THEORETICAL AND APPLIED MECHANICS
Volume 44 (2017) Issue 2, 229–236 DOI: https://doi.org/10.2298/TAM170926016D

DISCRIMINANTLY SEPARABLE POLYNOMIALS AND THE GENERALIZED KOWALEVSKI TOP

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Dedicated to the memory of Professor Aleksandar Bakša.

Abstract. The notion of discriminantly separable polynomials of degree two in each of three variables has been recently introduced and related to a class of integrable dynamical systems. Explicit integration of such systems can be performed in a way similar to Kowalevski's original integration of the Kowalevski top. Here we present the role of discriminantly separable polynomials in integration of yet another well known integrable system, the so-called generalized Kowalevski top - the motion of a heavy rigid body about a fixed point in a double constant field. We present a novel way to obtain the separation variables for this system, based on the discriminantly separable polynomials.

1. Introduction

In a series of recent papers starting with [3] the notion of discriminantly separable polinomials has been introduced and used in a treatise of certain continuous and discrete integrable systems. Let us recall here the definitions: a polynomial $F(x_1, \dots, x_n)$ is *discriminantly separable* if there exist polynomials $f_i(x_i)$ such that for every $i = 1, \dots, n$

$$\mathcal{D}_{x_i} F(x_1, \dots, \hat{x}_i, \dots, x_n) = \prod_{j \neq i} f_j(x_j),$$

where $\mathcal{D}_{x_i} F$ is the discriminant of F as a polynomial in x_i. The polynomial F is *symmetrically discriminantly separable* if $f_2 = f_3 = \cdots = f_n$, while it is *strongly discriminantly separable* if $f_1 = f_2 = f_3 = \cdots = f_n$. The motivation for studying this class of polynomials we found both in the famous Kowalevski top [17] and in pencils of conics, see [3]. It appears that the so-called fundamental Kowalevski

2010 *Mathematics Subject Classification:* Primary 37J35, 37K60; Secondary 70E17, 70E40, 39A10.

Key words and phrases: the generalized Kowalevski top, discriminantly separable polynomials, separation variables.

equation from [8, 17] is an symmetrically discriminantly separable polynomial in three variables, degree two in each of them:

$$Q(s,x_1,x_2) := (x_1-x_2)^2\Big(s-\frac{l_1}{2}\Big)^2 - R(x_1,x_2)\Big(s-\frac{l_1}{2}\Big) - \frac{1}{4}R_1(x_1,x_2) = 0,$$

where $R(x_1,x_2)$ and $R_1(x_1,x_2)$ are biquadratic polynomials in x_1 and x_2 given by

$$\begin{aligned} R(x_1,x_2) &= -x_1^2x_2^2 + 6l_1x_1x_2 + 2lc(x_1+x_2) + c^2 - k^2, \\ R_1(x_1,x_2) &= -6l_1x_1^2x_2^2 - (c^2-k^2)(x_1+x_2)^2 - 4lcx_1x_2(x_1+x_2) \\ &\quad + 6l_1(c^2-k^2) - 4c^2l^2. \end{aligned}$$

Beside the famous Kowalevski case, in [4] we also presented a whole class of integrable systems that can be explicitly integrated in the terms of the genus two theta-functions, in [5] and [7] we explicitly solved some already known integrable systems by using separation variables obtained via discriminantly separable polynomials. We showed also that the Sokolov case from [15, 19] can also be explicitly solved in separation variables originating from discriminantly separable polynomial.

In this paper we show an alternative way to obtain the separation variables from [10, 12–14].

We will use some of the properties of discriminantly separable polynomials from [3, 5]. The next lemma from [3] provides a generalization of the famous Kowalevski change of variables applicable to the whole class of discriminantly separable polynomials.

LEMMA 1.1. *For an arbitrary discriminatly separable polynomial $\mathcal{F}(x_3,x_1,x_2)$ of the second degree in each of the variables x_3, x_1, x_2, its differential is separable on the surface $\mathcal{F}(x_3,x_1,x_2)=0$:*

$$\frac{d\mathcal{F}}{\sqrt{f_3(x_3)f_1(x_1)f_2(x_2)}} = \frac{dx_3}{\sqrt{f_3(x_3)}} + \frac{dx_1}{\sqrt{f_1(x_1)}} + \frac{dx_2}{\sqrt{f_2(x_2)}}.$$

A geometric background of discriminatly separable polynomials was presented in [3] and [6], together with a classification of such polynomials.

