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LOCOMOTION OF MULTIBODY ROBOTIC SYSTEMS: DYNAMICS AND OPTIMIZATION

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ABSTRACT. Locomotion of multibody systems in resistive media can be based on periodic change of the system configuration. The following types of mobile robotic systems are examined in the paper: multilink snake-like systems; multibody systems in quasi-static motion; systems consisting of several interacting bodies; fish-like, frog-like, and boat-like systems swimming in fluids; systems containing moving internal masses. Dynamics of these systems subjected to various resistance forces, both isotropic and anisotropic, are investigated, including dry friction forces obeying Coulomb's law and forces directed against the velocity of the moving body and proportional to the velocity value or its square. Possible modes of locomotion and control algorithms are discussed. Optimization for various types of mobile robots is considered. Optimal values of geometrical and mechanical parameters as well as optimal controls are obtained that provide the maximum locomotion speed or minimum energy consumption. Results of experiments and computer simulation are discussed.

1. Introduction

Locomotion of mobile robotic systems along surfaces and inside media can be based on different principles. The most well-known locomotion systems use wheels, legs, tracks, propellers, and other external devices interacting with the outer media. However, mobile systems can move by means of special periodic change of their configuration. These types of locomotion can imitate motions of animals and insects such as snakes, fish, worms, etc. Other mobile robots controlled by the motion of internal masses have no direct analogues in nature.

In this paper based on research fulfilled recently in the Institute for Problems in Mechanics of the Russian Academy of Sciences, we describe and discuss results on dynamics and control of locomotion for certain types of mobile robotic systems. We consider systems that move on surfaces and inside media due to special periodic change of their configuration.

The following types of robotic systems are analyzed: multilink snake-like systems (Section 2); multibody systems in quasi-static motion (Section 3); systems

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consisting of several interacting bodies moving in various media (Section 4); fish-like, frog-like, and boat-like systems swimming in fluids (Section 5); systems containing movable internal masses (Section 6). Section 7 contains conclusions.

Locomotion of mobile systems can occur along surfaces in the presence of dry friction forces or inside resistive media. The resistance forces are directed against the velocity of the moving body and depend on the velocity value. Both linear and quadratic dependences are considered; the resistance forces may be either isotropic or anisotropic, i.e., dependent on the direction of motion.

Possible modes of locomotion, their dynamics and control algorithms are discussed. For various types of mobile robotic systems listed above, optimization problems are considered. Optimal values of geometrical and mechanical parameters as well as optimal controls are found that correspond to the maximum average locomotion speed or the minimum energy consumption.

Experimental devices are described that can serve as prototypes of mobile robots.

2. Snake-like locomotion

Motion of snakes and other limbless animals always attracted attention of scientists in the fields of mechanics and biomechanics. It is well-known that snakes can move fast in arbitrary directions over surfaces with different properties (Fig. 1). Motions of snakes in curved tubes were first analyzed in [1]. Different modes of

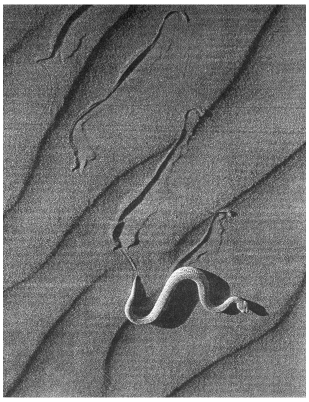

FIGURE 1. Snake in a desert

snakes locomotion were classified and described in [2] where motions along a horizontal plane are considered in the presence of vertical walls and obstacles. Multilink snake-like robot equipped with passive wheels was created and described in [3]. In a number of papers, see [4], snake-like locomotion is considered under the assumption that each link of the system does not slip in the direction perpendicular to the link. This restriction is fulfilled in the cases of vertical walls, obstacles, passive wheels, or if the friction between the moving system and the horizontal plane is anisotropic.

