use the standard l_2-approach with a quadratic functional. The objective function is $J = \int_0^T \left(||U^{(1)}||^2 + ||U^{(2)}||^2\right) dt$, which is required to be minimized. However, the system is ordinary differential equations with no analytical solution, it is discretized numerically to determine the approximate solution and study the trajectory. Hence the objective function can be used in its discretized form too, as submissions instead of integration, i.e. $J = \sum_{n=0}^{N-1} \left(||U_n^{(1)}||^2 + ||U_n^{(2)}||^2\right)$. To solve the problem, it is firstly solved for the linearized form of the system (Equation (4.2)), which is now considered to include the control terms as follows,

$$\dot{W} = J|_{L1} W + BU, \tag{6.1}$$

where $B = \left(\begin{smallmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{smallmatrix}\right)^T$. Then the minimization of J is determined for the discretized form of Equation (6.1), which is $W_{n+1} = W_n + \delta t (J|_{L_1} W_n + BU_n)$. After that, we apply the resulting minimized control values of U in the nonlinear system numerical solution. In this case, we obtain a terminal condition close to L_1. We take $T = 1$, $N = 5000$, and $\delta t = 1/5000$. The trajectory is shown in Figure 8 which is colored in blue. It is clear that the trajectory turned close to L_1 instead of escaping away far from L_1.

Simulation of such a control method under various initial conditions has effectiveness to keep a body for some space missions inside a close position to unstable Lagrange point till it completes its task, for example recording information or taking photos.

7. Conclusion

The behavior of a satellite trajectory near the equilibrium points of the Sun-Earth system is studied as a case of the circular restricted three body problem. The dynamical system is solved at a stationary state and Lagrange points are determined. Then, the linear stability of the system is discussed. We presented a satellite trajectory numerically and showed that the trajectory is unstable near L_1, L_2 and L_3, and its trajectory is unstable between L_4 or L_5 and Sun (closer to L_4 or L_5 respectively). Satellite trajectory is considered to be locally stable inside a very small neighborhood of L_4 or L_5. For the chaotic trajectory, quadratic programming is used to control it. L_1 is selected as an example and the control method is successfully applied to drive the trajectory close to L_1.

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ПОНАШАЊЕ ТРАЈЕКТОРИЈЕ САТЕЛИТА У БЛИЗИНИ ТАЧАКА РАВНОТЕЖЕ СИСТЕМА СУНЦЕ-ЗЕМЉА И ЊЕГОВА КОНТРОЛА

Резиме. У овом раду се проучава понашање трајекторије сателита у близини равнотежних тачака система Сунце-Земља. За диференцијалне једначине које описују кретање сателита у кружном ограниченом проблему три тела за систем Сунце-Земља одређују се Лагранжове тачке. Затим се проучава стабилност у свакој Лагранжовој тачки. Илустроване су трајекторије сателита који почиње да се креће у близини Лагранжове тачке, показујући стабилно и нестабилно понашање. Коначно, као пример, нестабилна трајекторија је контролисана коришћењем l_2-методе у L_1.

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