Here we consider the problem of motion of the Kowalevski top in a double constant force field. Such a system has a generic 4-dimensional invariant submanifold of the phase space, see [10]. The separating variables for that case were proposed in [13] and an explicit integration is performed in [12]. A geometrical approach to the proposed separation variables was given in [14]. In this paper we obtain the same separation variables as in [12, 13] but we use a different approach, based on discriminantly separable polynomials.

2. Equations of motion for the generalized Kowalevski top in the two constant fields

We consider the equations of motion of Kowalevski top, a rigid body fixed in the point O and the moments of inertia satisfy the conditions $I_1 = I_2 = 2I_3$, in two constant fields, see [2, 10, 11] for more details. In the reference system of the principal axes of inertia at the fixed point O, the equations of the motion take the following form:

$$
\begin{aligned}
2\dot\omega_1 &= \omega_2\omega_3 + \beta_3, & \dot\alpha_1 &= \alpha_2\omega_3 - \alpha_3\omega_2, & \dot\beta_1 &= \beta_2\omega_3 - \beta_3\omega_2,\\
2\dot\omega_2 &= -\omega_1\omega_3 - \alpha_3, & \dot\alpha_2 &= \alpha_3\omega_1 - \alpha_1\omega_3, & \dot\beta_2 &= \beta_3\omega_1 - \beta_1\omega_3,\\
\dot\omega_3 &= \alpha_2 - \beta_1, & \dot\alpha_3 &= \alpha_1\omega_2 - \alpha_2\omega_1, & \dot\beta_3 &= \beta_1\omega_2 - \beta_2\omega_1.
\end{aligned} \tag{2.1}
$$

Here $\vec\alpha = (\alpha_1, \alpha_2, \alpha_3)$ and $\vec\beta = (\beta_1, \beta_2, \beta_3)$ are the direction vectors of the force fields and $\vec\omega = (\omega_1, \omega_2, \omega_3)$ is the angular velocity. As in [10, 12], we consider the system of equations (2.1) together with the following geometric integrals:

$$
|\vec\alpha| = a, \quad |\vec\beta| = b, \quad \vec\alpha\cdot\vec\beta = 0. \tag{2.2}
$$

It is well known that the system of equations (2.1) under the assumptions (2.2) is completely integrable since it has three independent first integrals of motion in involution, see [1, 2, 12]:

$$
\begin{aligned}
H &= \omega_1^2 + \omega_2^2 + \tfrac{1}{2}\omega_3^2 - (\alpha_1 + \beta_2),\\
K &= (\omega_1^2 - \omega_2^2 + \alpha_1 - \beta_2)^2 + (2\omega_1\omega_2 + \alpha_2 + \beta_1)^2,\\
G &= \tfrac{1}{4}(M_\alpha^2 + M_\beta^2) + \tfrac{1}{2}\omega_3 M_\gamma - b^2\alpha_1 - a^2\beta_2,
\end{aligned} \tag{2.3}
$$

where by M_α, M_β and M_γ we denoted:

$$
\begin{aligned}
M_\alpha &= 2\omega_1\alpha_1 + 2\omega_2\alpha_2 + \omega_3\alpha_3,\\
M_\beta &= 2\omega_1\beta_1 + 2\omega_2\beta_2 + \omega_3\beta_3,\\
M_\gamma &= 2\omega_1(\alpha_2\beta_3 - \alpha_3\beta_2) + 2\omega_2(\alpha_3\beta_1 - \alpha_1\beta_3) + \omega_3(\alpha_1\beta_2 - \alpha_2\beta_1).
\end{aligned}
$$

Following [12, 13], we suppose that $a > b$ which can always be accomplished if $a \neq b$ by changing the order of vectors in the moving frame. Denote by

$$
p^2 = a^2 + b^2, \qquad r^2 = a^2 - b^2.
$$

Further, by F we denote the function which represents the following combination of the first integrals (2.3):

$$
F = (2G - p^2H)^2 - r^4K.
$$

Note that the phase space is diffeomorphic to $M^6 = \mathbb{R}^3 \times SO(3)$ due to geometric integrals (2.2). Denote, as in [13], by $N^4 \in M^6$ the subset of the set of critical points of the function F lying on the level $F = 0$. The new phase variables which generalize the Kowalevski complex variables [10, 17] are:

$$
\begin{aligned}
w_1 &= \omega_1 + i\omega_2 & w_2 &= \bar w_1 & w_3 &= \omega_3,\\
x_1 &= (\alpha_1 - \beta_2) + i(\alpha_2 + \beta_1) & x_2 &= \bar x_1,\\
y_1 &= (\alpha_1 + \beta_2) + i(\alpha_2 - \beta_1) & y_2 &= \bar y_1,\\
z_1 &= \alpha_3 + i\beta_3 & z_2 &= \bar z_1,
\end{aligned}
$$

where i is the imaginary unit. The equations of motion (2.1) written with respect to the imaginary time $\frac{d}{d(it)}$ become

$$
2w_1' = -w_1w_3 - z_1 \qquad 2w_2' = w_2w_3 + z_2 \qquad 2w_3' = y_2 - y_1,
$$

$$\begin{aligned} x_1' &= -z_1w_3 + z_1w_1 & x_2' &= x_2w_3 - z_2w_2, \\ y_1' &= -y_1w_3 + z_2w_1 & y_2' &= y_2w_3 - z_1w_2, \\ 2z_1' &= x_1w_2 - y_2w_1 & 2z_2' &= -x_2w_1 + y_1w_2, \end{aligned}$$

while geometric integrals (2.2) become

$$z_1^2 + x_1y_2 = r^2, \qquad z_2^2 + x_2y_1 = r^2, \qquad x_1x_2 + y_1y_2 + 2z_1z_2 = 2p^2. \tag{2.4}$$

Briefly reproducing a part of [13], we consider a linear combination of the first integrals G and H denoted by $M = \frac{1}{r^4}(2G - p^2H)$. On the level set $F = 0$, the relation $K = r^4M^2$ is satisfied. Assume $x_1x_2 \neq 0$. Introduce new variables u_1 and u_2 and the following function F_1:

$$\begin{aligned} u_1 &= \frac{x_2}{x_1}(w_1^2 + x_1), \qquad u_2 = \frac{x_1}{x_2}(w_2^2 + x_2), \\ F_1 &= \sqrt{x_1x_2\sqrt{x_3}} - \frac{1}{\sqrt{x_1x_2}}(x_2z_1w_1 + x_1z_2w_2). \end{aligned} \tag{2.5}$$

The first integrals H, K and M in the new variables take the following form:

$$\begin{aligned} H &= w_1w_2 + \frac{1}{2}w_3^2 - \frac{1}{2}(y_1 + y_2), \\ K &= u_1u_2, \\ M &= -\frac{1}{2r^4}F_1^2 + \frac{1}{2r^2}(u_1 + u_2). \end{aligned}$$

In [13], it was proved that the invariant set N^4, for $x_1x_2 \neq 0$ was specified by two following independent equations

$$F_1 = 0, \quad F_2 := u_1 - u_2 = 0.$$

The function

$$L = \frac{1}{\sqrt{x_1x_2}}\Big(\frac{w_1w_2}{x} + \frac{x_1x_2 + z_1z_2}{2r^2}\cdot(u_1 + u_2)\Big)$$

is a first integral of the reduced system on $N^4 \cap \{x_1x_2 \neq 0\}$ which is in involution with the first integral M, as it was proved in [13].

3. Discriminantly separable polynomial and separation variables

The differential equations for separating variables were introduced in [12, 13]. In this section we derive the differential equations for separating variables by using the discriminantly separable polynomials and the technique derived in [5]. The differential equations for u_1 and u_2 are

$$u_1' = -\Lambda u_1, \qquad u_2' = \Lambda u_2 \tag{3.1}$$

where Λ is function $\Lambda = w_3 - \frac{z_2w_2}{x_2} - \frac{z_1w_1}{x_1}$, and also

$$(x_1')^2 = \frac{x_1^2z_2^2w_2^2}{x_2^2}, \qquad (x_2')^2 = -\frac{x_2^2z_1^2w_1^2}{x_1^2}. \tag{3.2}$$

Following [12, 13] denote by $x^2 = x_1x_2$, $z^2 = z_1z_2$ and introduce the variables s_1 and s_2 by two one-parametric families of circles, see [14] for more details:

$$s_1 = \frac{x^2 + z^2 + r^2}{2x}, \qquad s_2 = \frac{x^2 + z^2 - r^2}{2x}. \tag{3.3}$$