In papers [5–8], the motion of a multilink snake-like system along a horizontal plane is considered in the absence of obstacles and wheels. We assume that the system consists of rigid links lying on the plane and connected consecutively by cylindrical joints P_i of masses m_i, $i = 1, 2, \dots, N$, with vertical axes (Fig. 2). The actuators installed at joints P_i can create torques M_i that act upon the neighboring

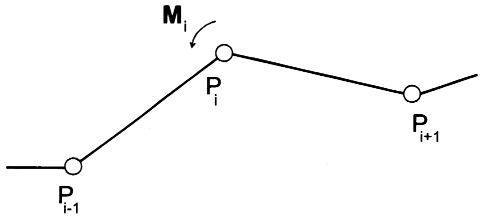

FIGURE 2. Multilink system on a horizontal plane

links. In the first approximation, we neglect masses of links compared to masses of joints. The dry friction force acting upon joint P_i obeys Coulomb's law:

$$\begin{aligned} \mathbf{F}_i &= -km_i g\mathbf{v}_i/|\mathbf{v}_i|, \quad \text{if} \quad \mathbf{v}_i \neq 0 \\ |\mathbf{F}_i| &\leqslant km_i g, \quad \text{if} \quad \mathbf{v}_i = 0. \end{aligned} \tag{2.1}$$

Here, $\mathbf{v}_i$ is the velocity of joint P_i, g is the acceleration of gravity, and k is the coefficient of friction.

Methods of control for linkages with two and three links are proposed [5, 7] that enable them to move in any prescribed direction on a plane. These motions consist of alternating simpler motions subdivided into slow and fast ones.

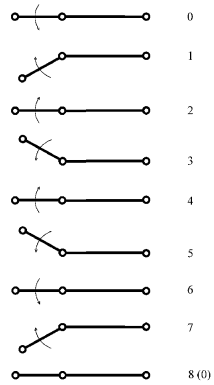

FIGURE 3. Phases of periodical motion of a two-link system

As an example, we describe here a periodic longitudinal locomotion of a two-member linkage consisting of three masses connected by two links [7]. The longer link is called a body, and the shorter one is a tail. At the beginning, the linkage is straight and stays at rest (Fig. 3). The period of motion comprises 8 phases, 4 slow and 4 fast ones. They alternate: slow phases start when the linkage is straight and at rest, see positions 0, 2, 4, 6 in Fig. 3. After each phase, the linkage comes to rest again. In slow phases, the body does not move, whereas the tail turns by a certain angle. Certain constraints on the angular velocity and acceleration of the tail must be fulfilled for slow phases. In fast phases, the linkage becomes straight in very short time. The torque M applied at the joint must be high enough: $M \gg mgka$, where m is the mass of the linkage, a is its total length.

During slow phases, the center of mass of the system moves forward (towards the body) and also sideways. During fast phases, the influence of the dry friction forces is negligible, and the center of mass practically does not move. To compensate lateral displacements of the center of mass and angular displacements of the linkage, all eight phases shown in Fig. 3 are required. As a result of this succession of phases, the linkage implements a longitudinal displacement during the period of motion. The value of the displacement as well as the conditions required for this motion are given in [7].

In a similar way, both longitudinal and lateral periodic motions of a three-link system as well as its rotation on the spot are constructed as a sequences of slow and fast phases [5].

It is shown that both the two-member and three-member linkages can be transferred from the initial position and configuration to any prescribed terminal position and configuration in the horizontal plane. The corresponding relationships are obtained, and the locomotion speed is evaluated [5, 7].

Experimental models of multilink mechanisms implementing the proposed motions are created and tested [9], see Fig. 4. The mechanisms perform the motions described above, and the experimental data correspond to the obtained theoretical results. However, since fast motions cannot be considered instantaneous for the experimental model, certain corrections are included into the calculations of the fast phases [10].

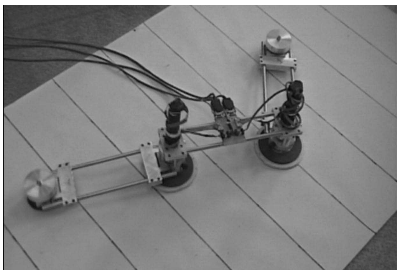

FIGURE 4. Experimental model of a three-link system

An important characteristic of multilink systems is their average locomotion speed that depends on such geometrical and mechanical parameters as lengths of the links, masses of the joints, angles of rotation of the links, and durations of phases. Optimal values of these parameters, under certain constraints imposed, are obtained that correspond to the maximum average locomotion speed [8, 11].

In particular, it is found that the lateral motion of a three-member linkage can be faster than its longitudinal motion. The two-member linkage moves slower than the three-member one with similar length and mass. This conclusion is quite clear: the three-member linkage is equipped with two actuators, whereas the two-member linkage has only one.

In addition to the average locomotion speed, another optimization criterion is important for snake-like motions, namely, the energy consumption per unit path. Multicriteria optimization of parameters for multilink systems with respect to two criteria (speed and energy consumption) is considered [10], and the respective Pareto sets are obtained.

As an example, let us consider some results of multicriteria optimization [10] for a two-member linkage consisting of a main body of length a_1 and a tail of length a_2. If the ratio $\lambda = a_2/a_1$ is sufficiently small ($\lambda < 0.3$), then, by increasing the energy consumption per unit path, the average speed of the longitudinal motion of the linkage can be increased. However, if $\lambda > 0.3$, then the speed cannot be increased even at the cost of the increase of the energy consumption. Thus, the value $\lambda = 0.3$ seems to be the most reasonable value for this ratio.

3. Quasi-static motions of multibody systems

If the motion of a multilink system along a plane is slow enough so that inertia forces are small compared to friction forces, then the inertia terms in the equations of motions can be neglected. In this case we have quasi-static motions that can be regarded as a sequence of equilibrium positions. Here, control forces created by actuators counterbalance friction forces.

Possible quasi-static motions of a multilink system over a horizontal plane are studied in [6,8,12,13]. It is shown that a two-link system cannot move progressively in a quasi-static mode; its motion is bounded by a circle of a certain radius [12]. For a three-link system, progressive quasi-static locomotion is possible under certain constraints imposed on the system parameters [13].

Let us consider a multilink system lying on a horizontal plane Oxy and consisting of N equal links connected by cylindrical joints with vertical axes. The links are straight rigid rods of length a, and their masses are negligible compared to the masses of the joints. The joints and end points of the chain P_i, $i = 0, 1, \ldots, N$, have the same mass m. The actuators installed at the joints $P_1, \ldots, P_{N-1}$ can create control torques about vertical axes.

Two types of longitudinal progressive motions of the multilink chain are proposed [6]. In these motions, at any instant of time, three or four links move, whereas all the other links stay at rest. These modes of locomotion are shown in Figs. 5 and 6, respectively.

At the beginning and at the end of a locomotion cycle, the multilink chain lies along the axis Ox and is at rest. First, in both modes of Figs. 5 and 6, the end point P_0 starts to move along the axis Ox so that the isosceles triangle $P_0P_1P_2$ forms. The angle α grows and reaches its maximal prescribed value α_0. Then other links begin to move as shown in Figs. 5 and 6. As a result, in both modes a wave travels along the chain from its left end to its right end. Certain differences exist

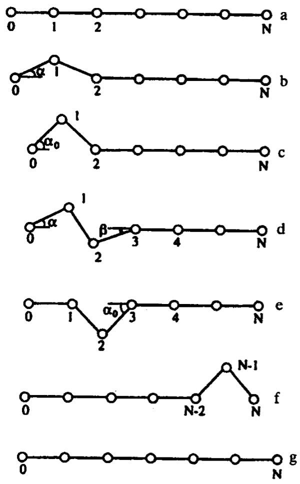

FIGURE 5. Wave-like motion with three moving links

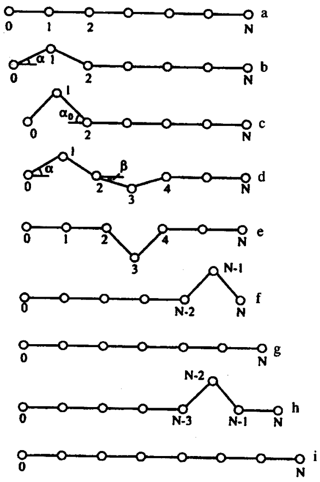

FIGURE 6. Wave-like motion with four moving links

for even and odd N in the case of four moving links (Fig. 6). At the end of the cycle, the chain becomes straight again. Its total displacement, for both modes of Figs. 5 and 6, is given by the formula $L = 2a(1 - \cos\alpha_0)$.

Comparing both modes of wave-like motions, we see that the motion with three moving links (Fig. 5) is simpler than one with four moving links (Fig. 6) but requires bigger angles between neighboring links for the same α_0 and L.

To justify the proposed kinematics of locomotion, the balance of forces is analyzed. The following proposition for the both locomotion modes is proved [6].

There exist such friction forces satisfying equations and inequalities (2.1), for moving joints and joints at rest, respectively, that the equilibrium equations are satisfied for all phases of motions shown in Figs. 5 and 6, if $N \geqslant 5$ or $N \geqslant 6$ for the motions with three or four moving links, respectively.

The required value of the torques M created by the actuators is evaluated. It does not exceed $2mgka$ and is much smaller than the torque $M \gg mgka$ required for fast dynamical motions.