By calculating $(z_1 \pm z_2)^2$ from (2.4) and by plugging s_1, s_2 instead of z_1, z_2 one gets:

$$(z_1 + z_2)^2 = \frac{1}{r^2}\big((x^2 + z^2 + r^2)^2 - 2x^2(p^2 + r^2)\big) = \frac{4x^2}{r^2}(s_1^2 - a^2),$$
$$(z_1 - z_2)^2 = \frac{1}{r^2}\big((x^2 + z^2 - r^2)^2 - 2x^2(p^2 - r^2)\big) = \frac{4x^2}{r^2}(s_2^2 - b^2).$$

The next step is to obtain x_1, x_2 as functions of s_1, s_2. From $M = -\frac{1}{2r^4}F_1^2 + \frac{1}{2r^2}(u_1 + u_2) = m$ and $F_1 = 0$ one gets $u_1 + u_2 = 2r^2m$ and further from $F_2 = 0$ one finally gets $u_1 = u_2 = r^2m$. Then from (3.1) we get $\Lambda = 0$ and

$$w_3 = \frac{z_2w_2}{x_2} + \frac{z_1w_1}{x_1}.$$

We rewrite the first integral L in the following form: $L = \frac{w_1w_2}{x} + \frac{x^2+z^2}{x}M$. We obtain a correlation between x_1, x_2 and s_1, s_2 that is based on the discriminantly separable polynomials. Thus, we get the differential equations for s_1 and s_2.

THEOREM 3.1. *The equations of motion on an arbitrary invariant submanifold $J = \{M = m, L = l\}$ are separated in the variables (3.3). The variables x_1, x_2 represent the roots of $\mathcal{F}(s_1, s_2, \tau) = 0$ as a quadratic equation in τ where $\mathcal{F}$ is the following discriminantly separable polynomial*

$$\mathcal{F}(s_1, s_2, \tau) := (s_1 - s_2)^2\tau^2 + \Psi(s_1, s_2)\tau + r^4,$$

with the polynomial $\Psi(s_1, s_2) = 4ms_1s_2 - 2l(s_1 + s_2) + \frac{1}{m}(l^2 - 1)$.

PROOF. Starting from $lx = w_1w_2 + (x^2 + z^2)m$ and (2.5) we get

$$\frac{x_1x_2(u_1 - x_2)(u_2 - x_1)}{x_1x_2} = l^2x^2 + (x^2 + z^2)^2m^2 - 2lx(x^2 + z^2)m.$$

Further we get

$$m^2(r^4 - (x^2 + z^2)^2) - (l^2 - 1)x^2 + 2lx(x^2 + z^2)m = (x_1 + x_2)mr^2.$$

From (3.3) we get the following relations

$$s_1 \cdot s_2 = \frac{(x^2 + z^2)^2 - r^4}{4x^2}, \quad s_1 - s_2 = \frac{r^2}{x}, \quad s_1 + s_2 = \frac{x^2 + z^2}{x}. \tag{3.4}$$

By replacing (3.4) into the previous relation we get

$$-4x^2m^2s_1s_2 - (l^2 - 1)x^2 + 2lx^2(s_1 + s_2)m = (x_1 + x_2)mr^2.$$

Denote by $\Psi(s_1, s_2) = 4ms_1s_2 - 2l(s_1 + s_2) + \frac{1}{m}(l^2 - 1)$ and recall that $x_1x_2 = x^2$ then we get:

$$x_1 + x_2 = -\frac{\Psi(s_1, s_2)r^2}{(s_1 - s_2)^2}, \qquad x_1x_2 = x^2 = \frac{r^4}{(s_1 - s_2)^2}.$$

Finally, we get x_1, x_2 in terms of s_1, s_2 as solutions of the quadratic equation in τ:

$$\mathcal{F}(s_1, s_2, \tau) := (s_1 - s_2)^2\tau^2 + \Psi(s_1, s_2)\tau + r^4 = 0.$$

The polynomial $\mathcal{F}(s_1, s_2, \tau)$ is discriminantly separable in three variables of degree two in each of them and its discriminants are:

$$\mathcal{D}_{s_1}(\mathcal{F})(s_2, \tau) = P(s_2)J(\tau), \qquad D_{s_2}(\mathcal{F})(s_1, \tau) = P(s_1)J(\tau),$$
$$D_\tau(\mathcal{F})(s_1, s_2) = P(s_1)P(s_2),$$

where $P(s) = \frac{r^2}{m}(l + 1 - 2ms)(l - 1 - 2ms)$ and $J(\tau) = 4\tau^2(r^2m - \tau)$.