Quasi-static two-dimensional motions of a three-body system along a horizontal plane are investigated [14]. Three point masses m_i, $i = 1, 2, 3$, are connected by massless rigid straight rods that can change their lengths (Fig. 7). The system is controlled by three linear actuators acting along the rods. Denote by k_i the coefficients of friction for masses m_i. It is shown that an arbitrary displacement of the three-body system in the horizontal plane is possible, if and only if the following triangle inequality is satisfied:

$$k_i m_i + k_j m_j \geqslant k_s m_s, \quad i, j, s = 1, 2, 3.$$

Under this condition, optimal displacements of the system are found that correspond to the minimum work against friction forces. In particular, if two masses stay at rest, the optimal motion of the third mass from a given initial point to the prescribed terminal one in the horizontal plane occurs either along a straight line or along a two-link broken line in the plane [14].

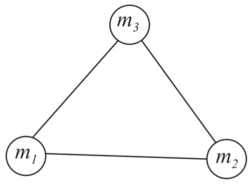

FIGURE 7. Three-mass system on a plane

4. Optimal motions of multibody chains

Multibody systems consisting of several masses connected consecutively can move progressively in resistive media under the influence of control forces created by actuators installed between neighboring bodies.

First, consider a system that consists of two interacting bodies of masses m and M, $m < M$, moving along the horizontal axis in the presence of dry friction forces obeying equation (2.1), see Fig. 8. The interaction forces acting upon masses m and M are F and $-F$, respectively, where F is bounded by condition $|F| \leqslant F_0$. The inequality $F_0 > Mgk$ is necessary to ensure the progressive motion of the two-mass system along a straight line.

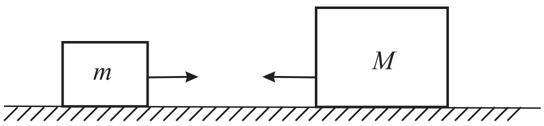

FIGURE 8. Two-mass system

Optimal periodic motions of this system are obtained under the condition that the maximum distance between masses does not exceed L [15]. For example, if $F_0 \to \infty$, the maximum locomotion speed of the two-mass system is

$$v = [gkLm(M-m)]^{1/2}(M+m)^{-1}. \tag{4.1}$$

The optimal ratio m/M that provides the maximum v in (4.1) is 1/3; in this case $v = (gkL)^{1/2}/4$. Similar results are obtained for more general situations [15].

If the actuator installed between the masses can rapidly bring the bodies to a state of constant relative velocity, then this velocity, instead of force F, can be considered as a control bounded by certain constraints. The analysis and optimization of such motions is given in [16].

Optimal progressive motions of a chain of $N \geqslant 3$ identical bodies along a straight line in the presence of dry friction forces (Fig. 9) are considered in [17]. The interaction forces acting between the neighboring bodies are assumed unbounded so that the velocities of the masses can change instantly. The motions corresponding to the maximum displacement of the whole chain for a given time interval are obtained.

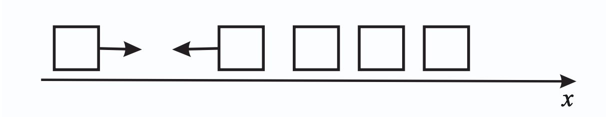

FIGURE 9. Chain of bodies

Let us consider now locomotion of a chain of N identical bodies in a resistive medium (Fig. 9). We assume that the resistance force F acting upon each body is directed against its velocity v and depends on the value of the velocity linearly:

$$F = -bv \tag{4.2}$$

or quadratically:

$$F = -cv|v|. \tag{4.3}$$

Coefficients b and c in formulas (4.2) and (4.3) can, in the case of anisotropic resistance, depend on the direction of motion:

$$\begin{aligned} b &= b_+, \quad c = c_+ \qquad \text{if} \quad v > 0, \\ b &= b_-, \quad c = c_-, \qquad \text{if} \quad v < 0. \end{aligned} \tag{4.4}$$

The actuators installed between each pair of neighboring bodies can exert impulses on these bodies. The algorithm of control is proposed that enables the chain to move progressively with a certain locomotion speed. According to this algorithm, one of the bodies always moves backwards for some period of time and thus transmits its momentum to all other bodies in order to support their progressive movement forward. Bodies moving backwards take turns: first, the last body in the chain moves backwards, then the preceding one, and quadratic so on, until the first body in the chain performs the backward movement. As a result, the whole chain moves progressively with an average locomotion speed that is evaluated. In the anisotropic case, such locomotion is possible both for the linear (4.2) and (4.3) resistance, whereas in the isotropic case the locomotion is impossible for the linear resistance ($b_+ = b_-$ in (4.4)) but possible for the quadratic one ($c_+ = c_-$ in (4.4)).

5. Locomotion in fluids

Resistance force acting upon a body moving in a fluid with velocity v is often approximated by a quadratic law (4.3). Under this assumption, certain simple models of motion imitating swimming of fish and frogs are considered. Unlike more complicated models considered in [2, 18–21], in our models we not only establish explicit relationships for main parameters of motions, but also find optimal controls for these motions.

As a first model, we consider a motion in a fluid of a rigid body to which a link OA (a tail) is attached by means of a cylindrical joint O, see Fig. 10. The mass of the tail OA can be neglected compared to mass m of the body. The fluid exerts quadratic resistance forces upon the body and the tail. The angle $\varphi(t)$ between the body and tail changes periodically with time: $\varphi(t + T) = \varphi(t)$, where T is the period.

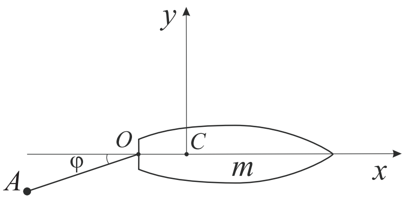

FIGURE 10. Fish-like system

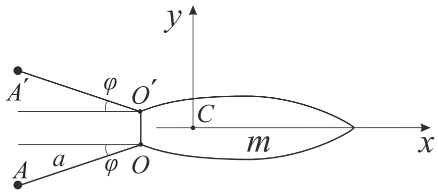

FIGURE 11. Body with two links attached

Another model with two links OA and $O'A'$ attached to the body is shown in Fig. 11. These links move symmetrically with respect to the axis Cx, where C is the center of mass of the body. Quadratic resistance forces directed against the axis Cx are applied to the bodies in Figs. 10 and 11. Also, such forces are applied to points A and A' and directed against the velocities of these points.

Under certain conditions such as the symmetry of the body with respect to the axis Cx, periodicity and symmetry of the oscillations of links, high frequency of these oscillations, the motions of models in Figs. 10 and 11 are described by the same equations.

The analysis performed in [22,23] permits to simplify drastically the equations of motion for these models. As a result of the procedure of averaging [24] and normalization, these equations are reduced to the following equation:

$$\frac{dv}{dt} = -\varepsilon(v^2 + I), \quad I = \int_0^1 \sin\varphi \frac{d\varphi}{dt}\left|\frac{d\varphi}{dt}\right| dt. \tag{5.1}$$

Here, v is the velocity of the body, t is a dimensionless time, $\varepsilon > 0$ is a dimensionless parameter; the angle $\varphi(t)$ changes periodically with period $T = 1$. For any periodic function $\varphi(t)$ given on the interval $t \in [0, 1]$, we can determine $v(t)$ from (5.1). If $I > 0$, then the velocity decreases ($v < 0$), and locomotion with a constant speed is impossible. If $I < 0$, then locomotion with a constant speed

$$v_* = (-I)^{1/2} > 0$$

is possible and stable.

Consider a simple piecewise linear time history of the angle $\varphi(t)$:

$$\begin{aligned} &\varphi(t) = \omega_+(t), && \text{if} \quad t \in (0, \theta), \\ &\varphi(t) = \omega_-(1 - t), && \text{if} \quad t \in (\theta, 1), \quad \theta \in (0, 1), \end{aligned} \tag{5.2}$$

where ω_+ and ω_- are constant angular velocities of the links, and θ is the instant of switch. For the case (5.2), we obtain from (5.1):

$$I = (1 - \cos\varphi_0)(\omega_+ + \omega_-), \quad \varphi_0 = \varphi(\theta) \in (0, \pi/2).$$

Hence, $I > 0$ for $\omega_+ > \omega_-$ and $I < 0$ for $\omega_+ < \omega_-$. Therefore, the locomotion speed v_* is positive, if and only if the angular velocity ω_+ of deflection of links OA and $O'A'$ from the axis Cx is smaller than the angular velocity ω_- of their return to the axis. Observations of swimming animals and fish corroborate this conclusion.

Optimal control laws for the angle $\varphi(t)$ bounded by the constraints imposed on the angular velocity:

$$-\omega_- \leqslant \dot{\varphi}(t) \leqslant \omega_+$$

are obtained that ensure the maximum average locomotion speed [22, 23, 25]. It occurs that the optimal law does not differ significantly from the simple piecewise linear law (5.2). More general case of the resistance force depending on the velocity in power s, namely,

$$F = -cv|v|^{s-1}$$

is also considered [23].