Now we use the Lemma 1.1:

$$\frac{ds_1}{\sqrt{P(s_1)}} + \frac{ds_2}{\sqrt{P(s_2)}} = \frac{dx_1}{\sqrt{J(x_1)}}, \qquad \frac{ds_1}{\sqrt{P(s_1)}} - \frac{ds_2}{\sqrt{P(s_2)}} = \frac{dx_2}{\sqrt{J(x_2)}}. \tag{3.5}$$

Further, from (3.2) we obtain:

$$(x_1')^2 = \frac{x_1^2 w_2^2}{x_2^2} z_2^2 = \frac{x_1^2 R_1^2}{x^2} z_2^2 = \frac{x_1^2 R_1^2}{r^2}\left(\sqrt{s_1^2 - a^2} - \sqrt{s_2^2 - b^2}\right)^2$$

and similarly

$$(x_2')^2 = \frac{x_2^2 R_2^2}{r^2}\left(\sqrt{s_1^2 - a^2} + \sqrt{s_2^2 - b^2}\right)^2,$$

where $R_i^2 = r^2m - x_i$ and $J(x_i) = 4x_i^2 R_i^2, \quad i = 1, 2$.

Now, from the previous relations we get

$$\frac{x_1'}{x_1 R_1} = \frac{1}{r}\left(\sqrt{s_1^2 - a^2} - \sqrt{s_2^2 - b^2}\right), \qquad \frac{x_2'}{x_2 R_2} = \frac{1}{r}\left(\sqrt{s_1^2 - a^2} + \sqrt{s_2^2 - b^2}\right). \tag{3.6}$$

From (3.6) and (3.5) we obtain

$$\begin{aligned}
\frac{2s_1'}{\sqrt{P(s_1)}} &= \frac{x_1'}{\sqrt{J(x_1)}} + \frac{x_2'}{\sqrt{J(x_2)}} = \frac{dx_1'}{2x_1R_1} + \frac{dx_2'}{2x_2R_2} \\
&= \frac{1}{2}\left(\frac{2}{r}\sqrt{s_1^2 - a^2}\right) = \frac{\sqrt{s_1^2 - a^2}}{r}, \\
\frac{2s_2'}{\sqrt{P(s_1)}} &= \frac{x_1'}{\sqrt{J(x_1)}} - \frac{x_2'}{\sqrt{J(x_2)}} = \frac{x_1'}{2x_1R_1} - \frac{x_2'}{2x_2R_2} \\
&= \frac{1}{2}\left(\frac{2}{r}\sqrt{s_2^2 - b^2}\right) = \frac{\sqrt{s_2^2 - b^2}}{r}.
\end{aligned}$$

Finally, in terms of the real time derivative we obtain the differential equations for s_1 and s_2 in the following form:

$$\begin{aligned}
\frac{ds_1}{dt} &= \frac{1}{2}\sqrt{(s_1^2 - a^2)}\sqrt{-4ms_1^2 + 4s_1 l - \frac{l^2 - 1}{m}}, \\
\frac{ds_2}{dt} &= \frac{1}{2}\sqrt{(b^2 - s_2^2)}\sqrt{4ms_2^2 - 4s_2 l + \frac{l^2 - 1}{m}}.
\end{aligned}$$

□

The phase variables for the system (2.1) in terms of the separation variables, the phase topology and bifurcation diagram for the generalized Kowalevski case are obtained in [13].

Acknowledgements. The authors are grateful to the referees for comments and suggestions. The research was partially supported by the Serbian Ministry of Science and Technological Development, Project 174020 *Geometry and Topology of Manifolds, Classical Mechanics and Integrable Dynamical Systems*.

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ДИСКРИМИНАНТНО СЕПАРАБИЛНИ ПОЛИНОМИ И УОПШТЕНА ЧИГРА КОВАЉЕВСКЕ

Резиме. Разматра се добро познати случај интеграбилног механичког проблема - уопштени проблем Коваљевске кретања крутог тела око непокретне тачке у пољу деловања две константне силе. Представља се нови начин раздвајања променљивих. Он се заснива на теорији дискриминантно сепарабилних полинома, коју су раније развили аутори. Раздвајање променљивих води до еплицитне интеграције једначина кретања полазног проблема.

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(Received 26.09.2017.)
(Revised 15.11.2017.)
(Available online 07.12.2017.)

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