More complicated five-link swimming model (Fig. 12) imitates a frog or a sportsman using breaststroke style. As a result of the analysis and computer simulation of this model, certain qualitative and quantitative features of its motion are established [26, 27]. It occurs that in order to increase the speed, it is necessary to shorten the time of straightening of limbs compared to the time of their bending. The knees should be unbended earlier than the hip joint so that the last phase of the motion should be fulfilled with straight limbs. These inferences are confirmed by observations.

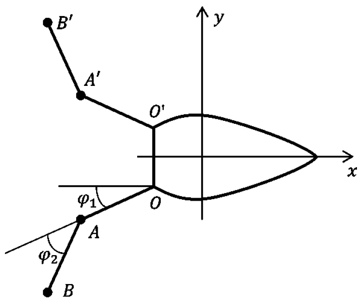

Figure 12. Frog-like model

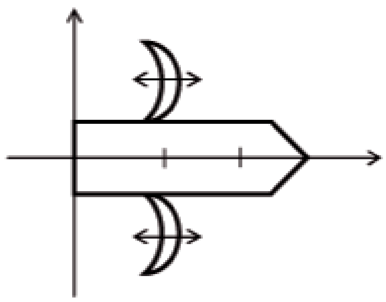

Figure 13. Boat-like model

A simple model of a rowing boat [28] shown in Fig. 13 consists of a main body and two auxiliary bodies (oars or fins) that are symmetric to each other and perform periodic translational movements relative to the main body. All three bodies are subjected to quadratic resistance forces described by equation (4.3). The analysis of equations of motion shows that, under certain conditions established in [28], the progressive locomotion of the system is possible; its velocity is positive and changes periodically.

6. Locomotion of systems containing internal movable masses

In this Section, we consider motion of bodies controlled by internal movable masses. This type of locomotion is possible on a horizontal plane in the presence of dry friction forces (Fig. 14) and in a resistive medium (Fig. 15). This kind of systems, contrary to those considered in previous Sections, is not encountered among living organisms.

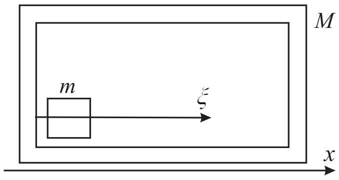

FIGURE 14. Body with a movable internal mass on a plane

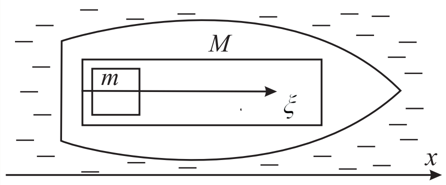

FIGURE 15. Body with a movable internal mass in a resistive medium

This principle of locomotion was considered in a number of papers, see [29–32]. It was applied to micro-robots [29, 32] and robots moving in tubes [31]. Various aspects of locomotion along surfaces are covered in [33].

Below we discuss mainly optimal motions of systems containing internal movable masses.

Suppose that the main body of mass M can move with velocity v along the axis x in a medium that exerts the resistance force $F(v)$ upon the body. The main body contains another body of mass m that interacts with the main one but does not interact with the external medium. The displacement of mass m relative to the body is denoted by ξ. The equations of motion of the two-body system can be written as follows

$$(M+m)\dot{v} = -m\dot{\xi} + F(v), \quad \dot{x} = v. \tag{6.1}$$

The resistance force $F(v)$ can take the form (2.1) for the dry friction, (4.2) and (4.3) for linear and quadratic resistance, respectively, or have more general form. For all cases, the inequality $F(v)v \leqslant 0$ must hold which means that the resistance force is always directed against the velocity.

The displacement $\xi(t)$ of the internal mass relative to the main body, relative velocity $\dot{\xi}(t)$, or acceleration $\ddot{\xi}(t)$ can be regarded as controls. We consider periodic relative motions with period T satisfying conditions

$$\xi(t+T) = \xi(t), \quad v(t+T) = v(t).$$

It is natural to impose a constraint upon a relative displacement of the internal mass:

$$0 \leqslant \xi(t) \leqslant L, \tag{6.2}$$

where L is a given value that determines the possible amplitude of displacement of body m relative to M.

The simplest controls that can be applied to the two-body system under consideration are piecewise constant time histories either of the relative velocity $\dot{\xi}(t)$ or relative acceleration $\ddot{\xi}(t)$.

These controls for the case of dry friction (2.1) are analyzed [34] under the initial condition $v(0) = 0$. Optimal periodic piecewise functions for the velocity $\dot{\xi}(t)$ and acceleration $\ddot{\xi}(t)$ are obtained that ensure the maximum average locomotion speed V. Note that in the case of the piecewise constant $\dot{\xi}(t)$, the main body moves forward ($v > 0$) and backwards ($v < 0$) during the period of motion, whereas for the piecewise constant acceleration $\ddot{\xi}(t)$, the phase of the forward motion of the main body ($v > 0$) alternates with the phase of rest ($v = 0$). The maximum average locomotion speed is given by

$$V_1 = 0.545\sqrt{\mu k L g}, \quad V_2 = \sqrt{\mu k L g}, \quad \mu = m(M+m)^{-1},$$

for the cases of optimal piecewise constant velocity $\dot{\xi}(t)$ and acceleration $\ddot{\xi}(t)$, respectively. Here, k and g are defined in (2.1), and L in (6.2). The case of the anisotropic dry friction is also analyzed [34].

Periodic piecewise control laws for the relative velocity $\dot{\xi}(t)$ and acceleration $\ddot{\xi}(t)$ in the case where the initial velocity $v(0)$ is arbitrary are also investigated [35]. For different anisotropic resistance forces: Coulomb's dry friction (2.1), linear (4.2) and quadratic (4.3) resistance, optimal controls are found that give the maximum locomotion speed.

If the resistance is anisotropic (see (4.4)), then the progressive locomotion is possible both for linear and quadratic cases. In the case of linear isotropic resistance, the progressive locomotion of the system is impossible: the system will only oscillate about some mean position. Similar fact is mentioned at the end of Section 4.

For the isotropic quadratic resistance (4.3), the progressive locomotion is possible, and its maximum speed is given by

$$V = -(cT)^{-1}\log(1 - \mu^2 c^2 L^2) > 0, \quad \mu c L < 1.$$

The motion of systems with internal movable masses are analyzed also for the general case of resistance force $F(v)$ in equation (6.1). Under certain general conditions, the optimal periodic laws for the relative acceleration $\ddot{\xi}(t)$ are found [36] that provide the maximum displacement of the system over the period T. The optimal motion includes intervals with maximum admissible relative acceleration $\ddot{\xi}(t)$ and also singular intervals with constant v.

It the main body contains two internal movable masses that can move along two perpendicular axes, horizontal and vertical, inside the main body (Fig. 16), then additional control possibilities appear. The mass moving along the vertical axis changes the normal reaction of the surface and thus changes the friction force. In this way, the average locomotion speed can be increased. Optimal motions of this system are studied [37].

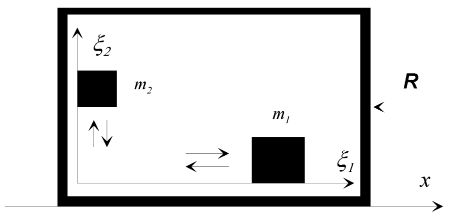

FIGURE 16. Body with two internal masses

Principle of locomotion based on motion of internal masses is implemented in a number of experimental devices. Experimental data obtained confirm theoretical conclusions and results of calculations made.

The model shown in Fig. 17 consists of a cart with passive wheels and an inverted pendulum installed on top of it. As a result of oscillations of the pendulum about the upper equilibrium position, the cart moves in the desired direction [38]. Thus, this model implements the same pattern as shown in Fig. 14.

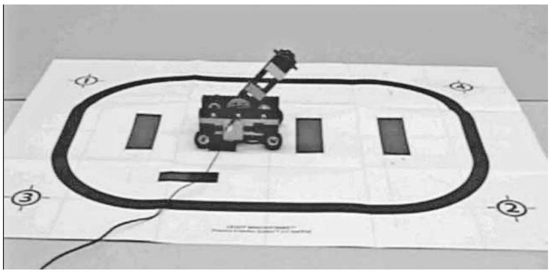

FIGURE 17. Cart with inverted pendulum

Fig. 18 shows "capsubot", or a vibro-robot that contains an electromagnetic actuator and an internal mass that performs longitudinal oscillations inside the robot [39]. The capsubot moves along a horizontal plane.

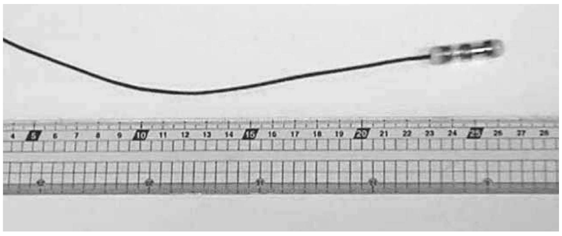

FIGURE 18. Capsubot

A cart with passive wheels shown in Fig. 19 carries rotating wheels with eccentric masses. As a result of the rotation of the wheels, the cart slides along a horizontal plane. In this model [40], the motion occurs that is similar to one described in [37]. Other examples of mobile systems with rotors are considered in [41] and [42].

Mini-robots that can move inside tubes are designed and made in Institute for Problems in Mechanics of Russian Academy of Sciences [31]. These vibro-robots shown in Fig. 20 consist of two parts that oscillate with respect to each other under the influence of an electromagnetic actuator. The robots can move inside straight and curved tubes, horizontal, vertical, and inclined, with diameters between 5 mm and 30 mm and with a speed from 10 to 30 mm/sec. Such robots can perform inspection and other operations in tubes.

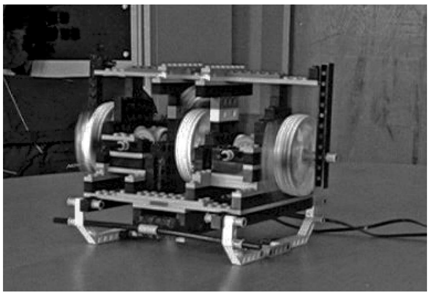

FIGURE 19. Cart with rotors

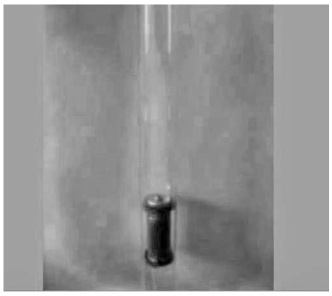

FIGURE 20. Robot moving in a tube

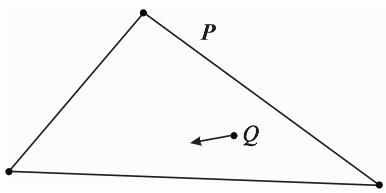

FIGURE 21. Two-dimensional motion of a body with internal mass

Up till now, we considered rectilinear motions of systems with internal moving masses. However, it is possible also to implement two-dimensional and three-dimensional motions of such systems on a plane or in resistive media. As an example, consider a two-dimensional motion of a rigid body P containing internal masses over a horizontal plane in the presence of dry friction forces obeying Coulomb's law (2.1). We assume that the body has three support points so that the system is statically determinate. As for internal masses, we assume that a point mass Q can move arbitrarily relative to body P along a horizontal plane parallel to the support plane (Fig. 21). It is shown that, under certain general conditions, there exist such motions of mass Q relative to body P that this body can be transferred from the initial position to any prescribed terminal position in the horizontal plane. Thus, the system is controllable by means of the internal mass. Another internal masses (a rotor and a point mass) that also make the system controllable are considered in [43].

7. Conclusions

For several types of locomotion systems, problems of dynamics and optimization of motion are examined. The mechanical systems under consideration can move inside various media; their motion is based on special periodic change of their configuration. Basic properties of motion for these robotic systems are considered, locomotion modes are described, and the speed of motion is evaluated. Optimal values of parameters and optimal controls for these systems are found that provide the maximum speed or minimum energy consumption per unit path. Most of locomotion types considered above are similar to those encountered in nature.

Systems containing internal moving masses have no direct biological analogues. These systems do not need external devices such as legs, wheels, tracks, propellers, etc. They can be even made hermetic; they need only energy source for movement of internal masses and, of course, the resistance force of the outer medium. Such robots may be useful for motions inside vulnerable or hazardous media.

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КРЕТАЊЕ РОБОТНИХ СИСТЕМА ВИШЕ ТЕЛА: ДИНАМИКА И ОПТИМИЗАЦИЈА

Резиме. Кретање система више тела у резистентним медијима може се заснивати на периодичном променом конфигурације система. У раду се испитују следећи типови покретних роботских система: системи вишевезних “попут-змија” система; система више тела у квази-статичком кретању; системи који се састоје од неколико интерактивних тела; “рибљи”, “жабљи” и системи попут “чамца” који пливају у течностима; системи који садрже покретне унутрашње масе. Испитана су динамика ових система подложних различитим силама отпорности, изотропним и анизотропним, укључујући силе сувог трења који поштују Колумбов закон и силе усмерене против брзине покретног тела и пропорционалне брзини или њеном квадрату. Дискутована су могућа стања кретања и контролних алгоритама. Разматран је проблем оптимизације за различите врсте покретних робота. Добијене су оптималне вредности геометријских и механичких параметара, као и оптималне контроле које обезбеђују максималну брзину покретања или минималну потрошњу енергије. Дискутовани су резултати експеримената и рачунарске симулације.

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